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SOME MORE RESULTS ON ONE MODULO THREE HARMONIC MEAN GRAPHS

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ABSTRACT

A graph G is said to be one modulo three harmonic mean graph if there is a function φ from the vertex set of G to $\{1, 3, 4, \ldots, 3q-2, 3q\}$ *with φ is one-one and φ induces a bijection φ* from the edge set of G to {1, 4, …, 3q – 2}, where* $\varphi^*(e = uv) = \left\lceil \frac{2\varphi(u)\varphi(v)}{\varphi(u)+\varphi(v)} \right\rceil$ *or* $\left\lfloor \frac{2\varphi(u)\varphi(v)}{\varphi(u)+\varphi(v)} \right\rfloor$ *and the function φ is called as one modulo three harmonic mean labeling of G. In this paper, we investicate one modulo three harmonic mean labeling for some graphs.*

Key words: *Graph, one modulo three harmonic mean labeling, one modulo three harmonic mean graph.*

1. INTRODUCTION

We begin with simple, finite, connected and undirected graph G(V, E) with p vertices and q edges. For a detailed survey of graph labeling we refer to Gallian[1]. For all other standared terminology and notations we follow Harary[2].

S. Somasundaram and R. Ponraj introduced mean labeling of graphs in [8]. S. Somasundaram and S.S Sandhya introduced the concept of harmonic mean labeling of graphs [6]. S.S Sandhya, C. Jayasekaran and C. David Raj investicated some new families of Harmonic mean Graphs in [4, 7]. V. Swaminathan and C. Sekar introduce the concept of one modulo three graceful labeling in [9]. C. David Raj, S.S Sandhya and C. Jayasekaran introduce the concept one modulo three harmonic mean labeling of graphs in [5] and investigated some of one modulo three harmonic mean graphs in [3].

We now give the following definitions which are useful for the present investigation.

Definition 1.1: A graph G is said to be one modulo three harmonic mean graph if there is a function φ from the vertex set of G to $\{1, 3, 4, \ldots, 3q-2, 3q\}$ with φ is one-one and φ induces a bijection φ* from the edge set of G to {1, 4, …, 3q – 2}, where $\varphi^*(e = uv) = \left\lceil \frac{2\varphi(u)\varphi(v)}{\varphi(u)+\varphi(v)} \right\rceil$ or $\left\lfloor \frac{2\varphi(u)\varphi(v)}{\varphi(u)+\varphi(v)} \right\rfloor$ and the function φ is called as one modulo three harmonic mean labeling of G

Definition 1.2: A Qudrilateral snake Q_n is obtained from a path $u_1u_2\ldots u_n$ by joining u_i and u_{i+1} to two new vertices v_i and w_i respectively and joining v_i and w_i for $1 \le i \le n-1$. That is, every edge of a path is replaced by a cycle C_4.

Definition 1.3: The corona $G_1 \odot G_2$ of two graphs G_1 and G_2 is defined as the graph G obtained by taking one copy of G_1 (which has p_1 vertices) and p_1 copies of G_2 and then joining the i^{th} vertex of G_1 to every vertices in the i^{th} copy of G_2.

Definition 1.4: The Cartesian product of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is a graph G(V, E) with $V = V_1 \times V_2$ and two vertices $u = (u_1, u_2)$ and $v = (v_1, v_2)$ are adjacent in $G_1 \times G_2$ whenever ($u_1 = v_1$ and u_2 is adjacent to v_2) or ($u_2 = v_2$ and u_1 is adjacent to v_1). It is denoted by $G_1 \times G_2$.

Definition 1.5: The product $P_m \times P_n$ is called a planar grid. The product $P_n \times K_2$ is called a ladder, and it is denoted by L_n.

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Definition 1.6: Given two graphs G_1 and G_2, their union will be another graph G such that $V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$ and $E(G_1 \cup G_2) = E(G_1) \cup E(G_2)$.

2. ONE MODULO THREE HARMONIC MEAN GRAPHS

In this section, we prove that $Q_n \odot K_1$, $P_m \cup (P_n \odot K_1)$, $(P_m \odot \overline{K}_2) \cup P_n$ and G_d are one modulo three harmonic mean graphs.

Theorem 2.1: $Q_n \odot K_1$ is one modulo three Harmonic mean graph.

Proof: Consider a path $u_1u_2\ldots.u_n$. Join u_i and u_{i+1} to the vertices v_i and w_i respectively and then join v_i and w_i, $1 \le i \le n-1$. The resultant graph is Q_n. Join u_i to x_i, $1 \le i \le n$, v_i to y_i and w_i to z_i, $1 \le i \le n-1$. The resultant graph is $Q_n \odot K_1$ whose edge set is $\{u_iu_{i+1}, u_iv_i, v_iy_i, v_iw_i, w_iz_i, u_{i+1}w_i\} / 1 \le i \le n-1\} \cup \{u_ix_i / 1 \le i \le n\}$. Define a function $\varphi : V(Q_n \odot K_1) \to \{1, 3, 4, \ldots, 3q-2, 3q\}$ by

$\varphi(x_1) = 1$; $\varphi(x_2) = 24$; $\varphi(x_3) = 42$; $\varphi(x_4) = 63$; $\varphi(x_i) = 21(i-1) + 3$, $5 \le i \le n$;
$\varphi\ u_1) = 6$; $\varphi(u_2) = 21$; $\varphi(u_3) = 43$; $\varphi(u_4) = 64$; $\varphi(u_i) = 21(i-1)$, $5 \le i \le n$;
$\varphi(v_1) = 10$; $\varphi(v_2) = 30$; $\varphi(v_3) = 51$; $\varphi(v_i) = 21(i-1) + 7$, $4 \le i \le n-1$;
$\varphi(y_1) = 3$; $\varphi(y_2) = 31$; $\varphi(y_i) = 21(i-1) + 6$, $3 \le i \le n-1$;
$\varphi(w_1) = 16$; $\varphi(w_2) = 37$; $\varphi(w_3) = 58$; $\varphi(w_i) = 21(i-1) + 18$, $4 \le i \le n-1$;
$\varphi(z_i) = 21i - 6$, $1 \le i \le n-1$.

Then φ induces a bijective function $\varphi^* : E(G) \to \{1, 4, 7, \ldots, 3q-2\}$, where

$\varphi^*(u_1u_2) = 10$; $\varphi^*(u_2u_3) = 28$; $\varphi^*(u_iu_{i+1}) = 21(i-1) + 10$, $3 \le i \le n-1$;
$\varphi^*(u_ix_i) = 21(i-1) + 1$, $1 \le i \le n$;
$\varphi^*(u_1v_1) = 7$; $\varphi^*(u_iv_i) = 21(i-1) + 4$, $2 \le i \le n-1$;
$\varphi^*(v_1y_1) = 4$; $\varphi^*(v_iy_i) = 21(i-1) + 7$, $2 \le i \le n-1$;
$\varphi^*(v_iw_i) = 21i - 8$, $1 \le i \le n-1$;
$\varphi^*(w_iz_i) = 21i - 5$, $1 \le i \le n-1$;
$\varphi^*(u_{i+1}w_i) = 21i - 2$, $1 \le i \le n-1$.

In the view of the above labeling pattern f provides one modulo three Harmonic mean labeling for $Q_n \odot K_1$. Hence the theorem.

Example 2.2: One modulo three Harmonic mean labeling of $Q_7 \odot K_1$ is given in figure 2.1.

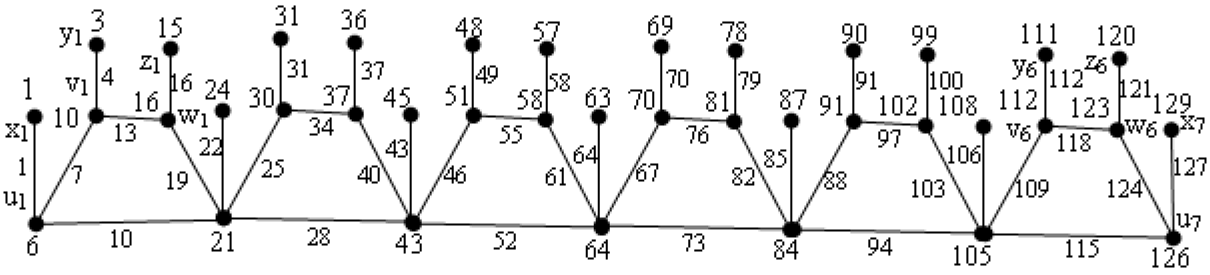

Figure-2.1: $Q_7 \odot K_1$

Theorem 2.3: $P_m \cup (P_n \odot K_1)$ is one modulo three harmonic mean graph.

Proof: Let $u_1u_2\ldots u_n$ be the path P_n and let v_i be the vertex which is joined to the vertex u_i of the path P_n, $1 \le i \le n$. The resultant graph is $P_n \odot K_1$. Let $s_1s_2\ldots s_m$ be the path P_m. Let $G = P_m \cup (P_n \odot K_1)$. Define a function $\varphi: V(G) \to \{1, 3, 4, 6, \ldots, 3q-2, 3q\}$ by

$\varphi(s_1) = 1$; $\varphi(s_i) = 3(i-1)$, $2 \le i \le m-1$; $\varphi(s_m) = 3(m-1) - 2$;
$\varphi(u_i) = 3(m-1) + 6i - 3$ for all odd i and $i \le n$;
$\varphi(u_i) = 3(m-1) + 6(i-1)$ for all even i and $i \le n$;
$\varphi(v_i) = 3(m-1) + 6(i-1)$ for all odd i and $i \le n$;
$\varphi(v_i) = 3(m-1) + 6(i-1) + 3$ for all even i and $i \le n$.

Then φ induces a bijective function $\varphi^*: E(G) \to \{1, 4, 7, \ldots, 3q-2\}$, where

$\varphi^*(w_iw_{i+1}) = 3i - 2$, $1 \le i \le n-1$;
$\varphi^*(u_iu_{i+1}) = 3(m-1) + 6i - 2$, $1 \le i \le n-1$;
$\varphi^*(u_iv_i) = 3(m-1) + 6i - 5$, $1 \le i \le n$.

Thus the edges get the distinct labels 1, 4, …, $3q-2$. Therefore, φ is one modulo three harmonic mean labeling. Hence $P_m \cup (P_n \odot K_1)$ is one modulo three harmonic mean graph.

Example 2.4: One modulo three Harmonic mean labeling of $P_7 \cup (P_6 \odot K_1)$ is given in figure 2.2.

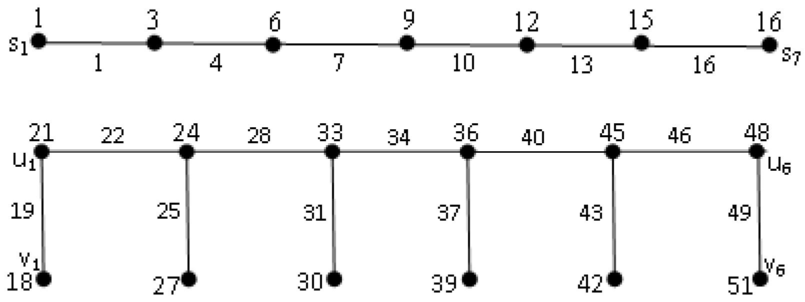

Figure-2.2: $P_7 \cup (P_6 \odot K_1)$

Theorem 2.5: $(P_m \odot \overline{K}_2) \cup P_n$ is one modulo three harmonic mean graph.

Proof: Let $u_1u_2 \ldots u_m$ be the path P_m and let v_i, w_i be the vertices which are joined to the vertex u_i of the path P_n, $1 \le i \le m$. The resultant graph is $(P_m \odot \overline{K}_2)$. Let $x_1x_2 \ldots x_n$ be the path P_n. Let $G = (P_m \odot \overline{K}_2) \cup P_n$. Define a function $\varphi : V(G) \to \{1, 3, 4, 6, \ldots, 3q-2, 3q\}$ by

$\varphi(v_1) = 1$; $\varphi(v_2) = 9$; $\varphi(v_i) = 9(i-1) - 2$, $3 \le i \le m$;
$\varphi(w_1) = 3$; $\varphi(w_2) = 12$; $\varphi(w_i) = 9i - 5$, $3 \le i \le m$;
$\varphi(u_1) = 4$; $\varphi(u_2) = 13$; $\varphi(u_i) = 9i - 6$, $3 \le i \le m$;
$\varphi(x_i) = 9m + 3(i-1) - 3$, $1 \le i \le m$.

Then φ induces a bijective function $\varphi^*: E(G) \to \{1, 4, 7, \ldots, 3q-2\}$, where

$\varphi^*(u_iv_i) = 9(i-1) + 1$, $1 \le i \le n$;
$\varphi^*(u_iw_i) = 9(i-1) + 4$, $1 \le i \le n$;
$\varphi^*(u_iu_{i+1}) = 9(i-1) + 7$, $1 \le i \le n-1$;
$\varphi^*(x_ix_{i+1}) = 9m + 3(i-1) - 2$, $1 \le i \le n-1$.

Thus the edges get the distinct labels 1, 4, …, $3q-2$. Therefore, φ is one modulo three harmonic mean labeling. Hence $P_m \cup (P_n \odot K_1)$ is one modulo three harmonic mean graph.

Example 2.6: One modulo three Harmonic mean labeling of $(P_5 \odot \overline{K}_2) \cup P_7$ is given in figure 2.3.

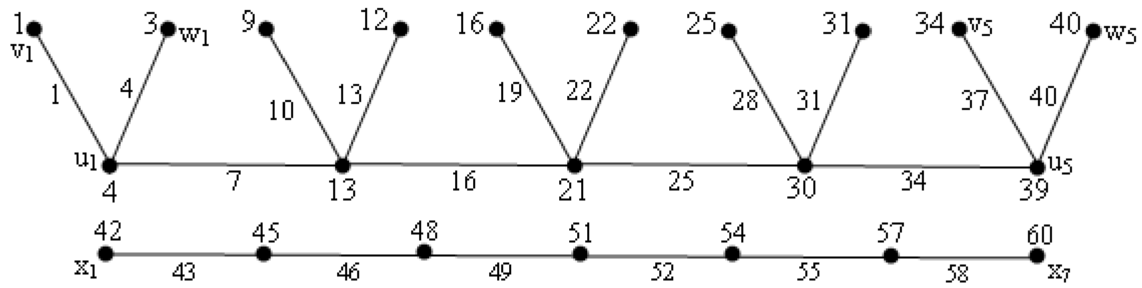

Figure-2.3: $(P_5 \odot \overline{K}_2) \cup P_7$

Construction: Let $L_n = P_n \times K_2$ be the ladder graph. $V(L_n) = \{u_i, v_i \,/\, 1 \le i \le n\}$ and $E(L_n) = \{u_iu_{i+1}, v_iv_{i+1} \,/ 1 \le i \le n-1\} \cup \{u_iv_i \,/ 1 \le i \le n\}$. The graph obtained from L_n by deleting the edge u_1v_1 is denoted by G_d.

Theorem 2.7: The graph G_d is one modulo three harmonic mean graph.

Proof: Let $V(G_d) = \{u_i, v_i \,/\, 1 \le i \le n\}$. $E(G_d) = \{u_iu_{i+1}, v_iv_{i+1} \,/\, 1 \le i \le n-1\} \cup \{u_iv_i \,/ 2 \le i \le n\}$. That is, $V(G_d) = V(L_n)$ and $E(G_d) = E(L_n) - \{u_1v_1\}$. Then G_d has $2n$ vertices and $3(n-1)$ edges. Define a function $\varphi : V(G_d) \to \{1, 3, 4, \ldots, 3q-2, 3q\}$ by

$\varphi(u_1) = 1$; $\varphi(u_2) = 9$; $\varphi(u_3) = 19$; $\varphi(u_i) = 9(i-1)$, $4 \le i \le n$;
$\varphi(v_1) = 3$; $\varphi(v_2) = 7$; $\varphi(v_i) = 9(i-1) - 3$, $3 \le i \le n$.

Then φ induces a bijective function $\varphi^*: E(G_d) \to \{1, 4, 7, \ldots, 3q - 2\}$, where

$\varphi^*(u_1u_2) = 1$; $\varphi^*(u_iu_{i+!}) = 9(i - 1) + 4, 2 \le i \le n - 1$;

$\varphi^*(v_1v_2) = 4$; $\varphi^*(v_iv_{i+1}) = 9(i - 1) + 1, 2 \le i \le n - 1$;

$\varphi^*(u_iv_i) = 9i - 2, 1 \le i \le n$.

Thus φ provides one modulo three harmonic mean labeling for G. Hence G_d is an one modulo three harmonic mean graph.

Example 2.8: One modulo three Harmonic mean labeling of G_d when n = 7 is given in figure 2.4.

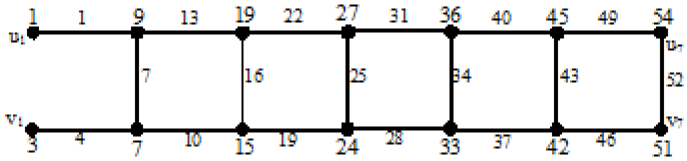

Figure-2.4: G_d

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