International Journal of Mathematical Archive-6(3), 2015, 184-189
Available online through www.ijma.info ISSN 2229 – 5046

ON THE HYPER-WIENER INDEX OF THORNY-COCKTAIL PARTY GRAPHS

Shigehalli V. S.[1] and Shanmukh Kuchabal*[2]

*[1]Professor, Department of Mathematics,
Rani Channamma University, Vidya Sangama, Belagavi- 591 156, India.*

*[2]Research Scholar, Department of Mathematics,
Rani Channamma University, Vidya Sangama, Belagavi- 591 156, India.*

(Received On: 19-02-15; Revised & Accepted On: 18-03-15)

ABSTRACT

Let G be the connected graph. The Wiener index $W(G)$ is the sum of all distances between vertices of G, whereas the hyper-Wiener index $WW(G)$ is defined as $WW(G) = W(G) + \frac{1}{2}\sum_{\{u,v\}\subseteq V\{G\}} d(u,v)^2$. In this paper we prove some general results on the hyper-Wiener index of thorny-cocktail party graphs.

Keywords: *thorny-cocktail party graphs, Wiener index and hyper-Wiener index.*

2000 Mathematics subject classification: *05C12.*

INTRODUCTION

In mathematical terms a graph is represented as $G = (V, E)$ where V is the set of vertices and E is the set of edges. Let G be an undirected connected graph without loops or multiple edges with n vertices, denoted by $1,2,\dots,n$. The topological distance between the vertices u and v of $V(G)$ is denoted by $d(u,v)$ and it is defined as the number of edges in a minimal path connecting the vertices u and v.

The Wiener index $W(G)$ of a connected graph G is defined as the sum the distances between all unordered pairs of vertices of G. It was put forward by Harold Wiener. The Wiener index is a graph invariant intensively studied both in mathematics and chemical literature, see for details[1, 6, 7, 8, 10 *and* 12].

The hyper-Wiener index was proposed by Randic [13] for a tree and extended by Klein *et al.* [2] to a connected graph. It is used to predict physicochemical properties of organic compounds. The hyper-Wiener index defined as,

$$WW(G) = \sum_{\{u,v\}\subseteq V(G)} \binom{d_{uv}+1}{2} = W(G) + \frac{1}{2}\sum_{\{u,v\}\subseteq V(G)} d(u,v)^2$$

The hyper-Wiener index is studied both from a theoretical point of view and applications. We encourage the reader to consult [4, 5 *and* 10] for further readings. The hyper-Wiener index of complete graph-K_p, path graph-P_n, star graph-$K_{1,(n-1)}$ and cycle graph C_n is given by the expressions

$$WW(K_n) = \frac{n(n-1)}{2}, WW(P_n) = \frac{n^4+2n^3-n^2-2n}{24}, \quad WW\left(K_{1,(n-1)}\right) = \frac{1}{2}(n-1)(3n-4)$$

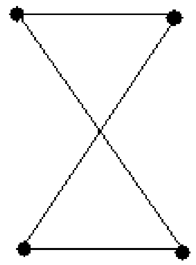
and

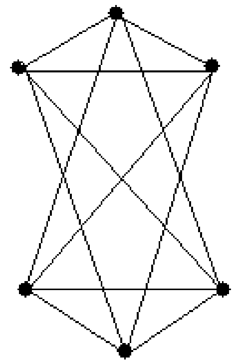
$$WW(C_n) = \begin{cases} \frac{n^2(n+1)(n+2)}{48}, & if\ n\ is\ even \\ \frac{n(n^2-1)(n+3)}{48}, & if\ n\ is\ odd \end{cases}$$

The cocktail party graph of order k, is the graph consisting of two rows of paired nodes in which all nodes but the paired ones are connected with a graph edge.

Corresponding Author: Shanmukh Kuchabal[2], [2]Research Scholar, Department of Mathematics, Rani Channamma University, Vidya Sangama, Belagavi- 591 156, India.*

Example: k=2 k=3





Let G be a connected n-vertex graph with vertex set $V(G)=\{v_1,v_2,\dots v_n\}$ and $P=(p_1,p_2,\dots,p_n)$ be an n-tuple of non-negative integers. The thorn graph G_p is the graph obtained by attached to the vertex V_i will be called the thorns of V_i. The concept of thorny graphs was introduced by Ivan Gutman.

MAIN RESULTS

Theorem 1: Let H be the cocktail party graph on k vertices. The graph G obtained by attaching s-number of pendent vertices to any one vertex of graph H with common vertex then its hyper-Wiener index given by

$$WW(G)=\frac{1}{2}[s(6k+3s-1)+k(4k+6s+2)]$$

Where p- total number of pendent vertices in G
k≥2

Proof: To find hyper-Wiener index of the graph, we need to find following two parts
To find Wiener index: $W(G)=\frac{1}{2}\sum_{\{u,v\}\subseteq V(G)} d(u,v)$

$$W(G)=\tfrac{1}{2}\{[1+3+\underbrace{2+2+\cdots+2}_{k+s-3\ times}]+\cdots+[1+3+\underbrace{2+2+\cdots+2}_{2k+s-3\ times}]$$

$$+[\underbrace{1+1+\cdots+1}_{2k+s-2\ times}+2]$$

$$+[\underbrace{2+2+\cdots+2}_{s+1\ times}+\underbrace{1+1+\cdots+1}_{2k-2\ times}]+\cdots+[\underbrace{2+2+\cdots+2}_{s+1\ times}+\underbrace{1+1+\cdots+1}_{2k-2\ times}]$$

$$+[\underbrace{1+1+\cdots+1}_{2k-2\ \ times}+2+\underbrace{3+3+\cdots+3}_{s\ times}]\}$$

$$W(G)=\tfrac{1}{2}\{[\underbrace{4k+2s-2+\cdots+4k+2s-2}_{s\ times}]+2k+s$$

$$+[\underbrace{2k+2s+\cdots+2k+2s}_{2y-2\ times}]+2y+3s\}$$

$$W(G)=\frac{1}{2}[s(4k+2s-2)+2k+s+(2k-2)(2k+2s)+2k+3s]$$

$$W(G)=\tfrac{1}{2}[s(4k+2s-2)+k(4k+4s)] \qquad \text{(i)}$$

To find $WW^*(G)$

$$WW^*(G)=\frac{1}{2}\sum_{\{u,v\}\subseteq V(G)} d(u,v)^2$$

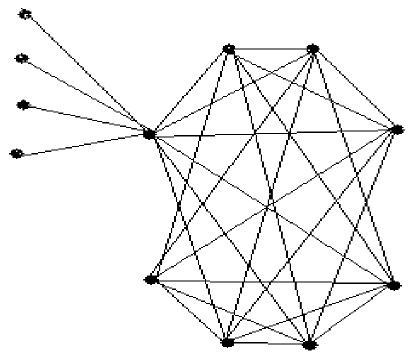
$$WW^*(G)=\tfrac{1}{2}\{[3+\underbrace{1+1+\cdots+1}_{2k+s-3\ times}]+\cdots+[3+\underbrace{1+1+\cdots+1}_{2k+s-3\ times}]+1$$

$$+[\underbrace{1+1+\cdots+1}_{s+1\ times}]+\cdots+[\underbrace{1+1+\cdots+1}_{s+1\ times}]+1+\underbrace{3+3+\cdots+3}_{s\ times}\}$$

$$WW^*(G)=\frac{1}{2}[(\underbrace{2k+s+\cdots+2k+s}_{s\ times})+1+(\underbrace{s+1+\cdots+s+1}_{2k-2\ times})+3s+1]$$

$$WW^*(G)=\frac{1}{2}[s(2k+s)+k(2s+2)+s] \qquad \text{(ii)}$$

Combining (i) and (ii) we get
$WW(G)=\frac{1}{2}[s(6k+3s-1)+k(4k+6s+2)]$



$s=4, k=4, W(G)=108\ and\ WW(G)=154.$

Theorem 2: Let H be the cocktail party graph on k vertices. The graph G obtained by attaching s-number of pendent vertices to each vertex of H with common vertex then its hyper-Wiener index given by

$$WW(G)=\frac{1}{2}[p(6p+12k+s-2)+k(4k+2s+2)]$$

Where $k\geq 2$

Proof: To find hyper-Wiener index of the graph, we need to find following two parts.

To find Wiener index: $W(G)=\frac{1}{2}\sum_{\{u,v\}\subseteq V(G)}d(u,v)$

$$W(G)=\frac{1}{2}\{[1+\underbrace{2+2+\cdots+2}_{s+2k-3\ times}+\underbrace{3+3+\cdots+3}_{p-2s+1\ times}+\underbrace{4+4+\cdots+4}_{s\ times}]$$

$$+\cdots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots+$$

$$+[1+\underbrace{2+2+\cdots+2}_{s+2k-3\ times}+\underbrace{3+3+\cdots+3}_{p-2s+1\ times}+\underbrace{4+4+\cdots+4}_{s\ times}]$$

$$+[\underbrace{1+1+\cdots+1}_{s+2k-2\ times}+\underbrace{2+2+\cdots+2}_{p-2s+1\ times}+\underbrace{3+3+\cdots+3}_{s\ times}]$$

$$+\cdots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots+$$

$$+[\underbrace{1+1+\cdots+1}_{s+2k-2\ times}+\underbrace{2+2+\cdots+2}_{p-2s+1\ times}+\underbrace{3+3+\cdots+3}_{s\ times}]\}$$

$$W(G)=\frac{1}{2}\{\underbrace{[3p+4k-2]+\cdots+[3p+4k-2]}_{p\ times}+\underbrace{[2p+2k]+\cdots+[2p+2k]}_{2k\ times}\}$$

$$W(G)=\frac{1}{2}[p(3p+4k-2)+2k(2p+2k)] \qquad \text{(i)}$$

To find $WW^*(G)$

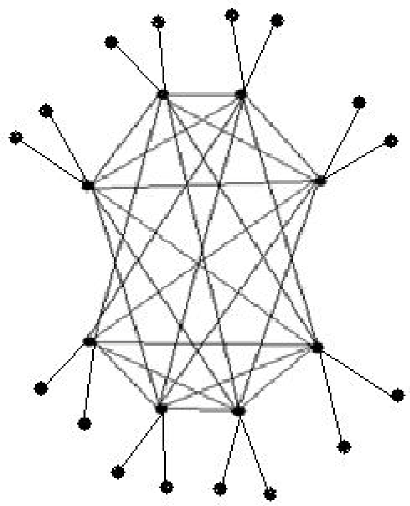
$$WW^*(G) = \frac{1}{2}\sum_{\{u,v\}\subseteq V(G)} d(u,v)^2$$

$$WW^*(G) = \frac{1}{2}\{[\underbrace{1+1+\cdots+1}_{s+2k-3\ times} + \underbrace{3+3+\cdots+3}_{p-2s+1\ times} + \underbrace{6+6+\cdots+6}_{s\ times}]$$
$$+\cdots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots+$$
$$+[\underbrace{1+1+\cdots+1}_{s+2k-3\ times} + \underbrace{3+3+\cdots+3}_{p-2s+1\ times} + \underbrace{6+6+\cdots+6}_{s\ times}]$$
$$+[\underbrace{1+1+\cdots+1}_{p-2s+1\ times} + \underbrace{3+3+\cdots+3}_{s\ times}]$$
$$+\cdots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots+$$
$$+[\underbrace{1+1+\cdots+1}_{p-2s+1\ times} + \underbrace{3+3+\cdots+3}_{s\ times}]\}$$

$$WW^*(G) = \frac{1}{2}\{\underbrace{[3p+2k+s]+\cdots+[3p+2k+s]}_{p\ times} + \underbrace{[p+s+1]+\cdots+[p+s+1]}_{2k\ times}\}$$

$$WW^*(G) = \frac{1}{2}[p(3p+2k+s) + 2k(p+s+1)] \qquad \text{(ii)}$$

Combining (i) and (ii) we get
$WW(G) = \frac{1}{2}[p(6p+12k+s-2) + k(4k+2s+2)]$



$s = 2, p = 16, k = 4, W(G) = 656\ and\ WW(G) = 1196$

Corollary 2.1: The wiener and hyper-wiener index of cocktail party graph is given by
$W(G) = 2k^2$

$WW(G) = k(2k+1)$

Proof: substituting s=0 and p=0 in the above theorem gives the result.

Theorem 3: Let H be the cocktail party graph on k vertices (k is even). The graph G obtained by attaching s-number of pendent vertices to alternative vertices of a graph H with common vertex then its hyper-Wiener index given by

$$WW(G) = \frac{1}{2}[p(6p+6k+s-2) + k(6p+s+4k+2)]$$

Where $k \geq 2$

Proof: To find hyper-Wiener index of the graph, we need to find following two parts.

To find Wiener index: $W(G) = \frac{1}{2}\sum_{\{u,v\}\subseteq V(G)} d(u,v)$

$$W(G)=\frac{1}{2}\{[1+\underbrace{2+2+\cdots+2}_{s+2k-3\ times}+\underbrace{3+3+\cdots+3}_{p-2s+1\ times}+\underbrace{4+4+\cdots+4}_{s\ times}]$$

$$+\cdots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots+$$

$$+[1+\underbrace{2+2+\cdots+2}_{s+2k-3\ times}+\underbrace{3+3+\cdots+3}_{p-2s+1\ times}+\underbrace{4+4+\cdots+4}_{s\ times}]$$

$$+[\underbrace{1+1+\cdots+1}_{s+2k-2\ times}+\underbrace{2+2+\cdots+2}_{p-2s+1\ times}+\underbrace{3+3+\cdots+3}_{s\ times}]$$

$$+\cdots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots+$$

$$+[\underbrace{1+1+\cdots+1}_{s+2k-2\ times}+\underbrace{2+2+\cdots+2}_{p-2s+1\ times}+\underbrace{3+3+\cdots+3}_{s\ times}]\}$$

$$+[\underbrace{1+1+\cdots+1}_{2k-2\ times}+\underbrace{2+2+\cdots+2}_{p+1\ times}]$$

$$+\cdots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots+$$

$$+[\underbrace{1+1+\cdots+1}_{k-2\ times}+\underbrace{2+2+\cdots+2}_{p+1\ times}]$$

$$W(G)=\frac{1}{2}\{\underbrace{[3p+4k-2]+\cdots+[3p+4k-2]}_{p\ times}+\underbrace{[2p+2k]+\cdots+[2p+2k]}_{k\ times}$$

$$+\underbrace{[2k+2p]+\cdots+[2k+2p]}_{k\ times}\}$$

$$W(G)=\frac{1}{2}\{p[3p+4k-2]+k[2p+2k]+k[2k+2p]\}$$

$$W(G)=\frac{1}{2}[p(3p+4k-2)+k(4p+4k)] \qquad \text{(i)}$$

To find $WW^*(G)$

$$WW^*(G)=\frac{1}{2}\sum_{\{u,v\}\subseteq V(G)} d(u,v)^2$$

$$WW^*(G)=\frac{1}{2}\{[\underbrace{1+1+\cdots+1}_{s+2k-3\ times}+\underbrace{3+3+\cdots+3}_{p-2s+1\ times}+\underbrace{6+6+\cdots+6}_{s\ times}]$$

$$+\cdots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots+$$

$$+[\underbrace{1+1+\cdots+1}_{s+2k-3\ times}+\underbrace{3+3+\cdots+3}_{p-2s+1\ times}+\underbrace{6+6+\cdots+6}_{s\ times}]$$

$$+[1+\underbrace{1+\cdots+1+3}_{p-2s+1\ times}+\underbrace{3+\cdots+3]}_{s\ times}$$

$$+\cdots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots+$$

$$+[\underbrace{1+1+\cdots+1}_{p-2s+1\ times}+\underbrace{3+3+\cdots+3}_{s\ times}]\}$$

$$+[\underbrace{1+1+\cdots+1}_{p+1\ times}]+\cdots+[\underbrace{1+1+\cdots+1}_{p+1\ times}]\}$$

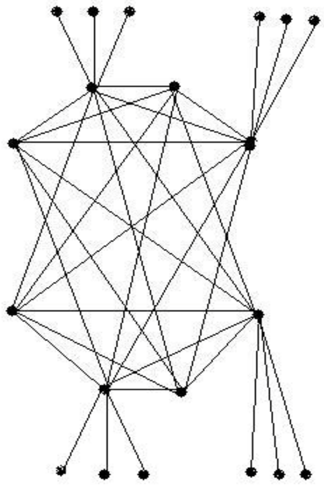
$$WW^*(G)=\frac{1}{2}\{\underbrace{[3p+2k+s]+\cdots+[3p+2k+s]}_{p\ times}+\underbrace{[p+s+1]+\cdots+[p+s+1]}_{k\ times}$$

$$+\underbrace{[p+1]+\cdots+[p+1]}_{k\ times}\}$$

$$WW^*(G) = \frac{1}{2}[p(3p + 2k + s) + k(p + s + 1) + k(p + 1)]$$

$$WW^*(G) = \frac{1}{2}[p(3p + 2k + s) + k(2p + s + 2)] \quad \text{(ii)}$$

Combining (i) and (ii) we get
$WW(G) = \frac{1}{2}[p(6p + 6k + s - 2) + k(6p + s + 4k + 2)]$



$s = 3, p = 12, k = 4, W(G) = 428$ *and* $WW(G) = 768$

REFERENCES

[1] D. Bonchev and D. H. Rouvary, Chemical Graph Theory, introduction and Fundamentals, 1991.
[2] D. J. Klein, I. Lukovits, I. Gutman, On the definition of the hyper-Wiener index for cycle-containing structures, J. Chem. Inf. Comput. Sci. 35 (1995).
[3] F. Buckley, F. Harary, Distances in Graphs, Addison-Wesley, Redwood, 1990.
[4] G. C. Cash, Polynomial expressions for the hyper-Wiener index of extended hydrocarbon networks, Comput. Chem. 25 (2001) 577-582.
[5] G. C. Cash, Relationship between the Hosaya Polynomial and the hyper-Wiener index, Appl. Math. Lett. 15 (2002) 893-895.
[6] H. B. Walikar, H. S. Ramane, V. S. Shigehalli, Wiener number of Dendrimers, In: Proc. National Conf. on Mathematical and Computational Models, (Eds. R. Nadarajan and G. Arulmozhi), Applied Publishers, New Delhi, 2003, 361-368.
[7] H. B. Walikar, V. S. Shigehalli, H. S. Ramane, Bounds on the Wiener number of a graph, MATCH comm. Math. Comp. Chem., 50 (2004), 117-132.
[8] H. Wiener, Structural determination of paraffin boiling points, J. Amer. Chem. Soc., 69 (1947), 17-20.
[9] I. Gutman, property of the Wiener number and its modifications, Indian J. Chem. 36A (1997) 128-132.
[10] I. Gutman, Relation between hyper-Wiener and Wiener index, Chem. Phys. Lett. 364 (2002) 352-356.
[11] J. Baskar Babujee and J. Senbagamalar, Wiener index of graphs using degree sequence, Applied Mathematical Sciences, Vol. 6, 2012, no: 88, 4387-4395.
[12] M. H. Khalifeh, H. Yousefi-Azari, A. R. Ashrafi, "The hyper-Wiener index of graph operations" computers and mathematics with applications 56 (2008) 1402-1407.
[13] Randic, M., Novel molecular description for structure-property studies, Chem. Phys. Lett., 211 (1993), 478-483.
[14] Shigehalli V. S. and Shanmukh kuchabal, hyper-wiener index of multi-thorn even cyclic graphs using cut-method, J. comp. and Math. Sci. Vol. 5(3), 304-308 (2014).
[15] Shigehalli V. S., D. N. Misale and shanmukh kuchabal, On the hyper-Wiener index of graph amalgamation, J. comp. and Math. Sci. Vol. 5(4), 352-356 (2014).

Source of support: Nil, Conflict of interest: None Declared