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Fuzzy r-Separation Axioms in Fuzzy Topological Spaces

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ABSTRACT

Here we study the fuzzy regularly open sets [1] and introduce fuzzy δ-open set. Using these sets we define and study the r-separation axioms for topological spaces. Several basic results were introduced and studied for r-separation axioms for topological spaces viz. fuzzy rT_i for i= 0, 1, 2, fuzzy r-regular, fuzzy r-normal spaces.

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Key words: *Fuzzy r-neighbourhood, Fuzzy regularly open sets, Fuzzy r-separation axioms, Fuzzy almost separation axioms.*

1. INTRODUCTION

The concept of Fuzzy set was introduced by Zadeh in 1965 to describe those phenomena which are imprecise, vague or fuzzy in nature. Fuzzy set theory has remarkable array of applications in almost all disciplines. Fuzzy set handle such situation by attributing a degree to which a certain object belongs to a set.

Fuzzy regularly open sets in a fuzzy topological space were introduced by Azad [1] in 1981. Using these concepts Sinhal and Rajvanshi in 1992 introduced almost separation axioms. All undefined fuzzy topological concepts are from [2]. In the present paper we have defined fuzzy r-separation axioms- fuzzy rT_i for i=0, 1, 2 fuzzy r-regularity and fuzzy r-normality in a fts.

2. PRELIMINARIES

We take I= [0, 1].For a fuzzy set $A \in I^X$, co A denotes its fuzzy complement. For $\alpha \in I$, $\underline{\alpha}$ denotes α valued constant fuzzy set. We denote the characteristic function of $Y \subseteq X$ as Y and regard it as an element of I^X.A fuzzy point x_r in X is a fuzzy set in X taking value $r \in (0, 1)$ at x and 0 elsewhere, x and r are called the support and value of x_r respectively. A fuzzy point x_r is said to belong to $A \in I^X$ if $r < A(x)$. Two fuzzy points are said to be distinct if their support are distinct. Given a fuzzy topological space (in short fts) (X, T), $A \in I^X$ is called regularly open if int cl (A) =A [1], also $A \in I^X$ is called regularly closed if co A is regularly open. Clearly A is regularly closed if and only if A=cl int A.

2.1. Proposition: [1] Intersection of two fuzzy regularly open sets is regularly open.

2.2. Proposition: [1] Closure of a fuzzy open set in a fts (X ,T) is fuzzy regularly closed and interior of a fuzzy closed set in X is fuzzy regularly open.

2.3. Definition: [3] Let (X, T) be an fts and x_r be a fuzzy point in X. A fuzzy set A is called a fuzzy r-neighbourhood if there exists a fuzzy regularly open set U such that $x_r \in U \subseteq A$. A fuzzy set A is called quasi r-neighbourhood of a fuzzy singleton x_r in X if there exist a fuzzy regularly open set U such that $x_r q U \subseteq A$.

2.4. Definition: [3] Let (X,T) be a fts ,then the set of all fuzzy regularly open sets forms a base for some topology on X. This fuzzy topology is called the fuzzy semi regularization topology of T and is denoted by T*, clearly $T^* \subseteq T$. (X, T*) is called the fuzzy semi regularization of (X, T).

Azad[1] defined an fts (X,T) to be fuzzy semi regular if and only if the fuzzy regularly open sets of X form a base for the fuzzy topology T on X. Then according to above(X, T *) is fuzzy semi regular if and only if T= T*.

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2.5. Definition: [1] A mapping f :(X, T_1) →(Y, T_2) is called fuzzy almost continuous if the inverse image of every fuzzy regularly open set in Y is fuzzy open in X.

2.6. Definition: [1] A mapping f:(X,T_1)→(Y,T_2) is called fuzzy almost open (closed) if for every fuzzy regularly open (closed) set U in X, f(U) is fuzzy open (closed) in Y.

2.7. Lemma: [3] If f: X→Y is a fuzzy almost continuous, fuzzy almost open map, then the inverse image of every fuzzy regularly open (closed) set is fuzzy regularly open (closed).

3. MAIN RESULTS

In this section we have defined fuzzy δ-open set and fuzzy r-separation axioms. Several results related to fuzzy δ-open set and fuzzy r- separation axioms have been established.

3.1. Definition: A fuzzy set A in a fts is said to be a fuzzy δ –open set in X if it can be expressed as a union of fuzzy regularly open sets in X. A fuzzy set A in X is called a fuzzy δ-closed set if it is a complement of a fuzzy δ –open set in X.

For a fuzzy set A in X, the δ-closure of A is defined as the intersection of all fuzzy δ-closed sets in X which contain A.

3.2. Definition: A fuzzy singleton x_r is called a δ-adherent point of a fuzzy set A in X if every quasi r-neighbourhood of x_r is quasi-coincident with A.

3.3. Proposition: Let (X, T) be an fts and (Y, T_Y) be its subspace where Y is fuzzy regularly open in X, then A∩Y is fuzzy regularly open in Y for any fuzzy regularly open set A in X.

Proof: First we show that if A is a fuzzy set in Y and Y is a fuzzy open in X then
$\mathrm{Int}_X A=\mathrm{int}_Y A$

Consider the two families, $\lambda_1 = \{U_Y \in T_Y: U_Y \subseteq A\}$
$\lambda_2 = \{U \in T: U \subseteq A\}$

Take any $U_Y \in \lambda_1$, then $U_Y = U\cap Y$ for some $U \in T$, but $Y\in T$,

Therefore, $U_Y \in T$ and hence $U_Y \in \lambda_2$.

Conversely, let $U \in \lambda_2$ then since $U \subseteq A \subseteq Y$, $U = U\cap Y$, hence $U \in T_Y$, implying that, $U \in \lambda_1$

Thus $\lambda_1 = \lambda_2$. Hence
$\cup \{ U_Y: U\in \lambda_1\} =\cup \{ U_Y: U\in \lambda_2\}$
$\Rightarrow \mathrm{int}_Y (A)= \mathrm{int}_X (A)$

Now we prove the proposition:

$$
\begin{aligned}
\mathrm{Int}_Y\, \mathrm{cl}_Y (A \cap Y) &= \mathrm{Int}_X\, \mathrm{cl}_Y (A\cap Y) \quad \text{(since Y is fuzzy open in X)}\\
&= \mathrm{Int}_X (Y\cap \mathrm{cl}_X (A\cap Y))\\
&= Y\cap \mathrm{int}_X\, \mathrm{cl}_X (A\cap Y) \quad \text{(since Y is fuzzy open in X)}\\
&= Y \cap(A\cap Y)\\
&= A\cap Y
\end{aligned}
$$

Hence A∩Y is fuzzy regularly open in Y.

3.4. Proposition: In a fts (X, T) a fuzzy set A is fuzzy δ-closed if and only if A= δ-adh A where δ-adh A denots the union of all its δ-adherent points.

Proof: First let us suppose that A is fuzzy δ-closed. To show that A= δ-adhA.

We first show that A⊆ δ-adhA.

Take any fuzzy point $x_r \in A$ then $r<A(x)$.

Consider any quasi-r-nbd of x_r, say N. Then there exists a fuzzy regularly open set U such that x_r q U⊆N. Now x_r q N and hence $r+N(x)> 1 \Rightarrow A(x)+N(x)>1$ since $A(x)>r$, so AqN

Implying that x_r is a δ-adherent point of A. hence $x_r \in$ δ-adh A .Thus A⊆ δ-adh A.

Next we show that δ-adh A⊆A .Take any δ-adherent point of A, say x_r.

Then we show that $x_r \in$ A. For, if not, then r>A(x) if and only if r-A(x) >0
⇔ r + (1-A(x)) >1
⇔x_r q co A

Now since A is fuzzy δ-closed, A=∩ F_i where each F_i is fuzzy regularly closed set containing A. Then coA= ∪co F_i where co F_i, for each i, is a is fuzzy regularly open set contained in A. Now x_r q co A if and only if x_r q (∪co F_i)

i.e r+∪co F_i(x)>1 if and only if ∪co F_i(x)>1-r

i.e. 1-r <∪co F_i(x)= $\sup_i$co F_i(x)
⇒1-r< co F_i(x) for some i, say i_1
⇒1<r+ co F_{i1}(x)

Thus x_r q co F_{i1}, ⇒co F_{i1}, is a fuzzy regularly open quasi-r nbd of x_r. As x_r is a δ-adherent point of A, we must have co F_{i1} q A which implies ∪ coFi q A or (coA) qA which is a contradiction.

Hence $x_r \in$ A implying that ∪{x_r: x_r ∈δ-adhA} ⊆A

i.e. δ-adhA⊆A

Hence A= δ-adh A

Conversely let us assume that A= δ-adhaA. Then we claim that A= ∩{F_i: Fi is a fuzzy regularly closed set containing A}

Obviously A⊆∩F_i .Now we show that ∩Fi⊆A, let x_r be a fuzzy singleton such that x_r⊆ ∩{F_i: Fi is a fuzzy regularly closed set containing A} -------- (1)

Take any fuzzy regularly open set U such that X_rqU. Then r+ U(x)>1 or r>1-U(x) i.e.r>coU(x) but this implies that x_r∉coU⇒coU is a fuzzy regularly closed set which does not contain A ie A⊈coU(Since in view of (1) , x_r∈F_i for each fuzzy regularly closed set F_i containing A)i.e. there must exist some z∈X such that co U(z)<A(z)
⇒1-U (z) < A (z)
⇒A (z) +U (z) >1
⇒UqA

Which further implies that x_r is a δ-adherent point of A and therefore since A= δ-adh A, we have x_r∈A thus ∩F_i⊆A therefore A=∩F_i and hence A is fuzzy δ-closed.

3.5. Definition: Fuzzy r - separation axioms in a fuzzy topological space. Now we define Fuzzy r - separation Axioms in a fuzzy topological space. A fts (X, T) is called

(a) Fuzzy r-T_0 if for every x, y in X, x≠y then there exists a fuzzy regularly open set U such that U(x)≠U(y).
(b) Fuzzy r-T_1 if for every x, y in X, x≠y then there exists a fuzzy regularly open set U such that U(x) =1, U(y) = 0.
(c) Fuzzy r-T_2 if for every pair of distinct fuzzy points x_r, y_s in X, then there exists fuzzy regularly open sets U and V such that x_r∈U, y_s∈ V and U∩V=∅.
(d) Fuzzy r-regular if for every fuzzy point x_r in X and each fuzzy closed set F such that x_r ∈coF, there exists fuzzy regularly open sets U and V such that x_r∈U, F⊆ V and U⊆coV.
(e) Fuzzy r-normal if for every pair of fuzzy closed sets F_1,and F_2 such that, F_1⊆coF_2, there exists fuzzy regularly open sets U and V such that F_1⊆U, F_2⊆ V and U⊆coV.

Now we prove some results related to fuzzy r-separation axioms.

Theorem: 3.6 If (X, T) is fuzzy rT_1 then every fuzzy singleton x_r in X is fuzzy closed.

Proof: We have to show that {x_r} is fuzzy closed or X-{x_r} is fuzzy open. Now choose any fuzzy point y_r in X-{x_r} then y ≠ x as (X, T) is rT_1 so ∃ regularly open set U such that U(x) = 0, U(y) =1 so y_r∈ U ⊆X-x_r, so X-{x_r} is fuzzy open i.e. {x_r} is fuzzy closed.

3.7. Theorem: Let (X, T) be an fts, then (X, T) is fuzzy rT_i if and only if its semi-regularization (X, T*) is fuzzy T_i for i=0, 1, 2.

Proof: Let (X,T)be a fuzzy r-T_0 space, then for every x, y in X, $x \neq y$ there exists fuzzy regularly open set U such that $U(x) \neq U(y)$.But every fuzzy regularly open set in (X,T) is fuzzy open in (X,T*) therefore U is a fuzzy open set in (X, T*) such that $U(x) \neq U(y)$ and hence (X, T*) is fuzzy T_0. Conversely, let (X, T*) be fuzzy T_0, then $\forall$ x, y $\in$ X, x≠y, $\exists$ U $\in$ T* such that $U(x) \neq U(y)$. Here U is of the form $\cup_{i\in A} U_i$ where each U_i is fuzzy regularly open set in X. hence, $\cup U_i(x) \neq \cup U_i(y)$ which implies that $\exists i \in A$ such that $U_i(x) \neq U_i(y)$ showing that (X, T) is fuzzy r T_0.

Next, let (X, T) be fuzzy r T_1, then $\forall$ x, y $\in$ X, x≠y, $\exists$ a fuzzy regularly open set U such that U(x)=1, U(y)=0. Obviously U $\in$ T* hence (X, T*) is fuzzy T_1. Conversely, if (X, T*) is fuzzy T_1 then $\exists$ U $\in$ T* such that U(x) =1, U(y) =0 and clearly any U is of the form $U = \cup_{i\in A} U_i$ thus $\cup_{i\in A} U_i (x) = 1$ thus $\sup_{i\in A} U_i(x) = 1$, so $U_i(x) = 1$ for some i And $\cup_{i\in A} U_i (y) = 0$ thus $\sup_{i\in A} U_i(y) = 0$ so $U_i(y) = 0$ so $\exists$ regularly open set U_i such that $U_i (x) = 1$, $U_i(y) = 0$ so (X, T) is rT_1.

Again, let (X, T) be fuzzy r T_2. Choose fuzzy points x_r, y_s in X,x≠y then $\exists$ fuzzy regularly open sets U, V in T such that $x_r \in U$, $y_s \in V$ and $U \cap V = \phi$. Now since U and V are fuzzy open in (X, T*) therefore (X, T*) be fuzzy T_2.

Conversely let (X, T*) be fuzzy T_2. Choose distinct fuzzy points x_r, y_s in X, then $\exists$ U, V $\in$ T* such that $x_r \in U$, $y_s \in V$, $U \cap V = \phi$. Here, $U = \cup_{i\in A1} U_i$, $V = \cup_{j\in A2} V_j$ where U_i's and V_j's are fuzzy regularly open in X. thus we have $x_r \in \cup_i U_i$, $y_s \in \cup_j V_j$ and$(\cup U_i) \cap (\cup V_j) = \phi$ which implies that $\exists$ $i \in A_1$ $j \in A_2$ such that $x_r \in U_i$, $y_s \in V_j$ and $U_i \cap V_j = \emptyset$ showing that (X, T) is fuzzy rT_2.

3.8. Theorem: Let f:(X, T)→(Y, S) be an injective, fuzzy almost continuous and fuzzy almost open mapping then (X, T) is fuzzy rT_i if (Y, s) is fuzzy rT_i for i=0,1,2.

Proof: Let f: (X, T)→(Y, S) be an injective, fuzzy almost continuous and fuzzy almost open map. First we show that (X, T) is fuzzy rT_0 if (Y, S) is fuzzy rT_0. To show this let x, y$\in$ X, $x \neq y$, since f is injective f(x), f(y) $\in$ Y, and f(x)≠f(y). Again since Y is fuzzy rT_0,$\exists$ a fuzzy regularly open set U such that

$U (f(x)) \neq U (f(y))$

$\Rightarrow f^{-1}(U) (x) \neq f^{-1}(U) (y)$

Since f is fuzzy almost continuous and fuzzy almost open map by using Lemma (2.7), $f^{-1}(U)$ is fuzzy regularly open in X. Hence $\exists$ a fuzzy regularly open set $f^{-1}(U)$ in X such that $f^{-1}(U) (x) \neq f^{-1}(U) (y)$ which implies that X is fuzzy rT_0

Next let x, y $\in$ X, where $x \neq y$ then f(x), f(y) $\in$ Y, f(x) ≠f(y). If (Y, S) is fuzzy rT_1, $\exists$ a fuzzy regularly open set U in Y such that U (f(x)) =1, U (f(y)) =0

$\Rightarrow f^{-1}(U(x)) =1, f^{-1}(U(y)) =0$

$\Rightarrow$ $f^{-1}(U)$ is fuzzy regularly open set in X such that $f^{-1}(U(x)) =1$, $f^{-1}(U(y)) =0$

Thus (X, T) is fuzzy rT_1.

Now we show that (X, T) is fuzzy rT_2 if (Y, S) is fuzzy rT_2. Let x_r, $y_s \in$ X, x≠y.

Since f is injective f (x_r), f (y_s) $\in$ Y, f $(x_r) \neq$ f (y_s)

Since Y is rT_2 $\exists$ fuzzy regularly open sets U and V such that f $(x_r) \in$ U, f $(y_s) \in$ V and $U \cap V = \emptyset$

This implies that $x_r \in f^{-1}(U)$, $y_s \in f^{-1}(V)$

Since f is fuzzy almost continuous and fuzzy almost open mapping by using Lemma $f^{-1}(U)$ and $f^{-1}(V)$ are fuzzy regularly open sets in X. Again $f^{-1}(U) \cap f^{-1}(V) = f^{-1}(U \cap V) = f^{-1}\emptyset = \emptyset$

Thus (X, T) is fuzzy rT_2.

3.9. Theorem: Every fuzzy regularly open subspace of
(i) A fuzzy rT_i space is fuzzy rT_i for i=0, 1, 2.
(ii) A fuzzy r-regular space is fuzzy r-regular

Proof: (i) first we show that every fuzzy regularly open subspace of a fuzzy rT_0- space is fuzzy rT_0. Let (X, T) be fuzzy rT_0 space and let (Y, T_Y) be its subspace where Y is fuzzy regularly open. Let x, y $\in$ Y$\subseteq$ X, $x \neq y$. Since X is fuzzy rT_0 $\exists$ a fuzzy regularly open set U such that $U (x) \neq U(y)$.

Now $U_Y = U \cap Y$ being intersection of two fuzzy regularly sets is again fuzzy regularly open. Also it can be easily seen that $U_Y(x) \neq U_Y(y)$ implying that (Y, T_Y) is fuzzy rT_0.

Next Let Y be fuzzy regularly open subspace of X and let $x, y \in Y \subseteq X$, where $x \neq y$, and Y is a subspace of X.

So $\exists$ a fuzzy regularly open set U in X such that U(x)=1 ,U(y)=0 (as X is rT_1)

Now take $U_Y = U \cap Y$ then $U_Y(x) = U \cap Y(x) = \inf\{U(x), Y(x)\} = 1$ and $U_Y(y) = U \cap Y(y) = \inf\{U(y), Y(y)\} = 0$

This U_Y is regularly open such that $U_Y(x) = 1$, $Uy(y) = 0$

Thus, (Y, T_Y) is fuzzy rT_1

Next, let (X, T) be fuzzy rT_2 and let (Y, T_Y) be its fuzzy regularly open subspace. Since X is fuzzy rT_2 $\forall$ $x_r, y_s \in Y \subseteq X$, $x \neq y$. $\exists$ fuzzy regularly open sets U and V such that $x_r \in U$, $Y_s \in V$ and $U \cap V = \emptyset$. Now $U_Y = U \cap Y$ and $V_Y = V \cap Y$ being intersections of fuzzy regularly open sets is itself fuzzy regularly open. Thus we have $\forall$ $x_r, y_s \in Y$, $\exists$ fuzzy regularly open sets U_Y, V_Y such that $x_r \in U_y$, $Y_s \in V_Y$ and $U_Y \cap V_Y = \emptyset$

Hence (Y, T_y) is fuzzy rT_2.

(ii): Let x_r be a fuzzy point in Y, where Y is a subspace of X and F is a fuzzy closed set in Y such that $x_r \in$ co F so $\exists$ fuzzy regularly open sets U and V such that $x_r \in U$, $F \subseteq V$ and $U \subseteq$ co V.

As X is fuzzy r regular and Y is fuzzy regularly open subspace of X, therefore $\exists$ fuzzy regularly open sets $U_Y = U \cap Y$, $V_Y = V \cap Y$ such that $x_r \in U \cap Y$, $F \subseteq V \cap Y$ and $U \cap Y \subseteq co(V \cap Y)$. Thus (Y, Ty) is fuzzy r regular

3.10. Theorem: Every fuzzy open subspace of
- (i) a fuzzy rT_i space is fuzzy T_i for i=0, 1, 2.
- (ii) a fuzzy r-regular space is fuzzy regular open

Proof: The Proof is on similar lines as in previous theorem.

3.11. Theorem: An fts (X, T) is fuzzy rT_2 if and only if the diagonal set Δ_x is fuzzy closed in $(X \times X, T^* \times T^*)$.

Proof: Let (X, T) be fuzzy rT_2. Consider $X \times X - \Delta_x$. Let $(x,y)_r$ be a fuzzy point in $X \times X - \Delta_x$. then x_r , y_r are distinct fuzzy points in X. therefore $\exists$ fuzzy regularly open sets U and V such that $x_r \in U$, $y_r \in V$ and $U \cap V = \emptyset$ Consider now $U \times V$, then $U \times V$ is a basic fuzzy open set in $(X \times X, T^* \times T^*)$ and is such that $(x, y)_r \in U \times V \subseteq X \times X - \Delta_x$ and hence $X \times X - \Delta_x$ is fuzzy open i.e. Δ_x is fuzzy closed in $(X \times X, T^* \times T^*)$.

Conversely, let Δ_x be fuzzy closed in $T^* \times T^*$, then $X \times X - \Delta_x$ is fuzzy open in $(X \times X, T^* \times T^*)$.

Now choose distinct fuzzy points x_r , y_s in X. let $s \leq r$. then $(x, y)_r$ is a fuzzy point in $X \times X - \Delta_x$ therefore $\exists$ a basic fuzzy open set $U \times V$ in $(X \times X, T^* \times T^*)$ such that $(x, y)_r \in U \times V \subseteq X \times X - \Delta_x$. here $U = \cup_{i \in A1} U_i$, $V = \cup_{j \in A2} V_j$ where each U_i and each V_j are fuzzy regularly open in X. so, $(x, y)_r \in \cup U_i \times \cup V_j \subseteq X \times X - \Delta_x$ which implies that $\exists$ iand j such that $(x, y)_r \in U_i \times V_j \subseteq X \times X - \Delta_x$ and hence we have found fuzzy regularly open sets U_i and V_j such that $x_r \in U_i$, $y_r \in V_j$ and $U_i \cap V_j = \phi$. Further, since $s \leq r$, we have $x_r \in U_i$, $y_s \in V_j$ and $U_i \cap V_j = \phi$ showing that (X, T*) is fuzzy rT_2.

3.12. Theorem: In an fts (X, T) the following are equivalent:
- (i) (X, T) is fuzzy r-regular
- (ii) For each fuzzy point x_r and each fuzzy open set V such that $x_r \in V$, $\exists$ a fuzzy regularly open set U such that $x_r \in U \subseteq cl\ U \subseteq V$.

Proof:

(i) ⇒ (ii)

Let x_r be a fuzzy point and let V be a fuzzy open set such that $x_r \in V$. then co V is a fuzzy closed set such that $x_r \in$ co (co V). Now since (X, T) is fuzzy r- regular, $\exists$ fuzzy regularly open sets U and W such that $x_r \in U$, co $V \subseteq W$ and $U \subseteq$ co W, thus $x_r \in U \subseteq coW \subseteq V$ but co W is a fuzzy closed set containing U, therefore $x_r \in U \subseteq cl\ U \subseteq V$

(ii)⇒ (i):

Let x_r be a fuzzy point and let F be a closed set such that $x_r \in$ co F. Then co F is a fuzzy open set such that $x_r \in$ co F, therefore $\exists$ a fuzzy regularly open set U such that $x_r \in U \subseteq cl\ U \subseteq$ co F consider now the fuzzy regularly open sets U and U_1 where $U_1 = 1 - cl\ U$, here U_1 is fuzzy regularly open . then $x_r \in U$, $F \subseteq U_1$ and $U \subseteq co\ U_1$ since for any $z \in X$, $U(z) + U_1(z) = U(z) + 1 - cl\ U(z)$ which is ≤ 1. Therefore (X, T) is fuzzy r-regular.

3.13. Theorem: in an fts (X, T) the following statements are equivalent:

(i) (X, T) is fuzzy r-normal.

(ii) ∀ Fuzzy closed set F and fuzzy open set V such that F⊆V ∃ fuzzy regularly open set U such that F⊆ U⊆cl U⊆V.

Proof:

(i) ⇒ (ii)

Let (X, T) be normal space. Let F be any fuzzy closed set and V be a fuzzy open set such that F⊆V, therefore ∃ fuzzy regularly open sets U and U_1 such that F⊆U, co V⊆ U_1, and U⊆co U_1, thus F⊆U⊆co U⊆V. since co U_1 is a fuzzy regularly closed set and hence a fuzzy closed set containing U, we have F⊆U⊆cl U⊆V.

(ii)⇒ (i)

Let F_1 and F_2 be any two fuzzy closed sets such that F_1⊆ co F_2, then ∃ a fuzzy regularly open set U such that F_1 ⊆U⊆ cl U⊆co F_2. Consider now the fuzzy regularly open sets U and U_1 where U_1 =1-cl U here U_1 is fuzzy regularly open. Then we have F_1 ⊆U, F_2 ⊆U_1 and U⊆co U_1 since for any z ∈ X, U (z) +U_1 (z) = U (z) +1-cl U (z) which is ≤1. Hence (X, T) is fuzzy r normal.

REFERENCES

1. Azad, K. K., on fuzzy semi-continuity, fuzzy almost continuity and fuzzy weakly continuity, J. Math. Anal. Appl. 82 (1981)14-32.
2. Lowen, R., Fuzzy topological spaces and fuzzy compactness, J. Math. Anal. Appl. 56(1976) 621-633.
3. Mukharjee, M. N. and Ghosh B., Fuzzy semi regularization topologies and fuzzy submaximal spaces, Fuzzy sets and systems 44(1991) 283-294.
4. Singhal H.K. and Rajvanshi N., Regularly open sets in fuzzy topological spaces, Fuzzy sets and systems 50 (1992) 343-353.
5. Zadeh, L. A., Fuzzy sets, Information and control 8 (1965) 338-353.

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