International Journal of Mathematical Archive-6(8), 2015, 207-211
Available online through www.ijma.info ISSN 2229 – 5046

INTERVAL VALUED FUZZY SUBSEMIRINGS OF A SEMIRING UNDER HOMOMORPHISM

[1]K. MURUGALINGAM*, [2]K. ARJUNAN

[1]Department of Mathematics,
Dr. Zakir Hussain College, Ilayangudi-630702, Tamilnadu, India.

[2]Department of Mathematics,
H. H. The Rajahs College, Pudukkottai – 622001, Tamilnadu, India.

(Received On: 18-07-15; Revised & Accepted On: 13-08-15)

ABSTRACT

In this paper, we study some of the properties of interval valued fuzzy subsemiring of a semiring under homomorphism and anti-homomorphism and prove some results on these.

2000 AMS Subject Classification: *03F55, 08A72, 20N25.*

Key Words: *Interval valued fuzzy subset, interval valued fuzzy subsemiring, pseudo interval valued fuzzy coset.*

INTRODUCTION

Interval valued fuzzy sets were introduced independently by Zadeh [11], Grattan-Guiness [4], Jahn [6], in the seventies, in the same year. An interval valued fuzzy set (IVF) is defined by an interval valued membership function. Jun.Y.B and Kin.K.H [7] defined an interval valued fuzzy R-subgroups of nearsemirings. Solairaju.A and Nagarajan.R [10] defined the charactarization of interval valued anti fuzzy Left h-ideals over hemisemirings. Azriel Rosenfeld [2] defined a fuzzy group. K.Murugalingam & K.Arjunan [8] defined an interval valued fuzzy subsemiring of a semiring. We introduce the concept of interval valued fuzzy subsemiring of a semiring under homomorphism and anti-homomorphism and established some results.

1. PRELIRMINARIES

1.1 Definition [8]: Let X be any nonempty set. A mapping $[M] : X \to D[0, 1]$ is called an interval valued fuzzy subset (briefly, IVFS) of X, where D[0,1] denotes the family of all closed subintervals of [0,1] and $[M](x) = [M^-(x), M^+(x)]$, for all x in X, where M^- and M^+ are fuzzy subsets of X such that $M^-(x) \leq M^+(x)$, for all x in X. Thus $M^-(x)$ is an interval (a closed subset of [0, 1]) and not a number from the interval [0, 1] as in the case of fuzzy subset. Note that [0] = [0, 0] and [1] = [1, 1].

1.2 Remark [8]: Let D^X be the set of all interval valued fuzzy subset of X, where D means D[0, 1].

1.3 Definition: Let [A] be an interval valued fuzzy subset of X. Then the following operations are defined as

(i) ?([*A*]) = {⟨ x, rmin{[½, ½], [A](x)} / for all $x \in X$}.
(ii) !([*A*]) = {⟨ x, rmax{[½,½], [A](x) } / for all $x \in X$}.
(iii) Q_α([*A*]) = {⟨x, rmin{α, [A](x)} / for all $x \in X$ and α in D[0, 1]}.
(iv) P_α([*A*]) = {⟨x, rmax{α, [A](x)} / for all $x \in X$ and α in D[0, 1]}.
(v) G_α([*A*]) = {⟨x, α [A](x)}⟩ / for all $x \in X$ and α in [0, 1]}.

1.4 Definition [8]: Let (R, +, ·) be a semiring. An interval valued fuzzy subset [M] of R is said to be an **interval valued fuzzy subsemiring** of R if the following conditions are satisfied:

(i) $[M](x+y) \geq$ rmin{[M](x), [M](y)}
(ii) $[M](xy) \geq$ rmin {[M](x), [M](y)} for all x and y in R.

Corresponding Author: [1]K. Murugalingam, [1]Department of Mathematics,*
Dr. Zakir Hussain College, Ilayangudi-630702, Tamilnadu, India.

1.5 Definition: Let (R, +, ·) and (R^l, +, ·) be any two semirings. Let f : R→ R^l be any function and [M] be an interval valued fuzzy subsemiring in R, [V] be an interval valued fuzzy subsemiring in f(R) = R^l, defined by [V](y) = $\sup_{x\in f^{-1}(y)}$ [M](x), for all x in R and y in R^l. Then [M] is called a pre-image of [V] under f and is denoted by f^{-1}([V]).

1.6 Definition: Let [M] be an interval valued fuzzy subsemiring of a semiring (R, +, ·) and a in R. Then the **pseudo interval valued fuzzy coset** $(a[M])^p$ is defined by $((a[M])^p)(x)$ = p(a)[M](x) for every x in R and for some p in P.

2. SOME PROPERTIES

2.1 Theorem: Let (R, +, ·) and (R^l, +, ·) be any two semirings. The homomorphic image of an interval valued fuzzy subsemiring of R is an interval valued fuzzy subsemiring of R^l.

Proof: Let f: R→R^l be a homomorphism. Let [M] be an interval valued fuzzy subsemiring of R. Let [V] be the homomorphic image of [M] under f. We have to prove that [V] is an interval valued fuzzy subsemiring of f(R) = R^l. Let f(x) and f(y) in R^l. Then [V](f(x) +f(y)) = [V](f(x+y)) ≥ [M](x+y) ≥ rmin{[M](x), [M](y)} which implies that [V](f(x)+f(y)) ≥ rmin{[V](f(x)), [V](f(y))}. And [V](f(x)f(y)) = [V](f(xy)) ≥ [M](xy) ≥ rmin {[M](x), [M](y)} which implies that [V](f(x)f(y)) ≥ rmin{[V](f(x)), [V](f(y))}. Hence [V] is an interval valued fuzzy subsemiring of a semiring R^l.

2.2 Theorem: Let (R, +, ·) and (R^l, +, ·) be any two semirings. The homomorphic pre-image of an interval valued fuzzy subsemiring of R^l is an interval valued fuzzy subsemiring of R.

Proof: Let f: R→R^l be a homomorphism. Let [V] be an interval valued fuzzy subsemiring of f(R) = R^l. Let [M] be the pre-image of [V] under f. We have to prove that [M] is an interval valued fuzzy subsemiring of R. Let x and y in R. Then [M](x+y) = [V](f(x+y)) = [V](f(x)+f(y)) ≥ rmin {[V](f(x)), [V](f(y))}= rmin{[M](x), [M](y)} which implies that [M](x+y) ≥ rmin{[M](x), [M](y)} for x and y in R. And [M](xy) = [V](f(xy)) = [V](f(x)f(y)) ≥ rmin{[V](f(x)), [V](f(y))} = rmin{[M](x), [M](y)} which implies that [M](xy) ≥ rmin{[M](x), [M](y)} for x and y in R. Hence [M] is an interval valued fuzzy subsemiring of the semiring R.

2.3 Theorem: Let (R, +, ·) and (R^l, +, ·) be any two semirings. The anti-homomorphic image of an interval valued fuzzy subsemiring of R is an interval valued fuzzy subsemiring of R^l.

Proof: Let f: R→ R^l be a anti-homomorphism. Let [M] be an interval valued fuzzy subsemiring of R. Let [V] be the homomorphic image of [M] under f. We have to prove that [V] is an interval valued fuzzy subsemiring of f(R) = R^l. Let f(x) and f(y) in R^l. Then [V](f(x)+f(y)) = [V](f(y+x)) ≥ [M](y+x) ≥ rmin{[M](x), [M](y)} which implies that [V](f(x)+f(y)) ≥ rmin{[V](f(x)), [V](f(y))}. And [V](f(x)f(y)) = V(f(yx)) ≥ [M](yx) ≥ rmin{[M](x), [M](y)} which implies that [V](f(x)f(y)) ≥ rmin {[V](f(x)), [V](f(y))}. Hence [V] is an interval valued fuzzy subsemiring of R'.

2.4 Theorem: Let (R, +, ·) and (R^l, +, ·) be any two semirings. The anti-homomorphic pre-image of an interval valued fuzzy subsemiring of R^l is an interval valued fuzzy subsemiring of R.

Proof: Let f: R → R^l be a anti-homomorphism. Let [V] be an interval valued fuzzy subsemiring of f(R) = R^l. Let [M] be the pre-image of [V] under f. We have to prove that [M] is an interval valued fuzzy subsemiring of R. Let x and y in R. Then [M](x+y) = [V](f(x+y)) = [V](f(y)+f(x)) ≥ rmin{[V](f(x)), [V](f(y))} = rmin{[M](x), [M](y)} which implies that [M](x+y) ≥ rmin{[M](x), [M](y)} for all x and y in R. And [M](xy) = [V](f(xy)) = [V](f(y)f(x)) ≥ rmin{[V](f(x)), [V](f(y))} = rmin{[M](x), [M](y)} which implies that [M](xy) ≥ rmin{[M](x), [M](y)} for all x and y in R. Hence [M] is an interval valued fuzzy subsemiring of the semiring R.

In the following Theorem ◦ is the composition operation of functions:

2.5 Theorem: Let [M] be an interval valued fuzzy subsemiring of a semiring H and f is an isomorphism from a semiring R onto H. Then [M]◦f is an interval valued fuzzy subsemiring of R.

Proof: Let x and y in R and [M] be an interval valued fuzzy subsemiring of the semiring H. Then ([M]◦f)(x+y) = [M](f(x+y)) = [M](f(x)+f(y)) ≥ rmin{[M](f(x)), [M](f(y)) ≥ rmin{([M]◦f)(x), ([M]◦f)(y)}which implies that ([M]◦f)(x+y) ≥ rmin{([M]◦f)(x), ([M]◦f)(y). And ([M]◦f)(xy) = [M](f(xy)) = [M](f(x)f(y)) ≥ rmin{[M](f(x)), [M](f(y))} ≥ rmin{([M]◦f)(x), ([M]◦f)(y)} which implies that ([M]◦f)(xy) ≥ rmin{([M]◦f)(x), ([M]◦f)(y)}. Therefore ([M]◦f) is an interval valued fuzzy subsemiring of a semiring R.

2.6 Theorem: Let [M] be an interval valued fuzzy subsemiring of a semiring H and f is an anti-isomorphism from a semiring R onto H. Then [M]◦f is an interval valued fuzzy subsemiring of R.

Proof: Let x and y in R and [M] be an interval valued fuzzy subsemiring of the semiring H. Then ([M]◦f)(x+y) = [M](f(x+y)) = [M](f(y)+f(x)) ≥ rmin{[M](f(x)), [M](f(y))}≥ rmin{([M]◦f)(x), ([M]◦f)(y)} which implies that ([M]◦f)(x+y)≥rmin{([M]◦f)(x), ([M]◦f)(y)}. And ([M]◦f)(xy)= [M](f(xy)) = [M](f(y)f(x)) ≥ rmin{[M](f(x)), [M](f(y))} ≥ rmin{([M]◦f)(x), ([M]◦f)(y)} which implies that ([M]◦f)(xy) ≥ rmin{([M]◦f)(x), ([M]◦f)(y)}. Therefore ([M]◦f) is an interval valued fuzzy subsemiring of R.

2.7 Theorem: Let [M] be an interval valued fuzzy subsemiring of a semiring R, then the pseudo interval valued fuzzy coset $(a[M])^p$ is an interval valued fuzzy subsemiring of the semiring R, for every a in R.

Proof: Let [M] be an interval valued fuzzy subsemiring of the semiring R. For every x and y in R, we have $((a[M])^p)(x+y)$ = p(a)[M](x+y) ≥ p(a) rmin{[M](x), [M](y)}= rmin {p(a)[M](x), p(a)[M](y)} = rmin{$((a[M])^p)(x)$, $((a[M])^p)(y)$}. Therefore $((a[M])^p)(x+y)$ ≥ rmin{$((a[M])^p)(x)$, $((a[M])^p)(y)$} for x and y in R. And $((a[M])^p)(xy)$ = p(a)[M](xy) ≥ p(a) rmin{[M](x), [M](y)} = rmin{p(a)[M](x), p(a)[M](y)}=rmin{$((a[M])^p)(x)$, $((a[M])^p)(y)$}. Therefore $((a[M])^p)(xy)$ ≥ rmin{$((a[M])^p)(x)$, $((a[M])^p)(y)$} for x and y in R. Hence $(a[M])^p$ is an interval valued fuzzy subsemiring of R.

2.8 Theorem [8]: If [M] and [N] are two interval valued fuzzy subsemirings of a semiring R, then their intersection [M]∩[N] is an interval valued fuzzy subsemiring of R.

2.9 Theorem: If [M] is an interval valued fuzzy subsemiring of a semiring R, then ?([M]) is an interval valued fuzzy subsemiring of R.

Proof: For every x and y in R, we have ?([M])(x+y) = rmin{[½,½], [M](x+y)} ≥ rmin{[½,½], rmin {[M](x), [M](y)}} = rmin{rmin {[½,½], [M](x)}, rmin {[½,½], [M](y)}} = rmin {?([M])(x), ?([M])(y)}. Therefore ?([M])(x+y) ≥ rmin {?A^+(x), ?A^+(y)} for all x and y in R. Also ?([M])(xy) = rmin {[½,½], [M](xy)} ≥ rmin {[½,½], rmin {[M](x), [M](y)}} = rmin {rmin{[½,½], [M](x)}, rmin {[½,½], [M](y)}} = rmin {?([M])(x), ?([M])(y)}. Therefore ?([M])(xy) ≥ rmin {?A^+(x), ?A^+(y) } for all x and y in R. Hence ?([M]) is an interval valued fuzzy subsemiring of R.

2.10 Theorem: If [M] is an interval valued fuzzy subsemiring of a semiring R, then !([M]) is an interval valued fuzzy subsemiring of R.

Proof: For every x and y in R, we have !([M])(x+y) = rmax{[½,½], [M](x+y)} ≥ rmax {[½,½], rmin{[M](x), [M](y)}} = rmin {rmax{[½,½], [M](x)}, rmax {[½,½], [M](y)}} = rmin {!([M])(x), !([M])(y)}. Therefore !([M])(x+y) ≥ rmin{!([M])(x), !([M])(y)} for all x and y in R. Also !([M])(xy) = rmax{[½,½], [M](xy)}≥ rmax {[½,½], rmin {[M](x), [M](y)}}= rmin{rmax {[½,½], [M](x)}, rmax {[½,½], [M](y)}}= rmin{!([M])(x), !([M])(y)}. Therefore !([M])(xy)≥ rmin{!([M])(x), !([M])(y)} for all x and y in R. Hence !([M]) is an interval valued fuzzy subsemiring of R.

2.11 Theorem: If [M] is an interval valued fuzzy subsemiring of a semiring R, then Q_α([M]) is an interval valued fuzzy subsemiring of R.

Proof: For every x and y in R, α in D[0, 1], we have Q_α([M])(x+y) = rmin {α, [M](x+y)}≥ rmin{α, rmin{[M](x), [M](y)}} = rmin {rmin {α, [M](x)}, rmin {α, [M](y)}} = rmin{Q_α([M])(x), Q_α([M])(y)}. Therefore Q_α([M])(x+y) ≥ rmin {Q_α([M])(x), Q_α([M])(y)} for all x and y in R. Also Q_α([M])(xy) = rmin {α, [M](xy)} ≥ rmin {α, rmin{[M](x), [M](y)}} = rmin {rmin {α, [M](x)}, rmin{α, [M](y)}} = rmin {Q_α([M])(x), Q_α([M])(y)}. Therefore Q_α([M])(xy) ≥ rmin {Q_α([M])(x), Q_α([M])(y)} for all x and y in R. Hence Q_α([M]) is an interval valued fuzzy subsemiring of R.

2.12 Theorem: If [M] is an interval valued fuzzy subsemiring of a semiring R, then P_α([M]) is an interval valued fuzzy subsemiring of R.

Proof: For every x and y in R, α in D[0, 1], we have P_α([M])(x+y) = rmax{α, [M](x+y)} ≥ rmax {α, rmin{[M](x), [M](y)}} = rmin{rmax {α, [M](x)}, rmax{α, [M](y)}} = rmin{P_α([M])(x), P_α([M])(y)}. Therefore P_α([M])(x+y) ≥ rmin {P_α([M])(x), P_α([M])(y)} for all x and y in R. Also P_α([M])(xy) = rmax{α, [M](xy)} ≥ rmax {α, rmin{[M](x), [M](y)}} = rmin {rmax {α, [M](x)}, rmax{α, [M](y)}} = rmin {P_α([M])(x), P_α([M])(y)}. Therefore P_α([M])(xy) ≥ rmin{ P_α([M])(x), P_α([M])(y)} for all x and y in R. Hence P_α ([M]) is an interval valued fuzzy subsemiring of R.

2.13 Theorem: If [M] is an interval valued fuzzy subsemiring of a semiring R, then G_α([M]) is an interval valued fuzzy subsemiring of R.

Proof: For every x and y in R, α in [0, 1], we have G_α([M])(x+y) = α [M](x+y) ≥ α (rmin{[M](x), [M](y)}) = rmin{α [M](x), α [M](y)}= rmin {G_α([M])(x), G_α([M])(y)}. Therefore G_α([M])(x+y) ≥ rmin {G_α([M])(x), G_α([M])(y)} for all x and y in R. Also G_α([M])(xy) = α [M](xy) ≥ α(rmin{[M](x), [M](y)})=rmin {α [M](x), α [M](y)} = rmin {G_α([M])(x), G_α([M])(y)}. Therefore G_α([M])(xy) ≥ rmin {G_α([M])(x), G_α([M])(y)} for all x and y in R. Hence G_α([M]) is an interval valued fuzzy subsemiring of R.

2.14 Theorem: If [M] and [N] are interval valued fuzzy subsemirings of a semiring R, then
?([M]∩[N]) = ?([M]) ∩ ?([N]) is also an interval valued fuzzy subsemiring of R.

Proof: By Theorem 2.8, 2.9, the statement of the Theorem is true.

2.15 Theorem: If [M] and [N] are interval valued fuzzy subsemirings of a semiring R, then
!([M]∩[N]) = !([M]) ∩!([N]) is also an interval valued fuzzy subsemiring of R.

Proof: By Theorem 2.8, 2.10, the statement of the Theorem is true.

2.16 Theorem: If [M] is an interval valued fuzzy subsemiring of a semiring R, then !(?([M])) = ?(!([M])) is also an interval valued fuzzy subsemiring of R.

Proof: By Theorem 2.9, 2.10, the statement of the Theorem is true.

2.17 Theorem: If [M] and [N] are interval valued fuzzy subsemirings of a semiring R, then
Q_α([M]∩[N] = Q_α([M]) ∩ Q_α([N]) is also an interval valued fuzzy subsemiring of R.

Proof: By Theorem 2.8, 2.11, the statement of the Theorem is true.

2.18 Theorem: If [M] and [N] are interval valued fuzzy subsemirings of a semiring R, then
P_α([M]∩[N]) = P_α([M]) ∩ P_α([N]) is also an interval valued fuzzy subsemiring of R.

Proof: By Theorem 2.8, 2.12, the statement of the Theorem is true.

2.19 Theorem: If [M] is an interval valued fuzzy subsemiring of a semiring R, then P_α(Q_α([M])) = Q_α(P_α([M])) is also an interval valued fuzzy subsemiring of R.

Proof: By Theorem 2.11, 2.12, the statement of the Theorem is true.

2.20 Theorem: If [M] and [N] are interval valued fuzzy subsemirings of a semiring R, then
G_α([M]∩[N]) = G_α([M]) ∩ G_α([N]) is also an interval valued fuzzy subsemiring of R.

Proof: By Theorem 2.8, 2.13, the statement of the Theorem is true.

REFERENCE

1. Akram.M and Dar.K.H, On fuzzy d-algebras, Punjab university journal of mathematics, 37, 61-76 (2005).
2. Azriel Rosenfeld, Fuzzy Groups, Journal of mathematical analysis and applications, 35, 512-517 (1971).
3. Biswas.R, Fuzzy subgroups and Anti-fuzzy subgroups, Fuzzy sets and systems, 35,121-124 (1990).
4. Grattan-Guiness, Fuzzy membership mapped onto interval and many valued quantities, Z.Math.Logik. Grundladen Math. 22, 149-160 (1975).
5. Indira.R, Arjunan.K and Palaniappan.N, Notes on IV-fuzzy rw-Closed, IV-fuzzy rw-Open sets in IV-fuzzy topological space, International Journal of Fuzzy Mathematics and Systems, Vol. 3, Num.1, pp 23-38 (2013).
6. Jahn.K.U., interval wertige mengen, Math Nach.68, 115-132 (1975).
7. Jun.Y.B and Kin.K.H, interval valued fuzzy R-subgroups of nearrings, Indian Journal of Pure and Applied Mathematics, 33(1), 71-80 (2002).
8. K.Murugalingam & K.Arjunan, A study on interval valued fuzzy subsemiring of a semiring, International Journal of Applied Mathematics Modeling, Vol.1, No.5, 1-6, (2013)
9. Palaniappan. N & K. Arjunan, Operation on fuzzy and anti fuzzy ideals, Antartica J.Math., 4(1): 59-64 (2007).
10. Solairaju.A and Nagarajan.R, Charactarization of interval valued Anti fuzzy Left h-ideals over Semirings, Advances in fuzzy Mathematics, Vol.4, No. 2, 129-136 (2009).

11. Zadeh.L.A, The concept of a linguistic variable and its application to approximation reasoning-1, Inform. Sci. 8, 199-249 (1975).

Source of support: Nil, Conflict of interest: None Declared