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RAINBOW CONNECTION NUMBER OF HELM GRAPH

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ABSTRACT

A path between two vertices is a rainbow path if no two edges in this path have the same color. If there is rainbow path between every pair of vertices then the coloring is rainbow coloring. Minimum number of colors required to achieve rainbow connection number. It is denoted by rc (G) where G is a graph. In this paper we found the rainbow connection number for Helm graph.

Key words: *Rainbow coloring, Rainbow path, Rainbow connection number, Helm graph.*

AMS Subject Classification: *05C15.*

1. INTRODUCTION.

All graphs in this paper are finite and simple. Let G be a non trivial connected graph on which an edge coloring c: E (G) → $\{c_1, c_2, c_3, \text{-----}, c_n\}$, n belongs to N, is defined where adjacent edges may be colored the same.

Definition: 1.1 A path in an edge colored graph is said to be a rainbow path if no two edges on the path have the same color.

Definition: 1.2 An edge colored graph is rainbow connected if there exists a rainbow path between every pair of vertices.

Definition: 1.3 The rainbow connection number of a graph G, denoted by rc (G) is the minimum number of colors that are required in order to make G rainbow connected.

Definition: 1.4 The wheel graph W_n is defined to be the join of $K_1 + C_n$. The vertex corresponding to K_1 is known as Apex vertex and vertices corresponding to cycle are known as rim vertices while the edges corresponding to cycle are known as rim edges [2]. We continue apex of wheel as the apex of respective graphs obtained from the wheel.

Definition: 1.5 The Helm graph H_n is the graph obtained from a wheel W_n by attaching a pendant edge to each rim vertex [2].

The concept of rainbow connection number was first introduced by Chartrand *et al.* [1] found rainbow connection numbers and strong rainbow connections for all complete multipartite graphs G as well as other classes of graphs, for every pair a, b of integers with a ≥ 3 and b ≥ (5a-6) / 3, there exists a connected graph G such that rc (G) = a and src (G) = b.

A.Sudhakaraiah *et al.* [3] shown rainbow connection number is less than or equal to the diameter of the graph G plus one.

R.J. Faudree *et al.* [4] proved that for d ≥ 4, d ≠ 5 the edges of the d-dimensional cube can be colored by d colors so that all quadrangles have four distinct colors.

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In 2011, Li *et al.* [5] attempted to bring together most of the results and papers that deal with the concept of rainbow connection which was introduced by Chartrand in 2008 [1], worked on strong rainbow connection number, Rainbow k-connectivity, k-rainbow index, rainbow vertex-connection number algorithms and computational complexity. Some conjectures, open problems, questions are also included.

The concept of rainbow connection has several interesting variants. Yuefang Sun [6] investigated the rainbow vertex-connection number of a graph according to some structural conditions of its complementary graph $\bar{G}$. Next, he investigated graphs with large rainbow vertex-connection numbers. And then, he derived a sharp upper bound for rainbow vertex-connection numbers of line graphs. For rainbow total-connection, he determined the precise values for rainbow total-connection numbers of some special graph classes including complete graphs, trees, cycles and wheels.

Motivated by these papers, we started to work on the rainbow connection number for certain classes of graphs.

2. RAINBOW COLORING OF HELM GRAPH.

In this section, we have given a coloring algorithm for Helm graph and we have proved it is a rainbow coloring in the following theorem.

2.1 Coloring algorithm

Input: H_n
$E \leftarrow \{vv_k\ (1 \le k \le n),\ v_k v_{k+1}\ (1 \le k \le n-1),\ v_n v_1,\ v_k v_{n+k}\ (1 \le k \le n)\}$
for k = 1 to n -1
{
$vv_k \leftarrow c_{k+1}$;
}
end for
$vv_n \leftarrow c_1$;
for k =1 to n
{
$v_k v_{k+n} \leftarrow c_k$;
}
end for
for k = 1 to n-1
{
i = k + 2;
if $i \le n$,
$v_k v_{k+1} \leftarrow c_i$;
else
$v_k v_{k+1} \leftarrow c_{i-n}$;
}
end for
$v_n v_1 \leftarrow c_2$;
end procedure
Output: edge colored H_n

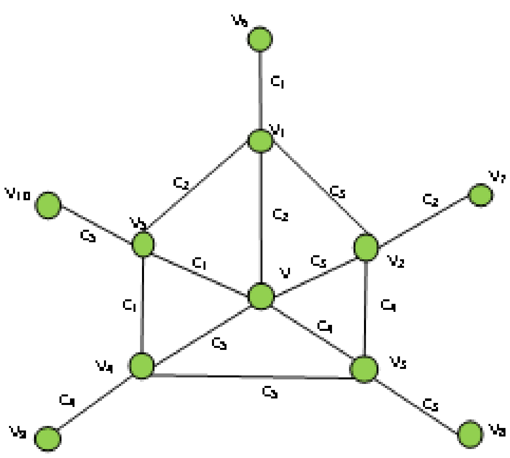

Figure 1: Helm graph H_5

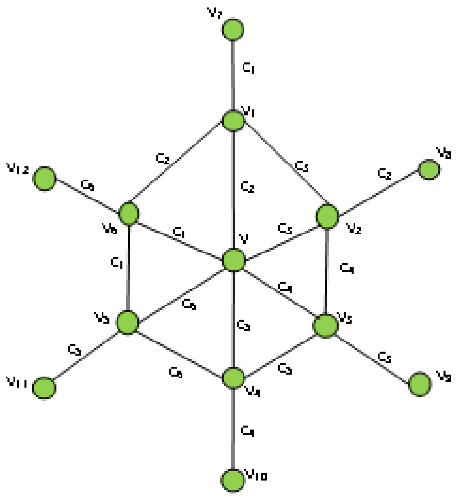

Figure 2: Helm graph H_6

2.2 Theorem: Rainbow connection number of the Helm graph H_n is n i.e. rc $(H_n) = n$, $n \geq 3$.

Proof: Color the edges of H_n as given in the coloring algorithm [2.1]. See the figures 1 and 2 for reference.

Case 1: Consider the vertices v and v_k $(1 \leq k \leq n\text{-}1)$. These are at a distance 1 and there is a rainbow path vv_k which is colored with c_{k+1} and for vertices v and v_n there is a rainbow path of length 1 colored with c_1. Therefore there is a rainbow path between v and v_n & v and v_k $(1 \leq k \leq n\text{-}1)$.

Case 2: Consider the vertices v and v_{n+k} $(1 \leq k \leq n\text{-}1)$ These vertices are at a distance 2 and there is a rainbow path v v_k v_{n+k} whose edges are colored with c_{k+1}, c_k and for vertices v and v_{2n}, there is a rainbow path v v_n v_{2n}, colored with c_1, c_n. Therefore there is a rainbow path between v and v_{n+k} $(1 \leq k \leq n\text{-}1)$ & v and v_{2n}.

Case 3: Consider the vertices v_j and v_k $(1 \leq j < k \leq n)$ there is a rainbow path v_j v v_k whose edges are colored with c_{j+1} c_{k+1} . Therefore there is a rainbow path between v_j and v_k $(1 \leq j < k \leq n)$.

Case 4: Consider the vertices v_k and v_{n+k} $(1 \leq k \leq n)$ and there is a rainbow path v_k v_{n+k} whose edges are colored with c_k. Therefore there is a rainbow path between vertices v_k and v_{n+k} $(1 \leq k \leq n)$.

Case 5: Consider the vertices v_{n+k} and v_{n+k+1} $(1 \leq k \leq n\text{-}2)$ and there is a rainbow path v_{n+k} v_k v_{k+1} v_{n+k+1} whose edges are colored with c_k , c_{k+2} , c_{k+1} . For the vertices v_{2n-1} and v_{2n} there is a rainbow path v_{2n-1} v_{n-1} v_n v_{2n} whose edges are colored with c_{n-1}, c_1, c_n . For the vertices v_{2n} and v_{n+1} there is a rainbow path v_{2n} v_n v_1 v_{n+1} whose edges are colored with c_n, c_2, c_1 .Therefore there is a rainbow path between v_{n+k} and v_{n+k+1} $(1 \leq k \leq n\text{-}2)$ & v_{2n-1} and v_{2n} & v_{2n} and v_{n+1}.

Case 6: Consider the vertices v_{n+j} and $v_{n+k}(1 \leq j < k \leq n)$ and there is a rainbow path v_{n+j} v_j v v_k v_{n+k} whose edges are colored with c_j, c_{j+1}, c_{k+1}, c_k . Therefore there is a rainbow path between v_{n+j} and v_{n+k} $(1 \leq j < k \leq n)$

In all the above cases, there exists a rainbow path between every pair of vertices of H_n. Therefore, the coloring given in the algorithm [2.1] is rainbow coloring of H_n. All Pendant edges should have different colors, since otherwise there is no rainbow path between these Pendant vertices. Therefore, we need minimum 'n' colors for rainbow coloring of H_n. Hence, rc $(H_n) = n$, for $n \geq 3$.

3. CONCLUSION

In this paper we found the rainbow connection number of the Helm graph H_n is 'n'.

REFERENCES

1. Chartrand G, Johns G.L, McKeon K, Ping Zhang, "Rainbow connection in graphs", Mathematica Bohemica, 133(2008), No 1, 85-98.
2. Joseph A.Gallian, "A Dynamics survey of graph labeling", Electronic journal of combinatorics, 18, 2011, # DS6.
3. Dr. Sudhakaraiah A, Gnana Deepika E, Ramalatha V, "Rainbow connection number and the Diameter of interval graphs", Journal of Mathematics, vol. 1, Issue 1, 2012, 36-43.

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4. Schelp R. H, Lesnaik L, Gyarfas A, Faudree R. J," Rainbow coloring the cube", Journal of Graph theory, vol. 17, No.5, 607-612 (1993).
5. Xueliang Li, Yuefang Sun, "Rainbow Connections of Graphs - A Survey*", arxiv: 1101. 5747 v2 [math.CO] Feb 2011.
6. Yuefang Sun, "On two variants of Rainbow Connection", Wseas transactions on Mathematics, Issue 3, vol. 12, March 2013.

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