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SHEAR FREE KALUZA-KLEIN COSMOLOGICAL MODEL WITH VARIABLE G AND Λ IN THE PRESENCE OF POLYTROPIC BULK VISCOUS FLUID

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ABSTRACT

In the present study, we have examined shear free polytropic bulk viscous five dimensional Kaluza-Klein type cosmological model with variable G and Λ. To get the deterministic model of the universe, here we have assumed that $\frac{\dot{K}}{K}=\frac{\dot{A}}{A}=\frac{m}{t^n}, \xi=\xi_0\rho^\alpha, P=l\rho^\gamma$. *Here* ρ *is energy density ,* ξ *is coefficient of bulk viscosity and P is pressure. We analyse the five dimensional cosmological models to show its physical applicability. Some physical consequences of the models have been discussed for exponential and power law cosmology.*

Keywords: *Five Dimensional, Cosmological model and Bulk Viscosity.*

INTRODUCTION

The idea of higher dimensions stems from the earlier attempts of Kaluza [1] and Klein [2] who were motivated by the desire to unify the fundamental forces of electromagnetism and Einstein gravity by introducing a compact fifth dimension. This theory which requires a higher dimensional framework was initially sought to explain the strong nuclear force but its peculiar properties made it a good candidate for studying quantum gravity with a hope of obtaining a unifying grand theory. In addition, studying models in higher dimensions provides a platform to understand the nature of earlier universe. It is belived that the universe, in its earlier epoch was dense and hot (a scenario better explained in higher dimensions), and as a result of expansion the extra dimensions have compactified to produce the present four dimensional universe[3].Various authors [4-6] constructed higher dimensional cosmological models in various theories of gravitation.

In recent years there has been considerable interest in the cosmological models with variable gravitational constant G and the cosmological constant Λ. Variation of the gravitational constant was first suggested by Dirac [7] in an attempt to understand the appearance of certain very large numbers, when atomic and cosmic world are compared. He postulates that the gravitational constant G decreases inversely with cosmic time. Canuto *et al.* [8, 9] made numerous suggestions based on different arguments that G is indeed time dependent. Beesham [10] has studied the creation with variable G and pointed out the variation of the form $G \sim t^{-1}$, originally proposed by Dirac [7].

On the other hand, Einstein introduced the cosmological constant Λ to account for a stable static universe as appeared to him at the time. When he later knew of the universal expansion he regretted its inclusion in his field equations. Now cosmologist believe that is not identically, but very close to zero. They relate this constant to the vaccum energy that first inflated a universe causing it to expand [11]. From the point of view of particle physics vacuum energy could correspond to quantum field that is diluted to its present small value. However, other cosmologists dictate a time variation of this constant in order to account for its present smallness [12]. The variation of its constant could resolve some of the standard model problems like G, the constant Λ is a gravity coupling and both should therefore be treated on an equal footing. The generalized Einstein's theory of gravitation with time dependent G and Λ has been proposed by Lau [13]. The possibility of variable G and Λ in Einsteins theory has also been studied by Dersarkissian [14]. This relation plays an important role in cosmology. Berman [15] has considered the Einstein field equations with perfect fluid and variable G and Λ for Robertson –Walker line element. Kalligas *et al.* [16] have studied FRW models with variable Λ and G and discussed the possible connection with power law time dependence of G. Recently some of us

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and others have studied cosmological models with variable G and Λ in a diversified fields [17,18,19,20,21,22,23, 24,25,26,27,28]. Thus the implication of time varying Λ and G are important to study the early evolution of the universe. Singh and Kotambkar [29] discussed cosmological models with variable G and Λ in space times of higher dimensions. Khadekar and Kamdi [30] have obtained exact solution of the Einstein's field equations in higher dimension with variable G and Λ.

To consider more realistic models one must take into account viscosity mechanisms and indeed, viscosity mechanisms has attracted the attention of many researchers. At the early stages of evolution of the universe, when radiation is in the form of photons as well as neutrino decoupled, the matter behaved like a viscous fluid. Bulk viscosity could arise in many circumstances and could lead to an effective mechanism of galaxy formation. Samdurkar and Sen investigated the effect of bulk viscosity on Bianchi Type V cosmological models with varying Λ in general relativity [31]. Murphy [32] constructed isotropic homogeneous spatially-flat cosmological model with a fluid containing bulk viscosity alone because the shear viscosity cannot exist due to assumption of isotropy. Padamanabhan and Chitre [33] have shown that the presence of bulk viscosity leads to inflationary like solutions in general relativity. Tiwari and Sonia [34] investigated the non-existance of shear in Bianchi type III string bulk viscous cosmological model in general relativity. Recently Ghate and Sandhya [35] have investigated bulk viscous cosmological model in higher dimension.

With this motivation the linking of varying G and Λ term has been considered within the frame work of general relativity. In this paper, we consider five dimensional Kaluza-Klein type cosmological model using polytropic bulk viscous fluid with variable G and Λ. To obtain the solution here we assume that $\frac{\dot{K}}{K}=\frac{\dot{A}}{A}=\frac{m}{t^n}, \xi=\xi_0\rho^{\alpha}, P=l\rho^{\gamma}$. where P is the pressure, ρ is the energy density and ξ is the coefficient of bulk viscosity. Here we have discussed the physical and geometrical aspects of the models for $n\neq 1$ i.e. exponential cosmology and n=1 i.e. power law cosmology.

1.2) Field Equations and their solution

We consider five dimensional Kaluza-Klein space time as

$$ds^2 = dt^2 - K^2\left(dx^2+dy^2+dz^2\right) - A^2 d\psi^2 \tag{1}$$

Where *K* & A are the functions of cosmic time t and ψ is Kaluza-Klein parameter.

The energy-momentum tensor in the presence of bulk stress has the form

$$T_i^j = (P+\rho)u_i u^j - P g_i^j \tag{2}$$

$$P = p - 4\xi H \tag{3}$$

ρ is the energy density, ξ is the coefficient of bulk viscosity, p is the equilibrium pressure and u_i is the fluid five-velocity such that $u^i u_i = -1$.

The Einstein field equations are given by

$$R_i^j - \frac{1}{2}R g_i^j = -8\pi G T_i^j + \Lambda g_i^j \tag{4}$$

Where G is gravitational constant and Λ is cosmological constant which depends on time t.

For the metric (1), Einstein field equation becomes

$$\frac{2\ddot{K}}{K}+\frac{\dot{K}^2}{K^2}+\frac{2\dot{K}\dot{A}}{KA}+\frac{\ddot{A}}{A} = -8\pi G P + \Lambda \tag{5}$$

$$\frac{3\ddot{K}}{K}+\frac{3\dot{K}^2}{K^2} = -8\pi G P + \Lambda \tag{6}$$

$$\frac{3\dot{K}^2}{K^2}+\frac{3\dot{K}\dot{A}}{KA} = 8\pi G\rho + \Lambda \tag{7}$$

The energy conservation equation $T^i_{j_{ji}} = 0$ takes the form

$$8\pi G\left[\dot{\rho}+(\rho+P)\right]\left\{\frac{3\dot{K}}{K}+\frac{\dot{A}}{A}\right\} + 8\pi\dot{G}\rho + \dot{\Lambda} = 0 \tag{8}$$

Which spilts into two equations as

$$\left[\dot{\rho}+(\rho+\mathrm{P})\right]\left\{\frac{3\dot{K}}{K}+\frac{\dot{A}}{A}\right\}=0 \tag{9}$$

and

$$8\pi\dot{G}\rho+\dot{\Lambda}=8\pi G\xi\left\{\frac{3\dot{K}}{K}+\frac{\dot{A}}{A}\right\}^2=0 \tag{10}$$

The expression for scalar of expansion, shear scalar and anisotropy parameter are defined as

$$\theta=4H \tag{11}$$

$$2\sigma^2=\sum_{i=1}^{4}H_i^2-\frac{1}{4}\theta^2 \tag{12}$$

$$\Delta=\frac{1}{4}\sum_{i=1}^{4}\left(\frac{H_i-H}{H}\right)^2 \tag{13}$$

Solution of Field Equations

Here let us assume

$$\xi=\xi_0\rho^{\alpha} \tag{14}$$

Where

$$\xi>0,\alpha=cons\tan t$$

To obtain the complete solution we assume the polytropic relation

$$\mathrm{P}=l\rho^{\gamma} \tag{15}$$

Where a and γ are constant

now taking γ=1, we get

$$\mathrm{P}=l\rho \tag{16}$$

Also we assume the solution of the system of the equations [36]as follows

$$\frac{\dot{\mathrm{K}}}{\mathrm{K}}=\frac{\dot{A}}{A}=\frac{m}{t^n} \tag{17}$$

Where m and n are constants

Case-I: Exponential cosmology

Now solving equation (17), we get

$$K=A=C\exp\left(\frac{mt^{1-n}}{1-n}\right),\qquad \text{for } n\neq 1 \tag{18}$$

Where C is integrating constant

Substituting eq (16) and (17) in (9), we get

$$\frac{\dot{\rho}}{\rho}=\frac{-(1+m)(3C+1)m}{t^n} \tag{19}$$

Which on integration gives

$$\rho=m_1\exp\left[\frac{-(1+m)(3C+1)t^{1-n}}{(1-n)}\right] \tag{20}$$

Differentiating above equation

$$\dot{\rho} = \frac{-m_1(1+m)(3C+1)}{(1-n)} \exp\left[\frac{-(1+m)(3C+1)t^{1-n}}{(1-n)}\right] \tag{21}$$

From equation (7), we get

$$\frac{-12nm^2}{t^{2n+1}} = 8\pi G\dot{\rho} + 8\pi G\xi\left[\frac{4m}{t^n}\right]^2 \tag{22}$$

Using equation (14) and (21) in (22), we get

$$G = \frac{-3nm^2}{2\pi t^{2n+1}} e^{\frac{(1+m)(3C+1)}{(1-n)}t^{(1-n)}} \left[\frac{16m\xi_0 m_1 m^2}{t^{2n}} e^{-2\frac{(1+m)(3C+1)}{(1-n)}t^{(1-n)}} - \frac{m_1(1+m)(3C+1)}{(1-n)}\right]^{-1} \tag{23}$$

$$\Lambda = \frac{6m^2}{t^{2n}} + \frac{12nm^2 m_1}{t^{(2n+1)}}\left[\frac{16\xi_0 m_1 m^2}{t^{2n}} e^{-2\frac{(1+m)(3C+1)}{(1-n)}t^{(1-n)}} - \frac{m_1(1+m)(3C+1)}{(1-n)}\right]^{-1} \tag{24}$$

From the equation (14) and (20), we get

$$\xi = \xi_0 m_1^\alpha \exp\left[\frac{-\alpha(1+m)(3C+1)t^{1-n}}{(1-n)}\right] \tag{25}$$

Hence the metric (1) becomes

$$ds^2 = dt^2 - C^2 \exp\left(\frac{2mt^{1-n}}{1-n}\right)\left(dx^2 + dy^2 + dz^2 + d\psi^2\right) \tag{26}$$

Therefore the Spatial volume and expansion scalar for the model (26) are given by

$$V = C^4 \exp\left[\frac{4mt^{1-n}}{(1-n)}\right] \tag{27}$$

$$\theta = \frac{4m}{t^n} \tag{28}$$

Also the expressions for coefficient of bulk viscosity ξ, density ρ, equilibrium pressure P and anisotropy parameter Δ are

$$\xi = \xi_0 m_1^\alpha \exp\left[\frac{-\alpha(1+m)(3C+1)t^{1-n}}{(1-n)}\right] \tag{29}$$

and $$\mathrm{P} = m_1 m_2 \exp\left[\frac{(1+m)(3C+1)t^{1-n}}{(1-n)}\right] \tag{30}$$

and $$\rho = m_1 \exp\left[\frac{(1+m)(3C+1)t^{1-n}}{(1-n)}\right] \tag{31}$$

And $$\Delta = 0 \tag{32}$$

Observations

For an expanding model we require m>0

- the spatial volume V decreases as time increases.
- At $t \to 0$ Hubble parameter H tends to infinity and when $t \to \infty$ Hubble parameter tends to zero
- When $t \to 0$, energy density $\rho \to \infty$ and when $t \to \infty$, $\rho \to 0$

- Here Anisotropic parameter vanishes for the model and $\sigma = 0$
- The model starts with a big bang at t=0 when n>0 and expansion scalar decreases as time t increases. However, when n<0 the expansion in the model increases as time decreases.
- As $\sigma = 0$, the model is isotropic in nature.
- Therefore the model (26) essentially gives an empty universe for large t. The model represents non-shearing, non-rotating and expanding universe with a big-bang start.
- Since $\xi = \xi_0 \rho^{\alpha}$, $\alpha > 1$ the model leads to the inflationary phases[37-40].

Case-II: Power law cosmology

For n=1, equation (17) reduces to

$$\frac{\dot{K}}{K} = \frac{\dot{A}}{A} = \frac{m}{t},$$

Hence

$$K = A = m_3 t^m \tag{33}$$

Therefore the model becomes

$$ds^2 = dt^2 - m_3^2 t^{2m}\left(dx^2 + dy^2 + dz^2 + d\psi^2\right) \tag{34}$$

For the model (34), the expressions for density, pressure, cosmological constant, gravitational constant, expansion scalar and spatial volume are as follows:

$$\rho = \frac{m_4}{t^{m(1+m)(3C+1)}} \tag{35}$$

$$P = \frac{m_2 m_4}{t^{m(1+m)(3C+1)}} \tag{36}$$

$$\Lambda = \frac{6m^2}{t^2}\left[\frac{2m_4}{16m^2 \xi t^{\{m(1+m)(3C+1)-1\}} - m m_4 (1+m)(3C+1)} + 1\right] \tag{37}$$

$$G = \frac{3m^2}{2\pi} t^{\{m(1+m)(3C+1)-2\}} . \left[m m_4 (1+m)(3C+1) - 16\xi_0 m^2 . t^{\{m(1+m)(3C+1)-1\}}\right]^{-1} \tag{38}$$

$$\theta = \frac{4m}{t} \tag{39}$$

$$V = m_3^4 t^{4k} \tag{40}$$

Observations

- For m>0, the spatial volume V is zero at t=0 and expansion scalar is infinite which shows that the universe starts evolving with zero volume and infinite rate of expansion at t=0.
- All the parameters θ, ρ, P, H tends to zero as $t \to 0$.
- As $\sigma = 0,$ the model is isotropic in nature

CONCLUSION

In this paper ,we have investigated five dimensional Kaluza-Klein space time type cosmological model in the presence of polytropic bulk viscous fluid with varying G and cosmological term. The role of bulk viscosity in the cosmic evolution, especially as its early stages seems tobe significant [27].

The model (26) starts with a big-bang at t=o when n> 0 the expansion scalar decreases as time increases However when n<0 the expansion in the model increases as the time increases. The model describes non shearing, non rotating and expanding universe with a big-bang starts. Furthermore the physical and geometrical aspects of the model are also discussed. As the time increases θ decreases and σ =0 implies that the shear scalar does not exist. As $\sigma = 0,$ the model is isotropic for large value of t. For the case n=1, the spatial volume V increases as time t increases. Since $\xi = \xi_0 \rho^{\alpha}$, $\alpha > 0$ the model leads to the inflationary solution.

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