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THE NUMBER OF MINIMUM CO – ISOLATED LOCATING DOMINATING SETS OF PATHS

S. MUTHAMMAI
Government Arts College for Women (Autonomous), Pudukkottai – 622 001, India.

N. MEENAL*
J. J. College of Arts and Science, Pudukkottai – 622 422, India.

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ABSTRACT

Let G (V, E) be a simple, finite, undirected connected graph. A non – empty set $S \subseteq V$ of a graph G is a dominating set, if every vertex in V – S is adjacent to atleast one vertex in S. A dominating set $S \subseteq V$ is called a locating dominating set, if for any two vertices v, w $\in$ V – S, $N(v) \cap S \neq N(w) \cap S$. A locating dominating set $S \subseteq V$ is called a co – isolated locating dominating set, if there exists atleast one isolated vertex in <V – S >. The co – isolated locating domination number γ_{cild} is the minimum cardinality of a co – isolated locating dominating set. The number of minimum co – isolated locating dominating sets in a graph G is denoted by $\gamma_{Dcild}(G)$. In this paper, the number γ_{Dcild} is obtained for a Path P_n, where $n \geq 3$.

Keywords: *Dominating set, locating dominating set, co – isolated locating dominating set.*

1. INTRODUCTION

Let G = (V, E) be a simple graph of order n. For v ∈ V(G), the neighborhood $N_G(v)$ (or simply N(v)) of v is the set of all vertices adjacent to v in G. The concept of domination in graphs was introduced by Ore [7]. A nonempty set S ⊆ V(G) of a graph G is a dominating set, if every vertex in V(G) – S is adjacent to some vertex in S. A special case of dominating set S is called a locating dominating set. It was defined by D. F. Rall and P. J. Slater in [8]. A dominating set S in a graph G is called a locating dominating set in G, if for any two vertices v, w ∈ V(G) – S, $N_G(v) \cap S$, $N_G(w) \cap S$ are distinct. The location dominating number of G is defined as the minimum number of vertices in a locating dominating set in G. A locating dominating set S ⊆ V(G) is called a co - isolated locating dominating set , if <V – S> contains atleast one isolated vertex. The minimum cardinality of a co – isolated locating dominating set is called the co – isolated locating domination number and is denoted by $\gamma_{cild}(G)$. The number of minimum co – isolated locating dominating sets in a graph G is denoted by $\gamma_{Dcild}(G)$. In this paper, the minimum number γ_{Dcild} of co-isolated locating dominating sets of Path P_n on n vertices, n ≥ 3, is obtained

2. PRIOR RESULTS

The following results are obtained in [3] & [4]

Theorem: 2.1 [3] For every non – trivial simple connected graph G, $1 \leq \gamma_{cild}(G) \leq n - 1$.

Theorem: 2.2 [3] $\gamma_{cild}(G) = 1$ if and only if $G \cong K_2$.

Theorem: 2.3 [3] $\gamma_{cild}(K_n) = n - 1$, where K_n is a complete graph on n vertices.

Theorem: 2.4 [3] $\gamma_{cild}(K_n - e) = n - 1$, where $e \in E(K_n)$

Observation: 2.1 [4] If S is an co – isolated locating dominating set of G(V, E) with | S | = k, then V(G) – S contains atmost $nC_1 + nC_2 + \ldots + nC_k$ vertices.

*Corresponding Author: N. Meenal**
J. J. College of Arts and Science, Pudukkottai – 622 422, India.

Theorem: 2.5 [4] If P_n is a path on n vertices, $n \geq 3$, then

$$\gamma_{cild}(P_n) = \begin{cases} \left\lfloor \frac{2n}{5} \right\rfloor & ; n \equiv 0 \ (mod\ 5) \\ 2\left\lfloor \frac{n}{5} \right\rfloor + 1; & n \equiv 1 \ or\ 2 \ (mod\ 5) \\ 2\left\lfloor \frac{n}{5} \right\rfloor + 2; & n \equiv 3 \ or\ 4 \ (mod\ 5) \end{cases}$$

3. MAIN RESULTS

Using the value of $\gamma_{cild}(P_n)$ given in Theorem 2.5., the minimum number of co-isolated locating dominating sets $\gamma_{Dcild}(P_n)$ of P_n, for all $n \geq 3$, are found in this section.

Observation: 3.1 Let $V(P_n) = \{v_1, v_2, \ldots, v_n\}$ with $deg(v_1) = deg(v_n) = 1$ and $deg(v_2) = deg(v_3) = \ldots = deg(v_{n-1}) = 2$ and let D be a minimum co – isolated locating dominating set of P_n. Then one of the following holds.

(i) $v_1, v_n \in D$
(ii) $v_1, v_{n-1} \in D$
(iii) $v_2, v_n \in D$
(iv) $v_2, v_{n-1} \in D$

It is sufficient to consider (i), (iii) and (iv), since the number of minimum co – isolated locating dominating sets of P_n containing v_2 and v_n is same as that of minimum co – isolated locating dominating sets containing v_1 and v_{n-1}.

Theorem: 3.2 For any integer $n \geq 1$, $\gamma_{Dcild}(P_{5n}) = 1$.

Proof: Let the labellings of vertices of P_{5n} be $v_1, v_2, v_3, \ldots, v_{5n-1}, v_{5n}$. Let D be a minimum co – isolated locating dominating set of P_{5n}. The theorem is proved by the method of induction on n. For n = 1, the following cases arise.

(i) If $v_1, v_5 \in D$, then $D = \{v_1, v_3, v_5\}$ and $|D| = 3$. But $\gamma_{cild}(P_5) = 2$. Therefore, D cannot be a minimum co-isolated locating dominating set of P_5.
(ii) If $v_2, v_5 \in D$, then $D = \{v_2, v_3, v_5\}$ or $\{v_2, v_4, v_5\}$, which is also not possible.
(iii) If $v_2, v_4 \in D$, then $D = \{v_2, v_4\}$ is the only minimum co – isolated locating dominating set of P_5 and $|D| = 2$ and hence $\gamma_{Dcild}(P_5) = 1$.

Similarly for n = 2, $D = \{v_2, v_4, v_7, v_9\}$ is the only minimum co – isolated locating dominating set of P_{10} and $|D| = 4$ and hence $\gamma_{Dcild}(P_{10}) = 1$. Assume that the theorem holds when n = k-1. That is, the result holds for all paths having $5(k-1)$ vertices. Therefore, $D = \{v_2, v_4, v_7, v_9, \ldots, v_{5k-8}, v_{5k-6}\}$ is the only minimum co – isolated locating dominating set of $P_{5(k-1)}$ with $|D| = 2(k-1)$ and $\gamma_{Dcild}(P_{5(k-1)}) = 1$. Let n = k. Consider the path P_{5k} on 5k vertices. Let $D' = D \cup \{v_{5k-3}, v_{5k-1}\}$ is a co – isolated locating dominating set of P_{5n}. Also, $|D'| = |D| + 2 = 2k$. Therefore, D' is the only minimum co – isolated locating dominating set of P_{5k}. It can be proved that, if $v_1, v_{5k} \in D'$ or $v_2, v_{5k} \in D'$, then D' will not be a minimum co – isolated locating dominating set of P_{5k}. Therefore, D' is the unique γ_{cild} – set of P_{5k}. Hence, $\gamma_{Dcild}(P_{5k}) = 1$.

By induction hypothesis, $\gamma_{Dcild}(P_{5n}) = 1$, for $n \geq 1$.

Theorem: 3.3 There are exactly n + 1 minimum co – isolated locating dominating sets of P_{5n+1} containing v_1 and v_{5n+1} with the labellings $v_1, v_2, v_3, \ldots, v_{5n-1}, v_{5n}, v_{5n+1}$ where $n \geq 1$.

Proof: Let the labellings of vertices of P_{5n+1} be $v_1, v_2, v_3, \ldots, v_{5n-1}, v_{5n}, v_{5n+1}$. The Theorem is proved by the method of induction on n. For n = 1, $D_1 = \{v_1, v_3, v_6\}$ and $D_2 = \{v_1, v_4, v_6\}$ are the only minimum co – isolated locating dominating sets of P_6 containing v_1 and v_6 since, $|D_1| = |D_2| = 3 = 2\left\lfloor \frac{5n+1}{5} \right\rfloor + 1$, (Theorem 2.5.).

For n = 2, $D_1 = \{v_1, v_3, v_6, v_8, v_{11}\}$, $D_2 = = \{v_1, v_4, v_6, v_8, v_{11}\}$ and $D_3 = \{v_1, v_4, v_6, v_9, v_{11}\}$ are the only minimum co – isolated locating dominating sets of P_{11} containing v_1 and v_{11}, since $|D_1| = |D_2| = |D_3| = 5 = 2\left\lfloor \frac{5n+1}{5} \right\rfloor + 1$. Therefore, the result is true for n = 1 and n = 2. Assume that the theorem holds when n = k-1. That is, the result holds for all paths having $5(k-1) + 1$ vertices. Let $D_1, D_2, D_3, \ldots, D_k$ be the only k minimum co – isolated locating dominating sets of $P_{5(k-1)+1}$ containing v_1 and v_{5k-4} with $|D_i| = 2\left\lfloor \frac{5(k-1)+1}{5} \right\rfloor + 1$. Let n = k. Consider the path P_{5k+1}.

Then $D_i' = D_i \cup \{v_{5k-2}, v_{5k+1}\}$; i = 1, 2, 3, …, k are the minimum co – isolated locating dominating sets of P_{5k} since, $|D_i'| = |D_i| + 2 = 2\left\lfloor \frac{5k}{5} \right\rfloor + 1$.

In addition, $D_{k+1}{}'$ = { $v_1, v_4, v_6, v_9, v_{11}, \ldots, v_{5k-1}, v_{5k+1}$} is also a minimum co – isolated locating dominating sets of P_{5k+1} containing v_1 and v_{5k+1} such that $D_{k+1}{}' \neq D_i{}'$; i = 1, 2, 3, …, k. Therefore, there are k + 1 minimum co – isolated locating dominating sets of P_{5k+1} containing v_1 and v_{5k+1}. By induction hypothesis, there are exactly n + 1 minimum co – isolated locating dominating sets of P_{5n+1} containing v_1 and v_{5n+1} with the labellings $v_1, v_2, v_3, \ldots, v_{5n-1}, v_{5n}, v_{5n+1}$, for all n ≥ 1.

Theorem: 3.4 There are exactly $\frac{n(n+3)}{2}$ minimum co – isolated locating dominating sets of P_{5n+1} containing v_1 and v_{5n} with the labellings $v_1, v_2, v_3, \ldots, v_{5n-1}, v_{5n}, v_{5n+1}$, where n ≥ 1.

Proof: Let the labellings of vertices of P_{5n+1} be $v_1, v_2, v_3, \ldots, v_{5n-1}, v_{5n}, v_{5n+1}$. The theorem is proved by the method of induction on n.

For n = 1, the sets A_{11} = {v_1, v_3, v_5} and A_{12} = {v_1, v_4, v_5} are the only minimum co – isolated locating dominating sets of P_6 containing v_1 and v_5 since, $|A_{11}| = |A_{12}| = 3 = 2\left\lfloor\frac{5n+1}{5}\right\rfloor + 1$, (Theorem 2.5).

Let $\mathcal{D}_1$= {$A_{1,1}, A_{1,2}$} and $|\mathcal{D}_1| = 2 = \frac{n(n+3)}{2}$, where n =1.

Let n = 2. In order to construct the minimum co – isolated locating dominating sets of P_{11} containing v_1 and v_{10}, the following sets are defined using $A_{1,1}$ and $A_{1,2}$.

Let $A_{2,1} = A_{1,1} \cup \{v_8, v_{10}\}$; $A_{2,2} = A_{1,2} \cup \{v_8, v_{10}\}$ and $B_{2,1} = (A_{1,1} - \{v_5\}) \cup \{v_6, v_8, v_{10}\}$;
$B_{2,2} = (A_{1,2} - \{v_5\}) \cup \{v_6, v_8, v_{10}\}$; $B_{2,3} = (A_{1,2} - \{v_5\}) \cup \{v_6, v_9, v_{10}\}$.

These are the only minimum co – isolated locating dominating sets of P_{11} containing v_1 and v_{10} since,
$|A_{2,1}| = |A_{2,2}| = |B_{2,1}| = |B_{2,2}| = |B_{2,3}| = 5 = 2\left\lfloor\frac{5n+1}{5}\right\rfloor + 1$.

Let $\mathcal{D}_2$ ={$A_{2,1}, A_{2,2}, B_{2,1}, B_{2,2}, B_{2,3}$} and $|\mathcal{D}_2| = 5 = \frac{n(n+3)}{2}$, where n = 2.

Let n = 3. In order to construct the minimum co – isolated locating dominating sets of P_{16} containing v_1 and v_{15}, the following sets are defined using the sets in $\mathcal{D}_2$.

Let $A_{3,1} = A_{2,1} \cup \{v_{13}, v_{15}\}$; $A_{3,2} = A_{2,2} \cup \{v_{13}, v_{15}\}$; $A_{33} = B_{2,1} \cup \{v_{13}, v_{15}\}$;
$A_{3,4} = B_{2,2} \cup \{v_{13}, v_{15}\}$; $A_{3,5} = B_{2,3} \cup \{v_{13}, v_{15}\}$ and $B_{3,1} = (B_{2,1} - \{v_{10}\}) \cup \{v_{11}, v_{13}, v_{15}\}$;
$B_{3,2} = (B_{2,2} - \{v_{10}\}) \cup \{v_{11}, v_{13}, v_{15}\}$; $B_{3,3} = (B_{2,3} - \{v_{10}\}) \cup \{v_{11}, v_{13}, v_{15}\}$;
$B_{3,4} = (B_{2,3} - \{v_{10}\}) \cup \{v_{11}, v_{14}, v_{15}\}$.

These are the only minimum co – isolated locating dominating sets of P_{16} containing v_1 and v_{15} since,
$|A_{3,i}| = |B_{2,j}| = 7 = 2\left\lfloor\frac{5n+1}{5}\right\rfloor + 1$; i = 1, 2, 3, 4, 5 and j = 1, 2, 3, 4.

Let $\mathcal{D}_3$ = {$A_{3,1}, A_{3,2}, A_{3,3}, A_{3,4}, A_{3,5}, B_{3,1}, B_{3,2}, B_{3,3}, B_{3,4}$} and $|\mathcal{D}_3| = 9 = \frac{n(n+3)}{2}$, where n = 3. Therefore, the result is true for n = 1, 2 and 3. Let n = k-1. Assume that the theorem holds for all paths having 5(k – 1) +1 vertices. Let $\mathcal{D}_{k-1}$ be the set of all minimum co – isolated locating dominating sets of $P_{5(k-1)+1}$ containing v_1 and $v_{5(n-1)}$.

Then, $\mathcal{D}_{k-1}$ ={$A_{(k-1),1}, A_{(k-1),2}, A_{(k-1),3}, \ldots, A_{(k-1),r}, B_{(k-1),1}, B_{(k-1),2}, \ldots, B_{(k-1),s}$}, where r = $\frac{(k-2)(k+1)}{2}$, s = k, k ≥ 3.

Also, $|\mathcal{D}_{k-1}| = r + s = \frac{(k-1)(k+2)}{2} = \frac{n(n+3)}{2}$, where n = k-1. and $|A_{(k-1),i}| = |B_{(k-1),j}| = 2\left\lfloor\frac{5(k-1)+1}{5}\right\rfloor + 1$; i = 1, 2, …, r and j = 1, 2, …, s. Let n = k. In order to construct the minimum co – isolated locating dominating set of P_{5k+1} containing v_1 and v_{5k}, the following sets are defined using the sets in $\mathcal{D}_{k-1}$.

Let $\mathcal{D}_k$ be the set of all minimum co – isolated locating dominating sets of P_{5k+1} containing v_1 and v_{5k}.

Then, $\mathcal{D}_n$ = {$A_{k,1}, A_{k,2}, A_{k,3}, \ldots, A_{k,\ell}, B_{k,1}, B_{k,2}, \ldots, B_{k,m}$},
where $\ell = \frac{(k-1)(k+2)}{2}$,
m = k + 1 (k ≥ 2) and
$A_{k,i} = A_{(k-1),i} \cup \{v_{5k-2}, v_{5k}\}$, i = 1, 2, ..., $\ell - k$, and
$A_{k,j} = B_{(k-1),(j-r)} \cup \{v_{5k-2}, v_{5k}\}$, j = ($\ell - k$ +1), ($\ell - k$ +2), …, ℓ.
$B_{k,i} = (B_{(k-1),i} - \{v_{5k-5}\}) \cup \{v_{5k-4}, v_{5k-2}, v_{5k}\}$; i = 1, 2, …., m-1 and
$B_{k,m} = (B_{(k-1),(m-1)} - \{v_{5k-5}\}) \cup \{v_{5k-4}, v_{5k-1}, v_{5k}\}$.

Also, $|A_{k,i}| = |B_{k,j}| = 2\left\lfloor\frac{5k}{5}\right\rfloor + 1$; i = 1, 2, …, ℓ and j = 1, 2, …, m and $|\mathcal{D}_n| = \ell + m = \frac{k(k+3)}{2}$. By induction hypothesis, there are exactly $\frac{n(n+3)}{2}$ minimum co – isolated locating dominating sets of P_{5n+1} containing v_1 and v_{5n} with the labellings $v_1, v_2, v_3, \ldots, v_{5n-1}, v_{5n}, v_{5n+1}$, for all n ≥ 1.

Theorem: 3.5 There are exactly $\frac{n(n+1)(n+5)}{6}$ minimum co – isolated locating dominating sets of P_{5n+1} containing v_2 and v_{5n} with the labellings $v_1, v_2, v_3, \ldots, v_{5n-1}, v_{5n}, v_{5n+1}$, where n ≥ 1.

Proof: Let the labellings of vertices of P_{5n+1} be $v_1, v_2, v_3, \ldots, v_{5n-1}, v_{5n}, v_{5n+1}$. The theorem is proved by the method of induction on n.

For n = 1, the sets $A_{1,1} = \{v_2, v_3, v_5\}$; $A_{1,2} = \{v_2, v_4, v_5\}$ are the only minimum co – isolated locating dominating sets of P_6 containing v_2 and v_5 , since $|A_{1,1}| = |A_{1,2}| = 3 = 2\left\lfloor\frac{5n+1}{5}\right\rfloor + 1$.

Let $\mathcal{D}_1 = \{A_{1,1}, A_{1,2}\}$. Then $|\mathcal{D}_1| = 2$.

For n = 2, the sets $A_{2,i} = A_{1,i} \cup\{v_8, v_{10}\}$; i = 1, 2 and $B_{2,i} = (A_{2,i} - v_5) \cup\{v_6, v_8, v_{10}\}$; i = 1, 2; $B_{2,3} = (A_{22} - \{v_5\}) \cup\{v_6, v_9, v_{10}\}$, $C_{2,1} = \{v_2, v_4, v_7, v_8, v_{10}\}$ and $E_{2,1} = \{v_2, v_4, v_7, v_9, {}_{10}\}$ are the only minimum co – isolated locating dominating sets of P_{11} containing v_2 and v_{10}, since $|A_{2,i}| = |B_{2,j}| = |C_{2,1}| = |E_{2,1}| = 5 = 2\left\lfloor\frac{5n+1}{5}\right\rfloor + 1$; i = 1, 2 and j = 1, 2, 3.

Let $\mathcal{D}_2 = \{A_{2,1}, A_{2,2}, B_{2,1}, B_{2,2}, B_{2,3}, C_{2,1}, E_{2,1}\}$. Then $|\mathcal{D}_2| = 7 = \frac{n(n+1)(n+5)}{6}$.

Let n = 3. In order to construct the minimum co – isolated locating dominating sets containing v_2 and v_{15} of P_{16} , the following sets are defined using the sets in $\mathcal{D}_2$.

Let $A_{3,i} = A_{2,i} \cup\{v_{13}, v_{15}\}$; i = 1, 2;
$A_{3,j} = A_{2,(j-2)} \cup\{v_{13}, v_{15}\}$; j = 3, 4, 5;
$A_{3,k} = C_{2,(k-5)} \cup\{v_{13}, v_{15}\}$; k = 6, 7;
$B_{3,i} = (B_{2,i} - \{v_{10}\}) \cup\{v_{11}, v_{13}, v_{15}\}$; i = 1, 2, 3;
$B_{3,4} = (B_{2,3} - \{v_{10}\}) \cup \{v_{11}, v_{14}, v_{15}\}$
$C_{3,i} = (C_{2,i} - \{v_{10}\}) \cup\{v_{11}, v_{13}, v_{15}\}$; i = 1, 2;
$E_{3,1} = (E_{2,1} - \{v_{10}\}) \cup \{v_{11}, v_{14}, v_{15}\}$;
$E_{3,2} = (E_{2,1} - \{v_{10}\}) \cup \{v_{12}, v_{14}, v_{15}\}$; and
$E_{3,3} = (E_{3,2} - \{v_{14}\}) \cup\{v_{13}\}$.

These are the only minimum co – isolated locating dominating sets of P_{16} containing v_2 and v_{15}, since $|A_{3,i}| = |B_{3,j}| = |C_{3,k}| = |E_{3,t}| = 7 = 2\left\lfloor\frac{5n+1}{5}\right\rfloor + 1$; i = 1, 2, …, 7; j = 1, 2, 3, 4, k = 1, 2, and t = 1, 2, 3.

Let $\mathcal{D}_3 = \{A_{31}, A_{32}, \ldots, A_{37}, B_{31}, B_{32}, B_{33}, B_{34}, C_{31}, C_{32}, E_{3,1}, E_{3,2}, E_{3,3}\}$. Then $|\mathcal{D}_3| = 16 = \frac{n(n+1)(n+5)}{6}$. Therefore the result is true for n = 1, 2 and 3.

Assume that the Theorem holds for n = k -1. That is, there are exactly $\frac{(k-1)k(k+4)}{6}$ minimum co – isolated locating dominating sets of $P_{5(k-1)+1}$ containing v_2 and $v_{5(k-1)}$ with the labellings $v_1, v_2, v_3, \ldots, v_{5(k-1)-1}, v_{5(k-1)}, v_{5(k-1)+1}$, where k ≥ 2.

Let $\mathcal{D}_{k-1}$ be the set of all minimum co – isolated locating dominating sets of $P_{5(k-1)+1}$ containing v_2 and $v_{5(k-1)}$.

Then $\mathcal{D}_{k-1} = \{A_{(k-1),1}, A_{(k-1),2}, A_{(k-1),3}, \ldots, A_{(k-1),r}, B_{(k-1),1}, B_{(k-1),2}, \ldots, B_{(k-1),k}, C_{(k-1),1}, C_{(k-1),2}, C_{(k-1),3}, \ldots, C_{(k-1),s}, E_{(k-1)1}, E_{(k-1)2}, \ldots, E_{(k-1)(k-1)}\}$, where $r = \frac{(k-2)(k-1)(k+3)}{6}$ and $s = \frac{k(k-3)}{2}$, where k ≥ 4.

Also $|\mathcal{D}_{k-1}| = r + k + s + (k-1) = \frac{(k-1)k(k+4)}{6}$ and $|A_{(k-1),i}| = |B_{(k-1),j}| = |C_{(k-1),p}| = |E_{(k-1),q}| = 2\left\lfloor\frac{5(k-1)+1}{5}\right\rfloor + 1$; i = 1, 2, …, r ; j = 1, 2,…., n ; p = 1, 2, …., s and q = 1, 2, …, k – 1.

Let n = k. In order to construct the minimum co – isolated locating dominating sets of P_{5k+1} containing v_2 and v_{5k}, the following sets are defined using the sets in $\mathcal{D}_{k-1}$. Let $\mathcal{D}_k$ be the set of all minimum co – isolated locating dominating sets of P_{5k+1} containing v_2 and v_{5k}. Then, $\mathcal{D}_k = \{A_{k,1}, A_{k,2}, A_{k,3}, \ldots, A_{k,l}, B_{k,1}, B_{k,2}, \ldots, B_{k,k+1}, C_{k,1}, C_{k,2}, \ldots, C_{k,m}, E_{k,1}, E_{k,2}, \ldots, E_{k,k}\}$,
where $\ell = \frac{(k-1)k(k+4)}{6}$, $m = \frac{(k+1)(k-2)}{2}$,

where k ≥ 2 and

$A_{n,\,i} = A_{(k-1),i} \cup \{v_{5k-2}, v_{5k}\}$, i = 1, 2, ..., r;
$A_{n,\,j} = B_{(k-1),(j-r)} \cup \{v_{5k-2}, v_{5k}\}$, j = r + 1, r + 2, …, r + k;
$A_{n,\,t} = C_{(k-1),(t-r-k)} \cup \{v_{5k-2}, v_{5k}\}$, t = r + k + 1, r + k + 2, …, r + k + s (= ℓ);
$B_{k,\,i} = (B_{(k-1),i} - \{v_{5k-5}\}) \cup \{v_{5k-4}, v_{5k-2}, v_{5k}\}$; i = 1, 2, …, k; and
$B_{k,\,k+1} = (B_{(k-1),k} - \{v_{5k-5}\}) \cup \{v_{5k-4}, v_{5k-1}, v_{5k}\}$ and
$C_{k,i} = (C_{(k-1),i} - \{v_{5k-5}\}) \cup \{v_{5k-4}, v_{5k-2}, v_{5k}\}$; i = 1, 2, …, m
$E_{k,h} = (E_{(k-1),h} - \{v_{5k-5}\}) \cup \{v_{5k-4}, v_{5k-1}, v_{5k}\}$; h = 1, 2, …, k -2 ;
$E_{k,(k-1)} = (E_{(k-1),(k-2)} - \{v_{5k-5}\}) \cup \{v_{5k-3}, v_{5k-1}, v_{5k}\}$; and
$E_{k,k} = (E_{k,(k-2)} - \{v_{5k-1}\}) \cup \{v_{5k-2}\}$

Also, $|A_{k,i}| = |B_{k,j}| = |C_{k,p}| = |E_{k,q}| = 2\left\lceil\frac{k}{5}\right\rceil + 1$; i = 1, 2, …,$\ell$ and j = 1, 2,…, k+1 ; p = 1, 2, …, m and q = 1, 2, …, k and $|\mathcal{D}_k| = \ell + k + 1 + m + k = \frac{k(k+1)(k+5)}{6}$. Therefore, there are $\frac{k(k+1)(k+5)}{6}$ minimum co – isolated locating dominating sets of P_{5k+1} containing v_2 and v_{5k}. The Theorem is true for n = k. By induction hypothesis, the theorem is true for all n ≥ 1.

Theorem: 3.6 For any integer n ≥ 1, $\gamma_{Dcild}(P_{5n+1}) = \frac{(n+3)(n^2+9n+2)}{6}$.

Proof: $\gamma_{Dcild}(P_{5n+1})$ is the sum of the number of minimum co – isolated locating dominating sets of P_{5n+1} containing

(i) v_1 and v_{5n+1}
(ii) v_1 and v_{5n}
(iii) v_2 and v_{5n+1}
(iv) v_2 and v_{5n}

(a) For (i), the number of minimum co-isolated dominating sets of P_{5n+1} containing v_1 and v_{5n+1} is (n + 1) by Theorem 3.3.
(b) For (ii), the number of minimum co-isolated dominating sets of P_{5n+1} containing v_1 and v_{5n} is $\frac{n(n+3)}{2}$ by Theorem 3.4.
(c) For (iii), the number of minimum co – isolated locating dominating sets of P_{5n+1} containing v_2 and v_{5n+1} is same as that of minimum co – isolated locating dominating sets containing v_1 and v_{5n} and hence it is $\frac{n(n+3)}{2}$).
(d) For (iv), the number of minimum co-isolated dominating sets of P_{5n+1} containing v_2 and v_{5n} is $\frac{n(n+1)(n+5)}{6}$, by Theorem 3.5.

Hence, $\gamma_{Dcild}(P_{5n+1}) = \frac{(n+3)(n^2+9n+2)}{6}$.

Remark: 3.7 The Recurrence relation is given by

$$\gamma_{Dcild}(P_{5n+1}) - \gamma_{Dcild}(P_{5(n-1)+1}) = \frac{(n+3)(n^2+9n+2)}{6} - \frac{(n+2)(n^2+7n-6)}{6}.$$
$$= \frac{7n^2+21n+18}{6}.$$

Therefore, $\gamma_{Dcild}(P_{5n+1}) = \gamma_{Dcild}(P_{5n-4}) + \frac{7n^2+21n+18}{6}$; n ≥ 2.

In the following, the number of minimum co-isolated locating dominating sets of P_{5n+2} is found.

Theorem: 3.8 There is no minimum co – isolated locating dominating set of P_{5n+2} containing v_1 and v_{5n+2} with the labellings $v_1, v_2, v_3, \ldots, v_{5n}, v_{5n+1}, v_{5n+2}$, where n ≥ 1.

Proof: On the contrary, let D be a minimum co – isolated locating dominating set of P_{5n+2} containing v_1 and v_{5n+2} with the labellings $v_1, v_2, v_3, \ldots, v_{5n}, v_{5n+1}, v_{5n+2}$. Then, $|D| = 2\left\lceil\frac{5n+2}{5}\right\rceil + 1$ (By Theorem 2.5.) and $D' = D - \{v_1, v_{5n+2}\}$ will be a minimum co – isolated locating dominating set of P_{5n} with the labellings $v_3, v_4, v_5, \ldots, v_{5n-1}, v_{5n}$. $|D'| = 2\left\lceil\frac{5n+2}{5}\right\rceil - 1$. Therefore, $\gamma_{cild}(P_{5n}) \le 2\left\lceil\frac{5n+2}{5}\right\rceil - 1$. But, $\gamma_{cild}(P_{5n}) = 2\left\lceil\frac{5n+2}{5}\right\rceil$, which is a contradiction. Hence, there is no minimum co – isolated locating dominating set of P_{5n+2} containing v_1 and v_{5n+2}.

Theorem: 3.9 There is exactly one minimum co – isolated locating dominating set of P_{5n+2} containing v_2 and v_{5n+2} with the labellings $v_1, v_2, v_3, \ldots, v_{5n}, v_{5n+1}, v_{5n+2}$, where n ≥ 1.

Proof: Clearly, $D = \{v_2, v_4, v_7, \ldots, v_{5n-1}, v_{5n+2}\}$ is a minimum co – isolated locating dominating set of P_{5n+2} containing v_2 and v_{5n+2}, which proves the existence and $|D| = 2\left\lceil\frac{5n+2}{5}\right\rceil + 1$. To prove the uniqueness, Let $D' = D - \{v_{5n+2}\}$. D' is a minimum co – isolated locating dominating set of P_{5n} with the labellingss $v_1, v_2, v_3, \ldots, v_{5n}$, since

$|D'| = |D|-1=2\left\lfloor\frac{5n+2}{5}\right\rfloor = \gamma_{cild}(P_{5n})$. But by Theorem 3.2, D' is the unique minimum co – isolated locating dominating set of P_{5n} and hence D is unique.

Theorem: 3.10 There are exactly n minimum co – isolated locating dominating sets of P_{5n+2} containing v_2 and v_{5n+1} with the labellings $v_1, v_2, v_3, \ldots, v_{5n}, v_{5n+1}, v_{5n+2}$, where $n \geq 1$.

Proof: Let the labellings of vertices of P_{5n+2} be $v_1, v_2, v_3, \ldots, v_{5n-1}, v_{5n}, v_{5n+2}$. The theorem is proved by the method of induction on n. For n = 1, $A_{1,1}=\{v_2, v_4, v_6\}$ is the only minimum co – isolated locating dominating set of P_7 containing v_2 and v_6, since, $|A_{1,1}| = 3 = 2\left\lfloor\frac{5n+2}{5}\right\rfloor + 1$. Let $\mathcal{D}_1=\{A_{1,1}\}$ and $|\mathcal{D}_1| = 1 = n$. Let n = 2. In order to construct the minimum co – isolated locating dominating sets of P_{12} containing v_2 and v_{11}, the following sets are defined using $\mathcal{D}_1$. Let $A_{2,1} = A_{1,1} \cup \{v_9, v_{11}\}$; and $B_{2,1} = (A_{1,1} - \{v_6\}) \cup \{v_7, v_9, v_{11}\}$. These are the only minimum co – isolated locating dominating sets of P_{12} containing v_2 and v_{11}, since, $|A_{2,1}| = |B_{2,1}| = 5 = 2\left\lfloor\frac{5n+2}{5}\right\rfloor + 1$. Let $\mathcal{D}_2=\{A_{2,1}, B_{2,1}\}$ and $|\mathcal{D}_2| = 2 = n$. Let n = 3. In order to construct the minimum co – isolated locating dominating sets of P_{17} containing v_2 and v_{16}, the following sets are defined using the sets in $\mathcal{D}_2$.

Let $A_{3,1} = A_{2,1} \cup \{v_{14}, v_{16}\}$; $A_{3,2} = B_{2,1} \cup \{v_{14}, v_{16}\}$ and $B_{3,1} = (B_{2,1} - \{v_{11}\}) \cup \{v_{12}, v_{14}, v_{16}\}$.

These are the only minimum co – isolated locating dominating sets of P_{17} containing v_2 and v_{16}, since,
$|A_{3,i}| = |B_{31}| = 7 = 2\left\lfloor\frac{5n+2}{5}\right\rfloor + 1$; i = 1, 2, where n = 3.

Let $\mathcal{D}_3=\{A_{3,1}, A_{3,2}, B_{3,1}\}$ and $|\mathcal{D}_3| = 3 = n$. Therefore, the result is true for n = 1, 2 and 3. Assume that the theorem holds for n = k-1. That is, there are exactly k-1 minimum co – isolated locating dominating sets of $P_{5(k-1)+2}$ containing v_2 and $v_{5(k-1)+1}$ with the labellings $v_1, v_2, v_3, \ldots, v_{5n}, v_{5(k-1)+1}, v_{5(k-1)+2}$, where $k \geq 4$. Let $\mathcal{D}_{k-1}$ be the set of all minimum co – isolated locating dominating sets of $P_{5(k-1)+2}$ containing v_2 and $v_{5(k-1)+1}$.

Also, $\mathcal{D}_{k-1} = \{A_{(k-1),1}, A_{(k-1),2}, A_{(k-1),3}, \ldots, A_{(k-1),(k-2)}, B_{(k-1),1}\}$, $|\mathcal{D}_{k-1}| = k-1$ and
$|A_{(k-1),i}| = |B_{(k-1),1}| = 2\left\lfloor\frac{5(k-1)+2}{5}\right\rfloor + 1$; i = 1, 2, …, (k – 1). The result is to be proved, when n = k. In order to construct the minimum co – isolated locating dominating sets of P_{5k+2} containing v_2 and v_{5k+1}, the following sets are defined using the sets in $\mathcal{D}_{k-1}$. Let $\mathcal{D}_k$ be the set of all minimum co – isolated locating dominating set of P_{5k+1}containing v_2 and v_{5k+1}.

Then, $\mathcal{D}_k = \{A_{k,1}, A_{k,2}, A_{k,3}, \ldots, A_{k,(k-1)}, B_{k,1}\}$ and $A_{k,i} = A_{(k-1),i} \cup \{v_{5k-1}, v_{5k+1}\}$; i = 1, 2, …, (k – 2) and $A_{k,(k-1)} = B_{(k-1),1} \cup \{v_{5k-1}, v_{5k+1}\}$ and $B_{k,1} = (B_{(k-1),1} - \{v_{5k-4}\}) \cup \{v_{5k-3}, v_{5k-1}, v_{5k+2}\}$.

Also, $|A_{k,i}| = |B_{k,1}| = 2\left\lfloor\frac{5k+2}{5}\right\rfloor + 1$; i = 1, 2, …, (k – 1) and $|\mathcal{D}_k| = k$. Therefore, there are exactly k minimum co – isolated locating dominating sets of P_{5k+2} containing v_2 and v_{5k+1} with the labellings $v_1, v_2, v_3, \ldots, v_{5k}, v_{5k+1}, v_{5k+2}$. By induction hypothesis, the theorem is proved for all $n \geq 1$.

Theorem: 3.11 For any integer $n \geq 1$, $\gamma_{Dcild}(P_{5n+2}) = n + 2$.

Proof: $\gamma_{Dcild}(P_{5n+2})$ is the sum of the number of minimum co – isolated locating dominating sets of P_{5n+2} containing

(i) v_1 and v_{5n+2}
(ii) v_2 and v_{5n+2}
(iii) v_1 and v_{5n+1}
(iv) v_2 and v_{5n+1}

(a) For (i), there is no minimum co-isolated dominating sets of P_{5n+2} containing v_1 and v_{5n+2}, by Theorem 3.8.
(b) For (ii), the number of minimum co-isolated dominating sets of P_{5n+2} containing v_2 and v_{5n+2} is 1, by Theorem 3.9.
(c) For (iii), the number of minimum co – isolated locating dominating sets of P_{5n+2} containing v_1 and v_{5n+1} is same as that of minimum co – isolated locating dominating sets containing v_2 and v_{5n+2} and hence it is 1.
(d) For (iv), the number of minimum co-isolated dominating sets of P_{5n+2} containing v_2 and v_{5n+1} is n, By Theorem 3.10. Hence, $\gamma_{Dcild}(P_{5n+2}) = n + 2$.

In the following, the number of minimum co-isolated locating dominating sets of P_{5n+3} is found.

Theorem: 3.12 There are exactly $\frac{n^2+5n+2}{2}$ minimum co – isolated locating dominating sets of P_{5n+3} containing v_1 and v_{5n+3} with the labellings $v_1, v_2, v_3, \ldots, v_{5n+2}, v_{5n+3}$, where $n \geq 2$.

Proof: Let the labellings of vertices of P_{5n+3} be $v_1, v_2, v_3, ..., v_{5n+2}, v_{5n+3}$. The theorem is proved by the method of induction on n. For n = 1, $A_{1,1}=\{v_1, v_3, v_5, v_8\}$; $A_{1,2}=\{v_1, v_3, v_6, v_8\}$; $A_{1,3}=\{v_1, v_3, v_6, v_8\}$; $A_{1,4}=\{v_1, v_4, v_6, v_8\}$ are the only minimum co – isolated locating dominating sets of P_8 containing v_1 and v_8, since, $|A_{1,i}| = 4 = 2\left\lfloor\frac{5n+3}{5}\right\rfloor + 2$, i = 1, 2, 3, 4, by Theorem 2.5. Let $\mathcal{D}_1=\{A_{1,1}, A_{1,2}, A_{1,3}\}$ and $|\mathcal{D}_1| = 4$. Let n = 2. In order to construct the minimum co – isolated locating dominating sets of P_{13} containing v_1 and v_{13}, the following sets are defined using the sets in $\mathcal{D}_1$.

Let $A_{2,i} = A_{1,i} \cup \{v_{10}, v_{13}\}$; i = 1, 2, 3, 4 and $A_{2,5} = (A_{1,4} - \{v_8\}) \cup \{v_9, v_{10}, v_{13}\}$;
$B_{2,1} = A_{1,3} \cup \{v_{11}, v_{13}\}$; $B_{2,2} = A_{1,4} \cup \{v_{11}, v_{13}\}$ and $B_{2,3} = (A_{1,4} - \{v_8\}) \cup \{v_9, v_{11}, v_{13}\}$. These are the only minimum co – isolated locating dominating sets of P_{13} containing v_1 and v_{13}, since $|A_{2,i}| = |B_{2,j}| = 6 = 2\left\lfloor\frac{5n+3}{5}\right\rfloor + 2$; i = 1, 2, …, 5 and j =1, 2, 3.

Let $\mathcal{D}_2=\{A_{2,1}, A_{2,2}, …, A_{2,5}, B_{2,1}, B_{2,2}, B_{2,3}\}$ and $|\mathcal{D}_2| = 8 = \frac{n^2+5n+2}{2}$. Let n = 3. In order to construct the minimum co – isolated locating dominating sets of P_{18} containing v_1 and v_{18}, the following sets are defined using the sets in $\mathcal{D}_2$.

Let $A_{3,i} = A_{2,i} \cup \{v_{15}, v_{18}\}$, for i = 1, 2, …, 5, and
$A_{3,j} = B_{2,(j-5)} \cup \{v_{15}, v_{18}\}$, for j = 6, 7, 8 and $A_{3,9} = B_{2,3} - \{v_{13}\} \cup \{v_{14}, v_{15}, v_{18}\}$;
$B_{3,i} = B_{2,i} \cup \{v_{16}, v_{18}\}$; i = 1, 2, 3;
$B_{3,4} = (B_{2,3} - \{v_{13}\}) \cup \{v_{14}, v_{16}, v_{18}\}$.

These are the only minimum co – isolated locating dominating sets of P_{18} containing v_1 and v_{18}, since,
$|A_{3,i}| = |B_{3,j}| = 8 = 2\left\lfloor\frac{5n+3}{5}\right\rfloor + 2$; i = 1, 2, …, 9 and j = 1, 2, ..., 4.

Let $\mathcal{D}_3=\{A_{3,1}, A_{3,2}, …, A_{3,9}, B_{3,1}, B_{3,2}, B_{3,3}, B_{3,4}\}$ and $|\mathcal{D}_3| = 13 = \frac{n^2+5n+2}{2}$, where n = 3. Therefore, the result is true for n=1,2 and 3. Assume that the theorem holds for n= k-1. That is, there are exactly $\frac{(k-1)^2+5(k-1)+2}{2} = \frac{k^2+3k-2}{2}$ minimum co– isolated locating dominating sets of $P_{5(k-1)+3}$ containing v_1 and $v_{5(k-1)+3}$ with the labellings $v_1, v_2, v_3, …, v_{5(k-1)+2}, v_{5(k-1)+3}$. Let $\mathcal{D}_{k-1}$ be the set of all minimum co – isolated locating dominating sets of $P_{5(k-1)+3}$ containing v_1 and $v_{5(k-1)+3}$.

Then, $\mathcal{D}_{k-1} = \{A_{(k-1),1}, A_{(k-1),2}, A_{(k-1),3}, …, A_{(k-1),r}, B_{(k-1),1}, B_{(k-1),2}, …, B_{(k-1),s}\}$ where $r = \frac{(k-1)(k+2)}{2}$; s = k, k ≥ 2.

Also, $|\mathcal{D}_{k-1}| = \frac{k^2+3k-2}{2}$ and $|A_{(k-1),i}| = |B_{(k-1),j}| = 2\left\lfloor\frac{5(k-1)+3}{5}\right\rfloor + 2$, i = 1, 2, …, r and j = 1, 2, …, s. The theorem is to be proved for n = k. In order to construct the minimum co – isolated locating dominating sets of P_{5k+3} containing v_1 and v_{5k+3}, the following sets are defined using the sets in $\mathcal{D}_{k-1}$. Let $\mathcal{D}_k$ be the set of all minimum co – isolated locating dominating sets of P_{5k+3} containing v_1 and v_{5k+3}.

Then, $\mathcal{D}_k = \{A_{k,1}, A_{k,2}, A_{k,3}, …, A_{k,l}, B_{k,1}, B_{k,2}, .., B_{k,m}\}$, where $l = \frac{k(k+3)}{2}$ and m = k + 1.
$A_{k,i} = A_{(k-1),i} \cup \{v_{5k}, v_{5k+3}\}$, for i = 1, 2, …, r;
$A_{k,j} = B_{(k-1),(j-r)} \cup \{v_{5k}, v_{5k+3}\}$, for j = r+1, r+2, …, r+s (= l);
$A_{k,(l+1)} = (B_{(k-1),k} - \{v_{5k-3}\}) \cup \{v_{5k-2}, v_{5k}, v_{5k+3}\}$;
$B_{k,i} = B_{(k-1),i} \cup \{v_{5k+1}, v_{5k+3}\}$; i = 1, 2, …, k;
$B_{k,(k+1)} = (B_{(k-1),k} - \{v_{5k-3}\}) \cup \{v_{5k-2}, v_{5k+1}, v_{5k+3}\}$.

Also, $|A_{k,i}| = |B_{k,j}| = 2\left\lfloor\frac{5k+3}{5}\right\rfloor + 2$; i = 1, 2, …, l; j = 1, 2, …, m and $|\mathcal{D}_k| = l + m = \frac{k^2+5k+2}{2}$. The theorem is proved for n = k. By induction hypothesis, there are exactly $\frac{n^2+5n+2}{2}$ minimum co – isolated locating dominating sets of P_{5n+3} containing v_1 and v_{5n+3} with the labellings $v_1, v_2, v_3, …., v_{5n+2}, v_{5n+3}$, for all n ≥ 2.

Theorem: 3.13 There are exactly $\frac{n^2+21n-24}{2}$ minimum co – isolated locating dominating sets of P_{5n+3} containing v_1 and v_{5n+2} with the labellings $v_1, v_2, v_3, …, v_{5n+2}, v_{5n+3}$, where n ≥ 3.

Proof: Let the labellings of vertices of P_{5n+3} be $v_1, v_2, v_3, ..., v_{5n+2}, v_{5n+3}$. The theorem is proved by the method of induction on n.

For n = 1, $A_{1,1}=\{v_1, v_2, v_5, v_7\}$; $A_{1,2}=\{v_1, v_3, v_5, v_7\}$; $A_{1,3}=\{v_1, v_3, v_6, v_7\}$ are the only minimum co – isolated locating dominating sets of P_8 containing v_1 and v_7, since $|A_{1,i}| = 4 = 2\left\lfloor\frac{5n+3}{5}\right\rfloor + 2$, by Theorem 2.5.

Let $\mathcal{D}_1 = \{A_{1,1}, A_{1,2}, A_{1,3}\}$ and $|\mathcal{D}_1| = 3$. Let n = 2. In order to construct the minimum co – isolated locating dominating sets of P_{13} containing v_1 and v_{12}, the following sets are defined using $\mathcal{D}_1$.

Let $A_{2,i} = A_{1,i} \cup \{v_{10}, v_{12}\}$; i = 1, 2, 3 and

$B_{2,1} = (A_{1,2} - \{v_7\}) \cup \{v_8, v_{10}, v_{12}\}$; $B_{2,2} = (A_{1,3} - \{v_7\}) \cup \{v_8, v_{10}, v_{12}\}$;

$B_{2,3} = (A_{1,1} - \{v_2, v_7\}) \cup \{v_4, v_8, v_{10}, v_{12}\}$; $B_{2,4} = (A_{1,1} - \{v_2, v_5, v_7\}) \cup \{v_4, v_6, v_8, v_{10}, v_{12}\}$;

$B_{2,5} = (A_{1,2} - \{v_2, v_5, v_7\}) \cup \{v_4, v_6, v_9, v_{10}, v_{12}\}$; $B_{2,6} = (A_{13} - \{v_7\}) \cup \{v_6, v_8, v_{11}, v_{12}\}$;

$B_{2,7} = (B_{2,4} - \{v_{10}\}) \cup \{v_{11}, v_{12}\}$ and $B_{2,8} = (B_{2,5} - \{v_{10}\}) \cup \{v_{11}, v_{12}\}$;

These are the only minimum co – isolated locating dominating sets of P_{13} containing v_1 and v_{12}, since $|A_{2,i}| = |B_{2,j}| = 6 = 2\left\lfloor\frac{5n+3}{5}\right\rfloor + 2$; i = 1, 2, 3 and j =1, 2, …, 8.

Let $\mathcal{D}_2 = \{A_{2,1}, A_{2,2}, A_{2,3}, B_{2,1}, B_{2,2}, \ldots, B_{2,8}\}$ and $|\mathcal{D}_2| = 11 = \frac{n^2+21n-24}{2}$, where n = 2.

Let n = 3. In order to construct the minimum co – isolated locating dominating set of P_{18} containing v_1 and v_{17}, the following sets are defined using the sets in $\mathcal{D}_2$.

$A_{3,i} = A_{2,i} \cup \{v_{15}, v_{17}\}$, for i = 1, 2, 3;

$A_{3,j} = B_{2,(j-3)} \cup \{v_{15}, v_{17}\}$, for j = 4, 5, …, 11;

$B_{3,i} = B_{2,i} - \{v_{12}\} \cup \{v_{13}, v_{15}, v_{17}\}$; i = 1, 2, …, 8;

$B_{3,j} = B_{2,(j-3)} - \{v_{12}\} \cup \{v_{13}, v_{16}, v_{17}\}$; j = 9, 10, 11; and $B_{3,12} = (B_{2,8} - \{v_{12}\}) \cup \{v_{14}, v_{15}, v_{17}\}$;

$B_{3,13} = (B_{2,8} - \{v_{12}\}) \cup \{v_{14}, v_{16}, v_{17}\}$.

These are the only minimum co – isolated locating dominating set of P_{18} containing v_1 and v_{18}, since $|A_{3,i}| = |B_{3,j}| = 8 = 2\left\lfloor\frac{5n+3}{5}\right\rfloor + 2$; i = 1, 2, …, 11 and j = 1, 2, ..., 13.

Let $\mathcal{D}_3 = \{A_{3,1}, A_{3,2}, \ldots, A_{3,11}, B_{3,1}, B_{3,2}, \ldots, B_{3,13}\}$ and $|\mathcal{D}_3| = 24 = \frac{n^2+21n-24}{2}$, where n = 3.

Let n = 4. In order to construct the minimum co – isolated locating dominating sets of P_{21} containing v_1 and v_{20}, the following sets are defined using the sets in $\mathcal{D}_3$.

Let $A_{4,i} = A_{3,i} \cup \{v_{20}, v_{22}\}$, for i = 1, 2, …,11;

$A_{4,j} = B_{3,(j-11)} \cup \{v_{20}, v_{22}\}$, for j = 12, 13, …, 24;

$B_{4,i} = B_{3,i} - \{v_{17}\} \cup \{v_{18}, v_{20}, v_{22}\}$, for i = 1, 2, …, 13;

$B_{4,14} = B_{3,13} - \{v_{18}\} \cup \{v_{19}, v_{20}, v_{22}\}$.

These are the only minimum co – isolated locating dominating set of P_{21} containing v_1 and v_{20}, since $|A_{3,i}| = |B_{3,j}| = 8 = 2\left\lfloor\frac{5n+3}{5}\right\rfloor + 2$; i = 1, 2, …, 24 and j = 1, 2, ..., 14.

Let $\mathcal{D}_4 = \{A_{4,1}, A_{4,2}, \ldots, A_{4,24}, B_{4,1}, B_{4,2}, \ldots, B_{4,14}\}$ and $|\mathcal{D}_4| = 38 = \frac{n^2+21n-24}{2}$, where n = 4. Therefore, the result is true for n = 2, 3 and 4. Assume that the theorem holds for n = k-1. That is, there are exactly $\frac{(k-1)^2+21(k-1)-24}{2}$ minimum co – isolated locating dominating sets of $P_{5(k-1)+3}$ containing v_1 and $v_{5(k-1)+2}$ with the labellings $v_1, v_2, v_3, \ldots, v_{5(k-1)+2}, v_{5(k-1)+3}$, where $k \geq 4$.

Let $\mathcal{D}_{k-1}$ be the set of all minimum co – isolated locating dominating sets of $P_{5(k-1)+3}$ containing v_1 and $v_{5(k-1)+2}$.

Then, $\mathcal{D}_{k-1} = \{A_{(k-1),1}, A_{(k-1),2}, A_{(k-1),3}, \ldots, A_{(k-1),r}, B_{(k-1),1}, B_{(k-1),2}, B_{(k-1),s}\}$ and $r = \frac{k^2+17k-62}{2}$; s = (k–1)+10 = k + 9, $k \geq 4$. Also, $|\mathcal{D}_{k-1}| = \frac{k-1)^2+21(k-1)-24}{2} = \frac{k^2+19k-44}{2}$; and $|A_{(k-1),i}| = |B_{(k-1),j}| = 2\left\lfloor\frac{5(k-1)+3}{5}\right\rfloor + 2$; i = 1, 2, …, r and j=1, 2, …, s. The theorem is to be proved for n = k. In order to construct the minimum co – isolated locating dominating set of P_{5k+3} containing v_1 and v_{5k+2} , the following sets are defined using the sets in $\mathcal{D}_{k-1}$. Let $\mathcal{D}_k$ be the set of all minimum co – isolated locating dominating sets of P_{5k+3} containing v_1 and v_{5k+2}.

Then, $\mathcal{D}_k = \{A_{k,1}, A_{k,2}, A_{k,3}, \ldots, A_{k,l}, B_{k,1}, B_{k,2}, \ldots, B_{k,m}\}$, where $l = \frac{n^2+19n-44}{2}$ and m = k + 10.

$A_{k,i} = A_{(k-1),i} \cup \{v_{5k}, v_{5k+2}\}$, i = 1, 2, …, r;

$A_{k,j} = B_{(k-1),(j-r)} \cup \{v_{5k}, v_{5k+2}\}$, j = r+1, r+2, …, r+s (= l);

$B_{k,i} = (B_{(k-1),i} - \{v_{5k-3}\}) \cup \{v_{5k-2}, v_{5k}, v_{5k+2}\}$; i = 1, 2, …, s;

$B_{k,(s+1)} = (B_{(k-1),s} - \{v_{5k-2}\}) \cup \{v_{5k-1}, v_{5k}, v_{5k+2}\}$.

Also, $|A_{k,i}| = |B_{k,j}| = 2\left\lfloor\frac{5k+3}{5}\right\rfloor + 2$; i = 1, 2, …, l; j = 1, 2, …, m and $|\mathcal{D}_k| = l +$ m $= \frac{k^2+21k-24}{2}$. The result is true for n = k. By induction hypothesis, there are exactly $\frac{n^2+21n-24}{2}$ minimum co – isolated locating dominating sets of P_{5n+3} containing v_1 and v_{5n+2} with the labellings $v_1, v_2, v_3, \ldots, v_{5n+2}, v_{5n+3}$, where n ≥ 3.

Theorem: 3.14 There are exactly $(n + 1)^2$ minimum co – isolated locating dominating sets of P_{5n+3} containing v_2 and v_{5n+2} with the labellings $v_1, v_2, v_3, \ldots, v_{5n+2}, v_{5n+3}$, where n ≥ 3.

Proof: Let the labellings of vertices of P_{5n+3} be $v_1, v_2, v_3, \ldots, v_{5n+2}, v_{5n+3}$.

For n = 2, $A_{2,1}=\{v_2, v_4, v_7, v_9, v_{10}, v_{12}\}$; $A_{2,2}=\{v_2, v_4, v_7, v_9, v_{11}, v_{12}\}$;$A_{2,3}=\{v_2, v_4, v_7, v_8, v_{10}, v_{12}\}$ and $A_{2,4}=\{v_2, v_4, v_7, v_8, v_{11}, v_{12}\}$ are the only minimum co – isolated locating dominating sets of P_{13} containing v_2 and v_{12}, since $|A_{2,i}| = 6 = 2\left\lfloor\frac{5n+3}{5}\right\rfloor + 2$ (By Theorem 2.5.). Let $\mathcal{D}_2 = \{A_{2,1}, A_{2,2}, A_{2,3}, A_{2,4}\}$ and $|\mathcal{D}_2| = 4$.

Let n = 3. In order to construct the minimum co – isolated locating dominating sets of P_{18} containing v_2 and v_{17}, the following sets are defined using $\mathcal{D}_2$.

Let $A_{3,i} = A_{2,i} \cup \{v_{15}, v_{17}\}$; and $B_{3,i} = (A_{2,i} - \{v_{12}\}) \cup \{v_{13}, v_{15}, v_{17}\}$; i = 1, 2, 3, 4;
$B_{3,5} = (A_{2,2} - \{v_{12}\}) \cup \{v_{13}, v_{16}, v_{17}\}$; $B_{3,6} = (A_{2,4} - \{v_{12}\}) \cup \{v_{13}, v_{16}, v_{17}\}$;
$B_{3,7} = (A_{2,2} - \{v_{12}\}) \cup \{v_{14}, v_{15}, v_{17}\}$; $B_{3,8} = (A_{2,2} - \{v_{12}\}) \cup \{v_{14}, v_{16}, v_{17}\}$.

Let D = $\{v_2, v_4, v_7, v_9\}$.
$B_{3,9} = D \cup \{v_{12}, v_{13}, v_{15}, v_{17}\}$; $B_{3,10} = D \cup \{v_{12}, v_{13}, v_{16}, v_{17}\}$; $B_{3,11} = D \cup \{v_{12}, v_{14}, v_{15}, v_{17}\}$; $B_{3,12} = D \cup \{v_{12}, v_{14}, v_{16}, v_{17}\}$;

These are the only minimum co – isolated locating dominating sets of P_{18} containing v_2 and v_{17}, since $|A_{2,i}| = |B_{2,j}| = 6 = 2\left\lfloor\frac{5n+3}{5}\right\rfloor + 2$; i = 1, 2, .., 4 and j =1, 2, …, 12. Let $\mathcal{D}_3 = \{A_{3,1}, \ldots, A_{3,4}, B_{3,1}, B_{3,2}, \ldots, B_{3,12}\}$ and $|\mathcal{D}_3| = 16 = (n + 1)^2$.

The theorem is proved by the method of induction on n, where n ≥ 4.

Let n = 4. In order to construct the minimum co – isolated locating dominating sets of P_{23} containing v_2 and v_{22}, the following sets are defined using the sets in $\mathcal{D}_3$.

Let $A_{4,i} = A_{3,i} \cup \{v_{20}, v_{22}\}$, i = 1, 2, 3, 4;
$A_{4,j} = B_{3,(j-4)} \cup \{v_{20}, v_{22}\}$, j = 5, 6, …, 16;

Let S = $\{v_{17}\} \cup \{v_{20}, v_{22}\}$;
$B_{4,1} = S\cup B_{3,1}$; $B_{4,2} = S\cup B_{3,3}$; $B_{4,3} = S\cup B_{3,4}$;
$B_{4,4} = S\cup B_{39}$; $B_{4,5} = S\cup B_{3,10}$; $B_{4,6} = S\cup B_{3,11}$; $B_{47} = S\cup B_{3,12}$;
$B_{4,8} = \{v_2, v_4, v_7\ v_8, v_{11}, v_{14}, v_{16}, v_{19}, v_{20}, v_{22}\}$; and
$B_{4,9} = \{ v_2, v_4, v_7\ v_8, v_{11}, v_{14}, v_{16}, v_{19}, v_{21}, v_{22}\}$. These are the only minimum co – isolated locating dominating sets of P_{23} containing v_2 and v_{22}, since $|A_{4,i}| = |B_{4,j}| = 10 = 2\left\lfloor\frac{5n+3}{5}\right\rfloor + 2$; i = 1, 2, …, 16 and j = 1, 2, ..., 9.

Let $\mathcal{D}_4=\{A_{4,1}, A_{4,2}, \ldots, A_{4,16}, B_{4,1}, B_{4,2}, \ldots, B_{4,9}\}$ and $|\mathcal{D}_4| = 25 = (n + 1)^2$. Therefore, the result is true for n = 4. Assume that the theorem holds for n = k - 1. Let $\mathcal{D}_{k-1}$ be the set of all minimum co – isolated locating dominating set of $P_{5(k-1)+3}$ containing v_2 and $v_{5(k-1)+2}$, then $\mathcal{D}_{k-1} = \{A_{(k-1),1}, A_{(k-1),2}, A_{(k-1),3}, \ldots, A_{(k-1),r}, B_{(k-1),1}, B_{(k-1),2}, \ldots, B_{(k-1),s}\}$, where
$r = (k-1)^2$; s = 2(k-1)+1 = 2k - 1, k ≥ 5 . Also, $|\mathcal{D}_{k-1}| = k^2$ and $|A_{(k-1),i}| = |B_{(k-1),j}| = 2\left\lfloor\frac{5(k-1)+3}{5}\right\rfloor + 2$; i = 1, 2, …, r and j = 1, 2, …, s. The theorem is to be proved for n = k. In order to construct the minimum co – isolated locating dominating set of P_{5k+3} containing v_2 and v_{5k+2}, the following sets are defined using the sets in $\mathcal{D}_{k-1}$. Let $\mathcal{D}_k$ be the set of all minimum co – isolated locating dominating sets of P_{5k+3} containing v_2 and v_{5k+2}. Then,
$\mathcal{D}_k = \{A_{k,1}, A_{k,2}, A_{k,3}, \ldots, A_{k,l}, B_{k,1}, B_{k,2}, \ldots, B_{k,m}\}$, where $l = k^2$ and m = 2k + 1.
$A_{k,i} = A_{(k-1),i} \cup \{v_{5k}, v_{5k+2}\}$; i = 1, 2, …, k^2;
$B_{k,j} = (B_{(k-1),j} - \{v_{5k-3}\}) \cup \{v_{5k}, v_{5k+2}\}$; j = 1, 2, …, 2k – 1;

The sets $B_{k,2k}$ and $B_{k,2k+1}$ are defined as follows.
If n is odd, $B_{k,2k} = \{v_2, v_4, v_7\ v_9, v_{12}, v_{14}, v_{15}, v_{17}, \ldots, v_{5k}, v_{5k+2}\}$;
$B_{k,2k+1} = \{v_2, v_4, v_7\ v_9, v_{12}, v_{14}, v_{15}, v_{17}, \ldots, v_{5k+1}, v_{5k+2}\}$.

If n is even, $B_{k,2k} = \{v_2, v_4, v_7\ v_8, v_{11}, v_{14}, v_{16}, v_{19}, \ldots, v_{5k}, v_{5k+2}\}$;
$B_{k,2k+1} = \{v_2, v_4, v_7\ v_8, v_{11}, v_{14}, v_{16}, v_{19}, \ldots, v_{5k+1}, v_{5k+2}\}$.

Also, $|A_{k,i}| = |B_{k,j}| = 2\left\lfloor\frac{5k+3}{5}\right\rfloor + 2$; i = 1, 2, …, l; j = 1, 2, …, m and $|\mathcal{D}_k| = l + m = (k+1)^2$. Therefore, there are exactly $(k+1)^2$ minimum co – isolated locating dominating sets of P_{5k+3} containing v_2 and v_{5k+2} with the labellings $v_1, v_2, v_3, \ldots, v_{5k+2}, v_{5k+3}$, where $k \geq 4$. By induction hypothesis, the result is true for $n \geq 4$. Also, for n = 3, the number of minimum co-isolated locating dominating sets of P_{18} is 16.

Theorem: 3.15 For any integer $n \geq 4$, $\gamma_{\text{Dcild}}(P_{5n+3}) = \frac{5n^2+51n-44}{2}$.

Proof: $\gamma_{\text{Dcild}}(P_{5n+3})$ is the sum of the number of minimum co – isolated locating dominating sets of P_{5n+3} containing

(i) v_1 and v_{5n+3}
(ii) v_1 and v_{5n+2}
(iii) v_2 and v_{5n+3}
(iv) v_2 and v_{5n+2}

(a) For (i), the number number of minimum co – isolated locating dominating sets of P_{5n+3} containing v_1 and v_{5n+3} is $\frac{n^2+5n+2}{2}$, by Theorem 3.12.
(b) For (ii), the number of minimum co – isolated locating dominating sets of P_{5n+3} containing v_1 and v_{5n+2} is $\frac{n^2+21n-24}{2}$, by Theorem 3.13.
(c) For (iii), the number of minimum co – isolated locating dominating sets of P_{5n+3} containing v_2 and v_{5n+3} is the same as the number $\frac{n^2+21n-24}{2}$.
(d) For (iv), the number of minimum co – isolated locating dominating sets of P_{5n+3} containing v_2 and v_{5n+2} is $(n+1)^2$, by Theorem 3.14.

Therefore, $\gamma_{\text{Dcild}}(P_{5n+3}) = \frac{(5n^2+51n-44)}{2}$.

Remark: 3.16 The Recurrence relation is given by

$$\gamma_{\text{Dcild}}(P_{5n+3}) - \gamma_{\text{Dcild}}(P_{5(n-1)+3}) = \frac{(5n^2+51n-44)}{2} - \frac{(5n^2+41n-90)}{2} = 5n + 23.$$

Therefore, $\gamma_{\text{Dcild}}(P_{5n+3}) = \gamma_{\text{Dcild}}(P_{5n-2}) + 5n + 23$; $n \geq 4$.

In the following, the number of minimum co-isolated locating dominating sets of P_{5n+4} is found.

Theorem: 3.17 There is exactly one minimum co – isolated locating dominating set of P_{5n+4} containing v_1 and v_{5n+4} with the labellings $v_1, v_2, v_3, \ldots, v_{5n+2}, v_{5n+3}, v_{5n+4}$, where $n \geq 1$.

Proof: Clearly, $D = \{v_1, v_4, v_6, v_9, \ldots, v_{5n+1}, v_{5n+4}\}$ is a minimum co – isolated locating dominating set of P_{5n+4} containing v_1 and v_{5n+4},which proves the existence. To prove the uniqueness, let $D' = D - \{v_{5n+4}\}$. D' is a minimum co – isolated locating dominating set of P_{5n+1} with the labellings $v_1, v_2, v_3, \ldots, v_{5n+1}, v_{5n+2}$ containing v_1 and v_{5n+1} , since, $|D'| = 2n + 1 = \gamma_{\text{cild}}(P_{5n+2})$. But by Theorem 3.9, D' is the unique minimum co – isolated locating dominating set of P_{5n+2} and hence D is unique.

Theorem: 3.18 There are exactly n+1 minimum co – isolated locating dominating sets of P_{5n+4} containing v_2 and v_{5n+4} with the labellings $v_1, v_2, v_3, \ldots, v_{5n+2}, v_{5n+3}, v_{5n+4}$ and $n \geq 0$.

Proof: Let the labellings of vertices of P_{5n+4} be $v_1, v_2, v_3, \ldots, v_{5n+2}, v_{5n+3}, v_{5n+4}$. The theorem is proved by the method of induction on n. For n = 0, $\{v_2, v_4\}$ is the only minimum co – isolated locating dominating set of P_4 containing v_2 and v_4.

For n = 1, $\{v_2, v_4, v_6, v_9\}$ and $\{v_2, v_4, v_7, v_9\}$ are the only minimum co – isolated locating dominating sets of P_9 containing v_2 and v_9.

Assume that the theorem holds for n = k-1. That is, there are exactly k minimum co – isolated locating dominating sets of $P_{5(k-1)+4}$ containing v_2 and $v_{5(k-1)+4}$ with the labellings $v_1, v_2, v_3, \ldots, v_{5(k-1)+2}, v_{5(k-1)+3}, v_{5(k-1)+4}$. Let the k sets be D_i, i = 1,2, …, k and $|D_i| = 2\left\lfloor\frac{5(k-1)+4}{5}\right\rfloor + 2$, by Theorem 2.5.

Assume n = k. Let $D_i' = D_i \cup \{v_{5i+1}, v_{5i+4}\}$; i = 1, 2, 3… k.

Since, $|D_i'| = |D_i| + 2 = 2\left\lfloor\frac{5(k-1)+4}{5}\right\rfloor + 2 + 2 = 2\left\lfloor\frac{5k+4}{5}\right\rfloor + 2$; D_i' are the minimum co – isolated locating dominating set of P_{5k+4}, i = 1, 2, …, k.

In addition, D = {v_2, v_4,v_7, v_9, …, v_{5k+2}, v_{5k+4}} is also a minimum co – isolated locating dominating set of P_{5k+4} since, $|D| = 2\left\lfloor\frac{5k+4}{5}\right\rfloor + 2$, which is different from D_i' for each i. Therefore, there are k + 1 minimum co – isolated locating dominating sets of P_{5k+4} containing v_2 and v_{5k+4}. Therefore, the theorem is proved for n = k. By induction hypothesis, there are exactly n+1 minimum co – isolated locating dominating sets of P_{5n+4} containing v_2 and v_{5n+4} with the labellings v_1, v_2, v_3, …., v_{5n+2}, v_{5n+3}, v_{5n+4} and $n \geq 0$.

Theorem: 3.19 There are exactly $\frac{(n+1)(n+2)}{2}$ minimum co – isolated locating dominating sets of P_{5n+4} containing v_2 and v_{5n+3} with the labellings v_1, v_2, v_3, …, v_{5n+2}, v_{5n+3}, v_{5n+4}, where $n \geq 1$.

Proof: Let the labellings of vertices of P_{5n+4} be v_1, v_2, v_3, ..., v_{5n+2}, v_{5n+3}, v_{5n+4}. The theorem is proved by the method of induction on n. For n = 1, $A_{1,1}$={v_2, v_3, v_6, v_8}; $A_{1,2}$={v_2, v_4, v_6, v_8}; $A_{1,3}$={v_2, v_4, v_7, v_8} are the only minimum co – isolated locating dominating sets of P_9 containing v_2 and v_8, since, $|A_{1,i}| = 4 = 2\left\lfloor\frac{5n+4}{5}\right\rfloor + 2$. Let $\mathcal{D}_1$={$A_{1,1}$, $A_{1,2}$, $A_{1,3}$} and $|\mathcal{D}_1| = 3 = \frac{(n+1)(n+2)}{2}$.

Let n = 2. In order to construct the minimum co – isolated locating dominating sets of P_{14} containing v_2 and v_{13}, the following sets are defined using $\mathcal{D}_1$.

Let $A_{2,i} = A_{1,i} \cup \{v_{11}, v_{13}\}$; i = 1, 2, 3 and
$B_{2,1} = (A_{1,2} - \{v_8\}) \cup \{v_9, v_{11}, v_{13}\}$; $B_{2,2} = (A_{1,3} - \{v_8\}) \cup \{v_9, v_{11}, v_{13}\}$ and
$B_{2,3} = (A_{1,3} - \{v_8\}) \cup \{v_9, v_{12}, v_{13}\}$.

These are the only minimum co – isolated locating dominating sets of P_{14} containing v_2 and v_{13}, since,
$|A_{2,i}| = |B_{2,i}| = 6 = 2\left\lfloor\frac{5n+4}{5}\right\rfloor + 2$.

Let $\mathcal{D}_2 = \{A_{2,1}, A_{2,2}, A_{2,3}, B_{2,1}, B_{2,2}, B_{2,3}\}$ and $|\mathcal{D}_2| = 6 = \frac{(n+1)(n+2)}{2}$.

Let n = 3. To construct the minimum co – isolated locating dominating sets of P_{16} containing v_2 and v_{16} the following sets are defined using the sets in $\mathcal{D}_2$.
$A_{3,i} = A_{2,i} \cup \{v_{16}, v_{18}\}$, i = 1, 2, 3;
$A_{3,j} = B_{2,(j-3)} \cup \{v_{16}, v_{18}\}$, j = 4, 5, 6;
$B_{3,i} = (B_{2,i} - \{v_{13}\}) \cup \{v_{14}, v_{16}, v_{18}\}$; i = 1, 2, 3; and
$B_{3,4} = (B_{2,3} - \{v_{13}\}) \cup \{v_{14}, v_{17}, v_{18}\}$.

These are the only minimum co – isolated locating dominating set of P_{19} containing v_2 and v_{18}, since
$|A_{3,i}| = |B_{3,j}| = 8 = 2\left\lfloor\frac{5n+4}{5}\right\rfloor + 2$; i = 1, 2, …, 6 and j = 1, 2, 3, 4.

Let $\mathcal{D}_3$={$A_{3,1}$, $A_{3,2}$, …, $A_{3,6}$, $B_{3,1}$, $B_{3,2}$, $B_{3,3}$, $B_{3,4}$} and $|\mathcal{D}_3| = 10 = \frac{(n+1)(n+2)}{2}$, when n = 3. Therefore, the result is true for n = 1, 2 and 3.

Assume that the theorem holds for n = k-1. That is, for all paths having 5(k – 1)+4 vertices. Let $\mathcal{D}_{k-1}$ be the set of all minimum co – isolated locating dominating sets of $P_{5(k-1)+2}$ containing v_2 and $v_{5(k-1)+1}$.
Then, $\mathcal{D}_{k-1}$ ={$A_{(k-1),1}$, $A_{(k-1),2}$, $A_{(k-1),3}$, …, $A_{(k-1),r}$, $B_{(k-1),1}$, $B_{(k-1),2}$, …, $B_{(k-1),s}$}, where $r = \frac{(n-1)n}{2}$; s = k, k ≥ 3. Also $|\mathcal{D}_{k-1}| = \frac{k(k+1)}{2}$ and $|A_{(k-1),i}| = |B_{(k-1),j}| = 2\left\lfloor\frac{5(k-1)+4}{5}\right\rfloor + 2$; i = 1, 2, …, r and j = 1, 2, …, s. Let n = k. In order to construct the minimum co – isolated locating dominating sets of P_{5k+4} containing v_2 and v_{5k+3}, the following sets are defined using the sets in $\mathcal{D}_{k-1}$. Let $\mathcal{D}_k$ be the set of all minimum co – isolated locating dominating set of P_{5k+4} containing v_2 and v_{5k+3}.

Then, $\mathcal{D}_k$ = {$A_{k,1}$, $A_{k,2}$, $A_{k,3}$, …, $A_{k,l}$, $B_{k,1}$, $B_{k,2}$, …, $B_{k,m}$}, where $l = \frac{k(k+1)}{2}$ and m = k + 1.
$A_{k,i} = A_{(k-1),i} \cup \{v_{5k+1}, v_{5k+3}\}$, ; for i = 1, 2, …, r;
$A_{k,j} = B_{(k-1),(j-r)} \cup \{v_{5k+1}, v_{5k+3}\}$, for j = r+1, r+2, …, r+s (= l);
$B_{k,i} = (B_{(k-1),i} - \{v_{5k-2}\}) \cup \{v_{5k-1}, v_{5k+1}, v_{5k+3}\}$, i = 1, 2, …, s; and
$B_{k,(k+1)} = (B_{(k-1),k} - \{v_{5k-2}\}) \cup \{v_{5k-1}, v_{5k+2}, v_{5k+3}\}$.

Also, $|A_{k,i}| = |B_{k,j}| = 2\left\lfloor\frac{5k+4}{5}\right\rfloor + 2$; i = 1, 2, …, l; j = 1, 2, …, m and $|\mathcal{D}_k| = l + m = \frac{(k+1)(k+2)}{2}$. Hence, the Theorem is proved for n = k. By induction hypothesis, there are exactly $\frac{(n+1)(n+2)}{2}$ minimum co – isolated locating dominating sets of P_{5n+4} containing v_2 and v_{5n+3} with the labellings $v_1, v_2, v_3, \ldots, v_{5n+2}, v_{5n+3}, v_{5n+4}$, for all n ≥ 1.

Theorem: 3.20 For any integer n ≥ 1, $\gamma_{Dcild}(P_{5n+4}) = \frac{(n^2+7n+8)}{2}$.

Proof: $\gamma_{Dcild}(P_{5n+4})$ is the sum of the number of minimum co – isolated locating dominating sets of P_{5n+4} containing

(i) v_1 and v_{5n+4}
(ii) v_1 and v_{5n+3}
(iii) v_2 and v_{5n+4}
(iv) v_2 and v_{5n+3}

(a). For (i), the number number of minimum co – isolated locating dominating sets of P_{5n+4} containing v_1 and v_{5n+4} is 1 , by Theorem 3.17.

(b). For (ii), the number of minimum co – isolated locating dominating sets of P_{5n+4} containing v_1 and v_{5n+3} is (n+1), by Theorem 3.18.

(c). For (iii), the number of minimum co – isolated locating dominating sets of P_{5n+4} containing v_2 and v_{5n+4} is the same as the number (n + 1)

(d). For (iv), the number of minimum co – isolated locating dominating sets of P_{5n+4} containing v_2 and v_{5n+3} is $\frac{(n+1)(n+2)}{2}$, by Theorem 3.19.

Hence, $\gamma_{Dcild}(P_{5n+4}) = \frac{(n^2+7n+8)}{2}$.

Remark 3.21: The Recurrence relation is given by

$$\gamma_{Dcild}(P_{5n+4}) - \gamma_{Dcild}(P_{5(n-1)+4}) = \frac{(n^2+7n+8)}{2} - \frac{(n^2+5n+2)}{2}.$$
$$= n + 3.$$

Therefore, $\gamma_{Dcild}(P_{5n+4}) = \gamma_{Dcild}(P_{5n-1}) + n + 3$.

4. CONCLUSION

In this paper, the number γ_{Dcild} is obtained for paths P_n, n ≥ 4 are studied.

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