International Journal of Mathematical Archive-6(3), 2015, 201-208
Available online through www.ijma.info ISSN 2229 – 5046

ON SOME PROPERTIES OF $\alpha b\hat{g}$-CLOSED SETS IN TOPOLOGICAL SPACES

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(Received On: 16-01-15; Revised & Accepted On: 12-02-15)

ABSRTACT

A new class of closed sets called $\alpha b\hat{g}$- closed sets in topological spaces is introduced. This class contains the class of all α-closed sets and is contained in the class of all $\alpha\hat{g}$ and $b\alpha\hat{g}$ closed sets. The inclusion relationships of this new class with other known classes of closed sets are investigated. Also new classes of spaces, based on the class of $\alpha b\hat{g}$-closed sets are introduced and their properties are analyzed.

Keywords: *α-closed sets, b-closed sets, $\hat{g}$-open sets, $b\hat{g}$ - open sets, $\alpha b\hat{g}$-closed sets, $\alpha b\hat{g}$-continuous, $\alpha b\hat{g}$-open map and $T^{c}_{\alpha b\hat{g}}$ space.*

AMS subject classification: *54H05, 54F65, 54C55.*

1. INTRODUCTION

Njastad [13] introduced α-closed sets and analyzed their properties. The class of b-open sets and $\hat{g}$-open sets are initiated by Andrijevic [3] and Veera Kumar [17] in 1996 and 2003 respectively. Recently Subasree and Maria Singam [16] introduced $b\hat{g}$-closed sets and studied their properties. Followed by these developments, Stella Irene Mary and NagaJothi [15] introduced $b\alpha\hat{g}$ -closed sets and analyzed their properties. In this article another new class of closed sets namely $\alpha b\hat{g}$-closed sets is introduced that satisfies the inclusion relation given below.

$\{\alpha - \text{closed sets}\} \subset \{\alpha b\hat{g} - \textit{closed sets}\} \subset \{b\alpha\hat{g} - \text{closed sets}\}$ and

$\{\alpha - \text{closed sets}\} \subset \{\alpha b\hat{g} - \textit{closed sets}\} \subset \{\alpha\hat{g} - \text{closed sets}\}$

Note that the class of $b\alpha\hat{g}$-closed sets and the class of $\alpha\hat{g}$-closed sets are independent of each other [15]. As an application of $\alpha b\hat{g}$ - closed sets new spaces such as $T^{c}_{\alpha b\hat{g}}$ - space, $T^{gs}_{\alpha b\hat{g}}$ - space, and $T^{b\alpha\hat{g}}_{\alpha b\hat{g}}$ – space are defined and their relationship with other known topological spaces are characterized.

2. PRELIMINARIES

Throughout this paper, (X, τ) denote a topological space with topology τ. For a subset A of X the interior of A and closure of A are denoted by int(A) and cl(A) respectively.

Definition 2.1.1: A subset A of a topological space (X, τ) is called

1. a semi open set [8] if $A \subseteq cl(int(A))$ and a semi closed set if $int(cl(A)) \subseteq A$.
2. a pre-open set [12] if $A \subseteq int(cl(A))$ and a pre-closed set if $cl(int(A)) \subseteq A$.
3. an α-open set [13] if $A \subseteq int(cl(int(A))$ and an α- closed set if $cl(int(cl(A)) \subseteq A$.
4. a b-open set [3] if $A \subseteq cl(int(A)) \cup int(cl(A))$ and a b-closed set if $int(cl(A)) \cap cl(int(A)) \subseteq A$.

The intersection of all semi-closed (resp α-closed, b-closed) sets of X containing A is called the semi-closure (resp. α-closure, b-closure) of A and is denoted by scl(A) (resp. αcl(A), bcl(A)).

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Definition 2.1.2: A subset A of a topological space (X, τ) is called,

(1) a generalized closed set (briefly g-closed) [7] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ). The complement of a g-closed set is called a g-open set.

(2) generalized semi-closed set (briefly gs-closed) [4] if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ).

(3) an α- generalized closed set (briefly αg-closed) [11] if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ).

(4) a generalized semi-pre closed set (briefly gsp-closed) [6] if spcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in(X, τ).

(5) a generalized pre-closed set (briefly gp-closed) [9] if pcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in(X, τ).

(6) a strongly g-closed set [14] if cl(intA) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ).

(7) a $\hat{g}$ -closed set [17] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is a semi-open set in (X, τ).

(8) a gb-closed set [2] if bcl (A) $\subseteq$ U whenever A $\subseteq$ U and U is open set in (X, τ).

(9) a b$\hat{g}$ - closed set [16] if bcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is $\hat{g}$-open set in (X, τ).

(10) a subset A of a topological space (X, τ) is said to be bα$\hat{g}$- closed set [15] if bcl(A) $\subseteq$ U, whenever A $\subseteq$ U and U is a α$\hat{g}$- open set in (X, τ).

Definition 2.1.3: A function f: (X, τ) $\rightarrow$ (Y, σ) is called

(1) gb- continuous [2] if f^{-1}(V) is gb-closed in (X, τ) for every closed set V of (Y, σ).

(2) α-generalized continuous (briefly αg-continuous) [10] if f^{-1}(V) is αg -closed in (X, τ) for every closed set V of (Y, σ).

(3) generalized semi continuous (briefly gs-continuous) [5] if f^{-1}(V) is gs-closed in (X, τ) for every closed set V of (Y, σ).

(4) generalized semi-pre continuous (briefly gsp -continuous) [6] if f^{-1}(V) is gsp -closed in (X, τ) for every closed set V of (Y, σ).

(5) generalized pre continuous (briefly gp -continuous) [9] if f^{-1}(V) is gp- closed in (X, τ) for every closed set V of (Y, σ).

(6) gb- open map [2] if f (U) is gb-open in (Y,σ), for every open set U of (X, τ).

(7) αg- open map [10] if f (U) is αg -open in (Y,σ), for every open set U of (X, τ).

(8) gs- open map [5] if f (U) is gs-open in (Y,σ), for every open set U of (X, τ).

(9) gp- open map [9] if f (U) is gp-open in (Y,σ), for every open set U of (X, τ).

(10) gsp- open map [6] if f (U) is gsp-open in (Y,σ), for every open set U of (X, τ).

(11) bα$\hat{g}$ - open map [15] if f (U) is bα$\hat{g}$ open in (Y, σ) for every open set U of (X, τ).

Definition 2.1.4: A space (X, τ) is called

(1) a $T_{1/2}$ - space [7] if every g-closed set in it is closed.

(2) a T_b – space [5] if every gs-closed set in it is closed.

(3) an $_{\alpha}T_b$ – space [5] if every αg-closed set in it is closed.

(4) a $T^c_{b\alpha\hat{g}}$ - space [15] if every bα$\hat{g}$ - closed set is closed.

3. αb$\hat{g}$- CLOSED SETS

In this section we introduce a new class of closed sets called αb$\hat{g}$- closed sets which lie between the class of α-closed sets and the class of αb$\hat{g}$ -closed sets.

Definition 3.1.1: A subset A of a topological space (X, τ) is said to be **αb$\hat{g}$ closed** if αcl(A) $\subseteq$ U, whenever A $\subseteq$ U, and U is b$\hat{g}$- open set in (X, τ).

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3.1 Relationship of $\alpha b\hat{g}$- closed sets with other classes of closed sets:

Theorem 3.1.1: Let A be a $\alpha b\hat{g}$- closed set in a topological space (X, τ). Then A is
(i) αg-closed (ii) gs –closed (iii) gp –closed (iv) gsp-closed (v) gb-closed (vi) $b\alpha\hat{g}$ *closed*.

Proof:

(i) Let A be $\alpha b\hat{g}$ -closed and U be an open set such that A $\subseteq$U, since every open set is $b\hat{g}$-open, A$\subseteq$U implies αcl(A) $\subseteq$ U. Hence A is αg-closed set.
(ii) Let A be $\alpha b\hat{g}$ closed set and U be an open set such that A$\subseteq$U. Since every open set is $b\hat{g}$-open, A$\subseteq$U implies scl(A)$\subseteq$αcl (A)$\subseteq$U. Hence A is gs-closed set.
(iii) Let A be $\alpha b\hat{g}$ closed set and U be an open set such that A$\subseteq$U. Since every open set is $b\hat{g}$- open, A$\subseteq$U implies pcl(A)$\subseteq$αcl (A)$\subseteq$U. Hence A is gp-closed set**.**
(iv) Let A be $\alpha b\hat{g}$- closed set and U be an open set such that A$\subseteq$U, since every open set is $b\hat{g}$-open, A$\subseteq$U implies spcl(A)$\subseteq$ αcl(A)$\subseteq$U. Hence A is gsp-closed set.
(v) Let A be $\alpha b\hat{g}$ closed set and U be an open set such that A$\subseteq$U. Since every open set is $b\hat{g}$-open, A$\subseteq$U implies bcl(A)$\subseteq$αcl (A)$\subseteq$U. Hence A is gb-closed set.
(vi) Let A be $\alpha b\hat{g}$ closed set and U is $\alpha\hat{g}$ - open, since every $\alpha\hat{g}$ open set is $b\hat{g}$ - open, and A is $\alpha b\hat{g}$- closed,
(vii) αcl(A) ⊆ U and bcl(A)$\subseteq$αcl(A) ⊆ U, implies A is $b\alpha\hat{g}$ - closed.

The converse part of the above Theorem need not true. This is proved in the following examples:

Example 3.1.1: Let X = {a, b, c}, τ ={X, ϕ, {a}, {b, c}} A= {a, b} is αg-closed, but not $\alpha b\hat{g}$-closed.

Example 3.1.2: Let X = {a, b, c}, τ ={X, ϕ, {a}, {b}, {a, b}}. A = {b} is gs-closed, but not $\alpha b\hat{g}$-closed.

Example 3.1.3: Let X = {a, b, c}, τ ={X, ϕ, {a}, {b, c}}. A= {c} is gp-closed, but not $\alpha b\hat{g}$-closed.

Example 3.1.4: Let X = {a, b, c}, τ ={X, ϕ, {a}, {b, c}}. A= {a, c} is gsp-closed, but not $\alpha b\hat{g}$ -closed.

Example 3.1.5: Let X = {a, b, c}, τ ={X, ϕ, {a, c}}. A= {c} is gb-closed, but not $\alpha b\hat{g}$ -closed.

Example 3.1.6: Let X = {a, b, c}, τ ={X, ϕ, {a}, {b}, {a, b}}. A= {a} is $b\alpha\hat{g}$ - closed but not $\alpha b\hat{g}$ -closed.

Remark: The following examples reveal that $\alpha b\hat{g}$-closed sets are independent from g-closed sets, $\hat{g}$-closed sets and strongly g-closed sets.

Example 3.1.7: i) X = {a, b, c}, τ = {X,ϕ, {a}, {b, c}}. A_1 = {c} is g-closed but not $\alpha b\hat{g}$ - closed. X = {a. b, c}, τ = {X,ϕ, {a}, {a, b}}. A_2 = {b} is $\alpha b\hat{g}$- closed but not g-closed.

Example 3.1.8: ii) X = {a. b, c}, τ = {X, ϕ, {a, c}}. A_1 = {b} is $\alpha b\hat{g}$-closed but not $\hat{g}$- closed. A_2 = {a, b} is $\hat{g}$- closed but not $\alpha b\hat{g}$-closed.

Example 3.1.9: iii) X = {a. b, c}, τ = {X, ϕ, {a}, {a, c}}. A_1 = {c} is $\alpha b\hat{g}$- closed but not strongly g-closed. A_2 = {a, b} is strongly g- closed but not $\alpha b\hat{g}$-closed.

Theorem 3.1.2: Every α- closed set is $\alpha b\hat{g}$- closed.

Proof: Let A be α- closed set and U be an $b\hat{g}$ -open set such that A$\subseteq$U, since αcl(A) = A, and αcl(A)$\subseteq$U, implies A is $\alpha b\hat{g}$- closed.

Corollary 3.1.3: Every closed set is $\alpha b\hat{g}$- closed set, but not conversely.

Proof: Let A be closed, then A is α-closed and αcl(A) = A. By Theorem 3.1.2, A is $\alpha b\hat{g}$- closed set.

Example 3.1.10: X = {a, b, c}, τ = {X, ϕ, {a}, {a, b}}. A= {b} is $\alpha b\hat{g}$ closed but not closed.

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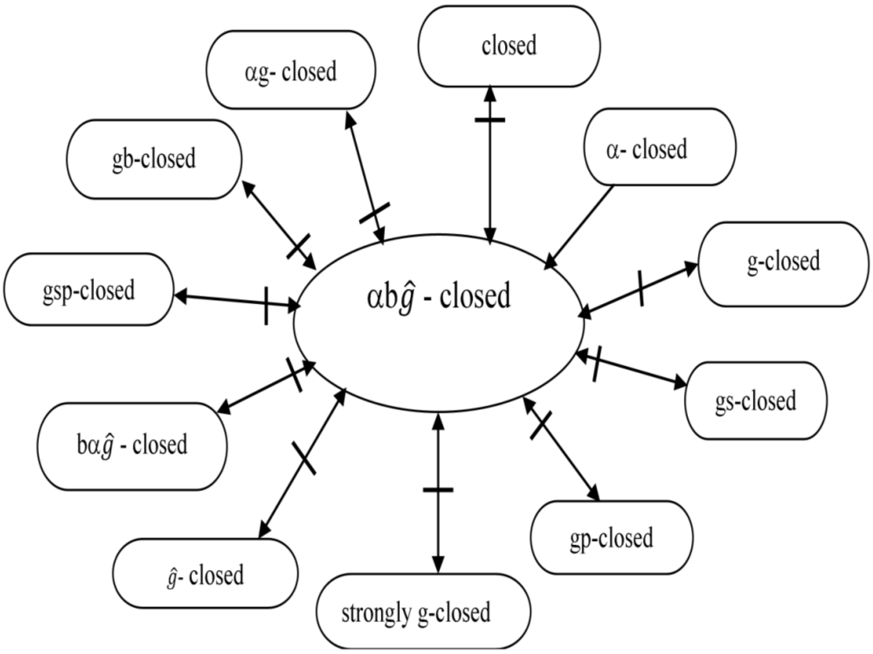
Relationships of $\alpha b\hat{g}$-closed sets with other closed sets are represented by the following diagram.



In the above diagram, A $\rightarrow$ B denotes A implies B, A $\not\leftrightarrow$ B represents, A and B are independent.
A $\leftrightarrow$B denotes B implies A, but A does not imply B. A $\leftrightarrow$ B means A implies B but B does not imply A.
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3.2 $\alpha b\hat{g}$ - Continuous functions:

We introduce the following definition.

Definition: A function f: (X, τ) $\rightarrow$ (Y, σ) is called $\alpha b\hat{g}$- continuous if $f^{-1}(V)$ is a $\alpha b\hat{g}$- closed set of (X, τ) for every closed set V of (Y,σ).

Theorem 3.2.1: Every continuous map f: (X, τ) $\rightarrow$ (Y, σ) is $\alpha b\hat{g}$ -continuous but not conversely.

Proof: Let V be a closed set in (Y, σ), then f^{-1} (V) is closed set in (X, τ). By Corollary 3.1.3, $f^{-1}(V)$ is $\alpha b\hat{g}$ -closed and hence f is $\alpha b\hat{g}$ $-$ccontinuous. The converse of the above Theorem is not true.

Example 3.2.1: Let X = {a, b, c} = Y, τ = {X, ϕ, {a}, {a, b}}, σ = {Y, ϕ, {a, c}}
Let f: (X, τ) $\rightarrow$ (Y, σ) be the identity map, then f is $\alpha b\hat{g}$- continuous but not continuous. For the closed set {b} in (Y, σ), f^{-1}{b} is $\alpha b\hat{g}$- closed in (X, τ), but not closed in (X, τ).

Theorem 3.2.2: Every $\alpha b\hat{g}$-continuous map is αg- continuous but not conversely.

Proof: Let V be a closed set in (Y, σ) then $f^{-1}(V)$ is $\alpha b\hat{g}$- closed in (X, τ), By Theorem 3.1.1, $f^{-1}(V)$ is αg- closed set in (X, τ). Hence f is αg-continuous. The converse of the above Theorem is not true.

Example 3.2.2: Let X = {a, b, c} = Y, τ = {X, ϕ, {a}, {b, c}}, σ = { Y, ϕ, {a, c}}

Let f: (X, τ) $\rightarrow$ (Y, σ) be the identity map. Then f is αg-continuous. For the closed set {b} in (Y, σ), $f^{-1}(b)$ = {b} is αg - closed in (X, τ), but not$\alpha b\hat{g}$ -closed in (X, τ). Hence f is not $\alpha b\hat{g}$-continuous.

Theorem 3.2.3: Every $\alpha b\hat{g}$ - continuous map is gs- continuous but not conversely.

Proof: Let V be a closed set in (Y, σ), then $f^{-1}(V)$ is $\alpha b\hat{g}$-closed set in (X, τ).By Theorem 3.1.1, $f^{-1}(V)$ is gs-closed set in (X, τ). Therefore, every $\alpha b\hat{g}$-continuous map is gs-continuous. The converse of the above Theorem is not true.

Example 3.2.3: Let X = {a, b, c} = Y, τ = {X, ϕ, {a},{b}, {a, b}}, σ = { Y, ϕ, {a}, {a, b},{b, c},{b}}.
Let f: (X, τ) $\rightarrow$ (Y, σ) be the identity map. Then f is gs-continuous. For the closed set {a} in (Y, σ), f^{-1}(a) = {a} is not $\alpha b\hat{g}$ - closed in (X, τ). Hence f is not $\alpha b\hat{g}$-continuous.

Theorem 3.2.4: Every $\alpha b\hat{g}$- continuous map is gp continuous but not conversely.

Proof: Let V be a closed set in (Y, σ), then f^{-1}(V) is $\alpha b\hat{g}$ -closed set in (X, τ).By Theorem 3.1.1, f^{-1}(V) is gp-closed set in (X, τ) and hence f is gp-continuous. The converse of the above Theorem is not true**.**

Example 3.2.4: Let X = {a, b, c} = Y, τ = {X, ϕ, {a}, {a, c}}, σ = { Y, ϕ, {a}}

Define f: (X, τ) $\rightarrow$ (Y, σ) by f (a) = c, f (b) = b, f (c) = a, f^{-1}(c) = a, f^{-1} (b) = b, f^{-1}(a) = c. Then f is gp-continuous. For the closed set {b, c} in (Y, σ), f^{-1} (b, c) = {a, b} is gp-closed in (X, τ), but not $\alpha b\hat{g}$ -closed in (X, τ). Hence f is not $\alpha b\hat{g}$-continuous.

Theorem 3.2.5: Every $\alpha b\hat{g}$- continuous map is gsp-continuous, but not conversely.

Proof: Let V be a closed set in (Y, σ), then f^{-1} (V) is $\alpha b\hat{g}$-closed set in (X, τ). By Theorem 3.1.1, f^{-1}(V) is gsp-closed set in (X, τ) and hence f is gsp-continuous. The converse of the above Theorem is not true.

Example 3.2.5: Let X = {a, b, c} = Y, τ = {X, ϕ, {a}, {b},{a, b}}, σ = {Y , ϕ, {a}, {b, c}}

Let f: (X, τ) $\rightarrow$ (Y, σ) be the identity map. Then f is gsp-continuous. For the closed set {a} in (Y, σ), f^{-1}(a) ={a} is gsp- closed in (X, τ), but not $\alpha b\hat{g}$ - closed in (X, τ). Hence f is not $\alpha b\hat{g}$-continuous.

Theorem 3.2.6: Every $\alpha b\hat{g}$ -continuous map is gb continuous but not conversely.

Proof: Let V be a closed set in (Y, σ), then f^{-1}(V) is $\alpha b\hat{g}$ -closed set in (X, τ). By Theorem 3.1.1, f^{-1}(V) is gb- closed set in (X, τ). Thus, f is gb- continuous. The converse of the above Theorem is not true.

Example 3.2.6: Let X = {a, b, c} = Y, τ = {X, ϕ, {a}, {a, b}}, σ = {Y, ϕ, {a}, {b}, {a, b}, {b, c}}. Define f: (X, τ) $\rightarrow$ (Y, σ) by f(a) = b, f(b) = a, f(c)= c, f^{-1} (b) = a,f^{-1} (a) = b, f^{-1} (c) = c. Then f is gb-continuous. But not $\alpha b\hat{g}$-continuous, for the closed set {b, c} in (Y, σ), f^{-1} (b, c) = {a, c} is gb- closed in (X, τ), but not $\alpha b\hat{g}$ - closed in (X, τ).

3.3 $\alpha b\hat{g}$-open maps

We introduce the following definition.

Definition 3.3.1: A map f: (X, τ)$\rightarrow$(Y, σ) is called an $\alpha b\hat{g}$- open map if f(U) is $\alpha b\hat{g}$- open in (Y, σ) for every open set U in (X, τ).

Theorem 3.3.1: Every open map is $\alpha b\hat{g}$- open map but not conversely.

Proof*:* Let f: (X, τ)$\rightarrow$(Y, σ) be an $\alpha b\hat{g}$- open map. Let U be an open set in (X, τ), then f(U) is an open set in (Y, σ). By Corollary 3.1.3, every open set is $\alpha b\hat{g}$-open set. Therefore, f is an $\alpha b\hat{g}$-open map. The converse of the above Theorem is not true.

Example 3.3.1: Let X = {a, b, c} = Y, τ ={X, φ, {a}, {a, c}}, σ = {Y, φ, {a}, {a, b}}

Let f: (X, τ) $\rightarrow$(Y, σ) be the identity map. Then f is $\alpha b\hat{g}$-open map. For the open set {a, c} in (X, τ), then f {a, c} = {a, c} is $\alpha b\hat{g}$ open in (Y, σ), but f{a, c}={a, c} is not open in (Y, σ). Therefore, f is not an open map.

Theorem 3.3.2: Every $\alpha b\hat{g}$-open map is αg- open map, but not conversely.

Proof*:* Let f: (X, τ)$\rightarrow$(Y, σ) be an $\alpha b\hat{g}$- open map. Let U be an open set in (X, τ), then f (U) is $\alpha b\hat{g}$-open in (Y, σ). By Theorem 3.1.1, f (U) is αg-open. Hence, every $\alpha b\hat{g}$-open map is an αg open map. The converse of the above Theorem is not true.

Example 3.3.2: Let X = {a, b, c} = Y, τ = {X, φ, {a}, {b}, {a, b}}, σ = {Y, φ, {a}, {a, c}}. Let f: (X, τ) $\rightarrow$ (Y, σ) be the identity map. Then f is αg-open map. For the open set {b} in (X, τ), f {b} = {b} is not $\alpha b\hat{g}$- open in (Y, σ). Therefore f is not an $\alpha b\hat{g}$- open map.

Theorem 3.3.3: Every $\alpha b\hat{g}$-open map is gs-open map. Converse need not true.

Proof: Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be an $\alpha b\hat{g}$- open map. Let U be an open set in (X, τ), then f (U) is $\alpha b\hat{g}$-open in (Y, σ). By theorem 3.1.1, f (U) is gs-open. Hence, every $\alpha b\hat{g}$-open map is gs-open map. The converse of the above Theorem is not true.

Example 3.3.3: Let X= {a, b, c} = Y, τ = {X, φ, {a}, {b}, {a, b}}, σ = {Y, φ, {a}, {b, c}}. Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be the identity map. Then f is gs-open. For the open set {b} in (X, τ), f{b}={b} is not $\alpha b\hat{g}$- open in (Y, σ). Therefore f is not an $\alpha b\hat{g}$- open map.

Theorem 3.3.4: Every $\alpha b\hat{g}$-open map is gp-open map, but not conversely.

Proof: Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be an $\alpha b\hat{g}$ open map. Let U be an open set in (X, τ), then f (U) is $\alpha b\hat{g}$-open in (Y, σ). By Theorem 3.1.1, f (U) is gp-open. Hence, every $\alpha b\hat{g}$-open map is gp-open map. The converse of the above Theorem is not true.

Example 3.3.4: Let X= {a, b, c} = Y, τ = {X, φ, {a}, {a, c}}, σ = {Y, φ, {a}, {a, b}}
Define f: $(X, \tau) \rightarrow (Y, \sigma)$ by f(a) = b f(b) = c f(c) = a. Then f is gp-open. For the open set {a} in (X, τ), f{a}={b} is not $\alpha b\hat{g}$- open in (Y, σ). Therefore f is not an $\alpha b\hat{g}$- open map.

Theorem 3.3.5: Every $\alpha b\hat{g}$-open map is gsp-open map. The converse part is not true.

***Proof*:** Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be an $\alpha b\hat{g}$-open map. Let U be an open set in (X, τ), then f (U) is $\alpha b\hat{g}$-open in (Y, σ). By Theorem 3.1.1, f (U) is gsp-open. Hence, every $\alpha b\hat{g}$-open map is a gsp-open map. The converse of the above Theorem is not true.

Example 3.3.5: Let X= {a, b, c} = Y, τ = {X, φ, {a}, {a, c}}, σ={Y, φ, {a}, {b}, {a, b}}. Define f: $(X, \tau) \rightarrow (Y, \sigma)$ by f(a)=b f(b)=a f(c)=c. Then f is gsp-open. For the open set {a, c} in (X, τ), f{a, c}={b, c} is not $\alpha b\hat{g}$- open in (Y, σ). Therefore f is not an $\alpha b\hat{g}$- open map.

Theorem 3.3.6: Every $\alpha b\hat{g}$-open map is gb-open map, but not conversely.

Proof: Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be an $\alpha b\hat{g}$- open map. Let U be an open set in (X, τ), then f (U) is $\alpha b\hat{g}$-open in (Y, σ). By Theorem 3.1.1, f(U) is gb-open. Hence, every $\alpha b\hat{g}$-open map is gb-open map. The converse of the above Theorem is not true.

Example 3.3.6: Let X = {a, b, c} = Y, τ = {X, φ, {a}, {b, c}}, σ = {Y, φ, {a}, {a, b}}
Define f: $(X, \tau) \rightarrow (Y, \sigma)$ by f(a) = b f(b) = c f(c) = a. Then f is gb-open. For the open set {a} in (X, τ), f{a} = {b} is not $\alpha b\hat{g}$- open in (Y, σ). Therefore f is not an $\alpha b\hat{g}$- open map.

Theorem 3.3.7: Every $\alpha b\hat{g}$-open map is $b\alpha\hat{g}$-open map. The reverse relation does not hold.

Proof: Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be an $\alpha b\hat{g}$- open map. Let U be an open set in (X, τ), then f (U) is $\alpha b\hat{g}$-open in (Y, σ). By Theorem 3.1.1, f (U) is $b\alpha\hat{g}$-open. Hence, every $\alpha b\hat{g}$-open map is $b\alpha\hat{g}$-open map. The converse of the above Theorem is not true.

Example 3.3.7: Let X = {a, b, c} = Y, τ = {X, φ, {a}, {b, c}}, σ = {Y, φ, {a, c}}
Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be the identity map. Then f is $b\alpha\hat{g}$-open map. For the open set {b, c} in (X, τ), f{b, c}={b, c} is not an $\alpha b\hat{g}$- open in (Y, σ). Therefore f is not an $\alpha b\hat{g}$- open map.

3.4 Applications of $\alpha b\hat{g}$ - closed sets:

As an application of $\alpha b\hat{g}$ - closed sets we introduce new spaces namely
$T^{c}_{\alpha b\hat{g}}$ - space, $T^{gs}_{\alpha b\hat{g}}$ - space, and $T^{b\alpha\hat{g}}_{\alpha b\hat{g}}$ - space.

Definition 3.4.1: A topological space (X, τ) is called,

1. a $T^{c}_{\alpha b\hat{g}}$ - space if every $\alpha b\hat{g}$ - closed set is closed.
2. a $T^{gs}_{\alpha b\hat{g}}$ - space if every gs- closed set is closed $\alpha b\hat{g}$ - closed.
3. a $T^{b\alpha g}_{\alpha b\hat{g}}$ - space if every $b\alpha\hat{g}$ - closed set is $\alpha b\hat{g}$ - closed.

Theorem 3.4.1: Every T_b - space is $T^c_{\alpha b\hat{g}}$- space, converse is not true.

Proof: Let A be $\alpha b\hat{g}$ - closed. By Theorem 3.1.1, A is gs- closed and in a T_b - space, A is closed. So, A is in $T^c_{\alpha b\hat{g}}$- space.

Example 3.4.1: Let X = {a, b, c}, τ = {X, ϕ, {a}, {b, c}}

In (X, τ) every $\alpha b\hat{g}$ - closed set is closed. Hence X is a $T^c_{\alpha b\hat{g}}$ - space but not a T_b - Space, since {b} is gs closed, but not closed.

Theorem 3.4.2: Every T_b - space is $T^{gs}_{\alpha b\hat{g}}$- space.

Proof: Let A be gs- closed. In a T_b - space, A is closed and hence $\alpha b\hat{g}$- closed.

Theorem 3.4.3: Every ${}_\alpha T_b$ - space is $T^{\alpha g}_{\alpha b\hat{g}}$- space.

Proof: Let A be αg- closed. In ${}_\alpha T_b$ - space, A is closed and hence $\alpha b\hat{g}$- closed.
So, A is in $T^{\alpha g}_{\alpha b\hat{g}}$- space.

Theorem 3.4.4: Every $T^c_{b\alpha\hat{g}}$ - space is $T^{b\alpha g}_{\alpha b\hat{g}}$- space, but converse does not hold.

Proof: Let A be $b\alpha\hat{g}$ - closed. In a $T^c_{b\alpha\hat{g}}$- space, A is closed and hence $\alpha b\hat{g}$- closed.
So, A is in $T^{b\alpha g}_{\alpha b\hat{g}}$ - space.

Example 3.4.2: Let X = {a, b, c}, τ = {X, ϕ, {a}}.

In (X, τ), every $\alpha b\hat{g}$ -closed set is $b\alpha\hat{g}$ -closed, hence X is $T^{b\alpha g}_{\alpha b\hat{g}}$ - space, but not a $T^c_{b\alpha\hat{g}}$ - space, since {c} is $b\alpha\hat{g}$ - closed, but not closed.

Theorem 3.4.5: Every $T^c_{b\alpha\hat{g}}$ - space is $T^c_{\alpha b\hat{g}}$ - space, but converse is not true.

Proof: The Theorem follows, since every $\alpha b\hat{g}$- closed set is $b\alpha\hat{g}$ - closed set.

Example 3.4.3: Let X = {a, b, c}, τ = {X, ϕ, {a}, {b}, {a, b}}.
The $\alpha b\hat{g}$-closed sets are {X, φ, {c}, {a, c}, {b, c}}. In (X, τ) every $\alpha b\hat{g}$- closed set is closed, hence X is a $T^c_{\alpha b\hat{g}}$ - space, but not a $T^c_{b\alpha\hat{g}}$ - space, since {b} is $b\alpha\hat{g}$ - closed, but not closed.

Theorem 3.4.6: Let (X, τ) be $T^{gs}_{\alpha b\hat{g}}$ - space and a $T^c_{\alpha b\hat{g}}$ - space then it is a $T_{1/2}$ - space. The converse part is not true.

Proof: Let A be g- closed and hence gs- closed. In $T^{gs}_{\alpha b\hat{g}}$- space, A is $\alpha b\hat{g}$-closed and in $T^c_{\alpha b\hat{g}}$ - space, A is closed. Hence (X, τ) is $T_{1/2}$ - space**.**

Example 3.4.4: Let X = {a, b, c}, τ = {X, φ, {a}, {b}, {a, b}}. The g-closed sets are {X, φ, {c}, {b, c}, {a, c}}. Every g-closed set is closed. Hence X is a $T_{1/2}$ -space, but not $T^{gs}_{\alpha b\hat{g}}$-space, since {a} is gs-closed, not $\alpha b\hat{g}$-closed.

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Source of support: Nil, Conflict of interest: None Declared

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