# International Journal of Mathematical Archive-6(4), 2015, 88-91 NMA Available online through www.ijma.info ISSN 2229-5046 <br> RAINBOW CONNECTION NUMBER OF HELM GRAPH 

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#### Abstract

A path between two vertices is a rainbow path if no two edges in this path have the same color. If there is rainbow path between every pair of vertices then the coloring is rainbow coloring. Minimum number of colors required to achieve rainbow connection number. It is denoted by rc (G) where $G$ is a graph. In this paper we found the rainbow connection number for Helm graph.


Key words: Rainbow coloring, Rainbow path, Rainbow connection number, Helm graph.
AMS Subject Classification: 05C15.

## 1. INTRODUCTION.

All graphs in this paper are finite and simple. Let $G$ be a non trivial connected graph on which an edge coloring c: $E(G) \rightarrow\left\{c_{1}, c_{2}, c_{3}, \ldots, c_{n}\right\}$, $n$ belongs to $N$, is defined where adjacent edges may be colored the same.

Definition: 1.1 A path in an edge colored graph is said to be a rainbow path if no two edges on the path have the same color.

Definition: 1.2 An edge colored graph is rainbow connected if there exists a rainbow path between every pair of vertices.

Definition: 1.3 The rainbow connection number of a graph $G$, denoted by rc $(G)$ is the minimum number of colors that are required in order to make $G$ rainbow connected.

Definition: 1.4 The wheel graph $W_{n}$ is defined to be the join of $K_{1}+C_{n}$. The vertex corresponding to $K_{1}$ is known as Apex vertex and vertices corresponding to cycle are known as rim vertices while the edges corresponding to cycle are known as rim edges [2]. We continue apex of wheel as the apex of respective graphs obtained from the wheel.

Definition: 1.5 The Helm graph $H_{n}$ is the graph obtained from a wheel $W_{n}$ by attaching a pendant edge to each rim vertex [2].

The concept of rainbow connection number was first introduced by Chartrand et al. [1] found rainbow connection numbers and strong rainbow connections for all complete multipartite graphs $G$ as well as other classes of graphs, for every pair a , b of integers with $\geqq 3$ and $\mathrm{b} \geq(5 \mathrm{a}-6) / 3$, there exists a connected graph $G$ such that $\mathrm{rc}(\mathrm{G})=\mathrm{a}$ and $\operatorname{src}(G)=b$.
A.Sudhakaraiah et al. [3] shown rainbow connection number is less than or equal to the diameter of the graph G plus one.
R.J. Faudree et al. [4] proved that for $d \geq 4, d \neq 5$ the edges of the d-dimensional cube can be colored by d colors so that all quadrangles have four distinct colors.

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In 2011, Li et al. [5] attempted to bring together most of the results and papers that deal with the concept of rainbow connection which was introduced by Chartrand in 2008 [1], worked on strong rainbow connection number, Rainbow k-connectivity, k-rainbow index, rainbow vertex-connection number algorithms and computational complexity. Some conjectures, open problems, questions are also included.

The concept of rainbow connection has several interesting variants. Yuefang Sun [6] investigated the rainbow vertexconnection number of a graph according to some structural conditions of its complementary graph $\overline{\mathrm{G}}$. Next, he investigated graphs with large rainbow vertex-connection numbers. And then, he derived a sharp upper bound for rainbow vertex-connection numbers of line graphs. For rainbow total-connection, he determined the precise values for rainbow total-connection numbers of some special graph classes including complete graphs, trees, cycles and wheels.

Motivated by these papers, we started to work on the rainbow connection number for certain classes of graphs.

## 2. RAINBOW COLORING OF HELM GRAPH.

In this section, we have given a coloring algorithm for Helm graph and we have proved it is a rainbow coloring in the following theorem.

### 2.1 Coloring algorithm

Input: $\mathrm{H}_{\mathrm{n}}$
$\mathrm{E} \leftarrow\left\{\mathrm{vv}_{\mathrm{k}}(1 \leq \mathrm{k} \leq \mathrm{n}), \mathrm{v}_{\mathrm{k}} \mathrm{v}_{\mathrm{k}+1}(1 \leq \mathrm{k} \leq \mathrm{n}-1), \mathrm{v}_{\mathrm{n}} \mathrm{v}_{1}, \mathrm{v}_{\mathrm{k}} \mathrm{v}_{\mathrm{n}+\mathrm{k}}(1 \leq \mathrm{k} \leq \mathrm{n})\right\}$
for $\mathrm{k}=1$ to $\mathrm{n}-1$
\{
$\mathrm{vv}_{\mathrm{k}} \leftarrow \mathrm{c}_{\mathrm{k}+1} ;$
\}
end for
$\mathrm{v}_{\mathrm{n}} \leftarrow \mathrm{c}_{1}$;
for $\mathrm{k}=1$ to n
\{
$\mathrm{v}_{\mathrm{k}} \mathrm{V}_{\mathrm{k}+\mathrm{n}} \leftarrow \mathrm{c}_{\mathrm{k}} ;$
\}
end for
for $\mathrm{k}=1$ to $\mathrm{n}-1$
\{
$\mathrm{i}=\mathrm{k}+2$;
if $\mathrm{i} \leq \mathrm{n}$,
$\mathrm{v}_{\mathrm{k}} \mathrm{v}_{\mathrm{k}+1} \leftarrow \mathrm{c}_{\mathrm{i}}$;
else
$\mathrm{v}_{\mathrm{k}} \mathrm{V}_{\mathrm{k}+1} \leftarrow \mathrm{c}_{\mathrm{i}-\mathrm{n}} ;$
$\}$
end for
$\mathrm{v}_{\mathrm{n}} \mathrm{V}_{1} \leftarrow \mathrm{c}_{2} ;$
end procedure
Output: edge colored $\mathrm{H}_{\mathrm{n}}$


Figure 1: Helm graph $\mathrm{H}_{5}$


Figure 2: Helm graph $\mathrm{H}_{6}$
2.2 Theorem: Rainbow connection number of the Helm graph $H_{n}$ is $n$ i.e. rc $\left(H_{n}\right)=n, n \geq 3$.

Proof: Color the edges of $\mathrm{H}_{\mathrm{n}}$ as given in the coloring algorithm [2.1]. See the figures 1 and 2 for reference.
Case 1: Consider the vertices $v$ and $v_{k}(1 \leq k \leq n-1)$. These are at a distance 1 and there is a rainbow path $v_{\mathrm{k}}$ which is colored with $c_{k+1}$ and for vertices $v$ and $v_{n}$ there is a rainbow path of length 1 colored with $c_{1}$. Therefore there is a rainbow path between $v$ and $v_{n} \& v$ and $v_{k}(1 \leq k \leq n-1)$.

Case 2: Consider the vertices $v$ and $v_{n+k}(1 \leq k \leq n-1)$ These vertices are at a distance 2 and there is a rainbow path $v v_{k} v_{n+k}$ whose edges are colored with $c_{k+1}, c_{k}$ and for vertices $v$ and $v_{2 n}$, there is a rainbow path $v v_{n} v_{2 n}$, colored with $c_{1}, c_{n}$. Therefore there is a rainbow path between $v$ and $v_{n+k}(1 \leq k \leq n-1) \& v$ and $v_{2 n}$.

Case 3: Consider the vertices $\mathrm{v}_{\mathrm{j}}$ and $\mathrm{v}_{\mathrm{k}}(1 \leq \mathrm{j}<\mathrm{k} \leq \mathrm{n})$ there is a rainbow path $\mathrm{v}_{\mathrm{j}} \mathrm{v} \mathrm{v}_{\mathrm{k}}$ whose edges are colored with $\mathrm{c}_{\mathrm{j}+1} \mathrm{c}_{\mathrm{k}+1}$. Therefore there is a rainbow path between $\mathrm{v}_{\mathrm{j}}$ and $\mathrm{v}_{\mathrm{k}}(1 \leq \mathrm{j}<\mathrm{k} \leq \mathrm{n})$.

Case 4: Consider the vertices $\mathrm{v}_{\mathrm{k}}$ and $\mathrm{v}_{\mathrm{n}+\mathrm{k}}(1 \leq \mathrm{k} \leq \mathrm{n})$ and there is a rainbow path $\mathrm{v}_{\mathrm{k}} \mathrm{v}_{\mathrm{n}+\mathrm{k}}$ whose edges are colored with $c_{k}$. Therefore there is a rainbow path between vertices $v_{k}$ and $v_{n+k}(1 \leq k \leq n)$.

Case 5: Consider the vertices $v_{n+k}$ and $v_{n+k+1}(1 \leq k \leq n-2)$ and there is a rainbow path $v_{n+k} v_{k} v_{k+1} v_{n+k+1}$ whose edges are colored with $c_{k}, c_{k+2}, c_{k+1}$. For the vertices $v_{2 n-1}$ and $v_{2 n}$ there is a rainbow path $v_{2 n-1} v_{n-1} \quad v_{n} \quad v_{2 n}$ whose edges are colored with $c_{n-1}, c_{1}, c_{n}$. For the vertices $v_{2 n}$ and $v_{n+1}$ there is a rainbow path $v_{2 n} v_{n} v_{1} V_{n+1}$ whose edges are colored with $c_{n}, c_{2}, c_{1}$.Therefore there is a rainbow path between $v_{n+k}$ and $v_{n+k+1}(1 \leq k \leq n-2) \& v_{2 n-1}$ and $v_{2 n} \& v_{2 n}$ and $v_{n+1}$.

Case 6: Consider the vertices $v_{n+j}$ and $v_{n+k}(1 \leq j<k \leq n)$ and there is a rainbow path $v_{n+j} v_{j} v v_{k} v_{n+k}$ whose edges are colored with $c_{j}, c_{j+1}, c_{k+1}, c_{k}$. Therefore there is a rainbow path between $v_{n+j}$ and $v_{n+k}(1 \leq j<k \leq n)$

In all the above cases, there exists a rainbow path between every pair of vertices of $H_{n}$. Therefore, the coloring given in the algorithm [2.1] is rainbow coloring of $\mathrm{H}_{\mathrm{n}}$. All Pendant edges should have different colors, since otherwise there is no rainbow path between these Pendant vertices. Therefore, we need minimum ' $n$ ' colors for rainbow coloring of $H_{n}$. Hence, rc $\left(H_{n}\right)=n$, for $n \geq 3$.

## 3. CONCLUSION

In this paper we found the rainbow connection number of the Helm graph $H_{n}$ is ' $n$ '.

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