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THE EFFECTS OF HRF OF B-S DISTRIBUTION BY CBSM ON CORTISOL DUE TO STRESS IN HEALTHY SAMPLES

[1]Geetha. T and [2]Karthika. K*

[1]Assistant Professor, Department of Mathematics,
K. N. Govt. Arts College for Women, Thanjavur, Tamil Nadu, India.

[2]Research Scholar, Department of Mathematics,
K. N. Govt. Arts College for Women, Thanjavur, Tamil Nadu, India.

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ABSTRACT

In this paper, we discuss about the hazard rate function of Birnbaum–Saunders (B-S) Distribution. Particularly, the hazard function (hrf) of B-S Distribution is an upside down function for all values of the shape parameter. In reliability and survival analysis, as it is often of interest to determine the point at which the hazard function reaches its maximum. In application part, we discuss about the long-term effects of cognitive-behavioral stress management training on Cortisol stress responses in healthy men and women. Salivary Cortisol responses were taken repeatedly before and after the stress test. Subjects in the CBSM group showed significantly reduced Cortisol stress responses. However, the magnitude of the CBSM effect on Cortisol responses was smaller in women than in men. The hazard rate function has been applied to the Cortisol levels to check whether men or women have the lower Cortisol level by CBSM.

Keywords: *Cortisol, CBSM, stress, Birnbaum-Saunders distribution, hazard function.*

1. INTRODUCTION

The two-parameter Birnbaum-Saunders (BS) distribution [1] was originally proposed by Birn-baum and Saunders as a failure time distribution for fatigue failure caused under cyclic loading. The cumulative distribution function (CDF) of a two-parameter BS random variable T is of the form

$$F(t;\alpha,\beta) = \Phi\left[\frac{1}{\alpha}\left\{\left(\frac{t}{\beta}\right)^{1/2} - \left(\frac{\beta}{t}\right)^{1/2}\right\}\right],\quad 0 < t < \infty \quad \alpha,\beta > 0 \tag{1}$$

Where $\Phi(.)$ is the standard normal CDF. The parameters α and β in (1) are the shape and scale parameters, respectively. Although the BS distribution was originally proposed as a failure time distribution for fatigue failure under the assumption that the failure is due to development and growth of a dominant crack, a more general derivation was provided by Desmond. It is known that the density function of the BS distribution is unimodal. Though several articles have been published in the last three decades regarding different inferential methods for the parameters of the BS distribution and their properties, yet the shape of the hazard function has not been examined possibly due to its complex form. In this paper, we discuss about the hazard function of the BS distribution[2, 3] is indeed an upside down function of $t > 0$ for all values of the shape parameter α and scale parameter β. It is not uncommon to model survival and failure time data by distributions which have monotone hazard function. But in many practical situations, the hazard function is not monotone and in fact it increases up to a point and then decreases [5, 11].

1.1 BIRNBAUM-SAUNDERS DISTRIBUTION

The probability density function (PDF) of a two-parameter BS random variable T corresponding to the CDF in (1) is given by

$$f(t;\alpha,\beta) = \frac{1}{2\alpha\beta\sqrt{2\pi}}\left[\left(\frac{\beta}{t}\right)^{1/2} + \left(\frac{\beta}{t}\right)^{3/2}\right]\exp\left[-\frac{1}{2\alpha^2}\left(\frac{t}{\beta} + \frac{\beta}{t} - 2\right)\right],\quad 0 < t < \infty \quad \alpha,\beta > 0 \tag{2}$$

*Corresponding Author: [2]Karthika. K**
[2]Research Scholar, Department of Mathematics,
K. N. Govt. Arts College for Women, Thanjavur, Tamil Nadu, India.

Consider now the monotone transformation

$$X = \frac{1}{2}\left[\left(\frac{T}{\beta}\right)^{1/2} - \left(\frac{T}{\beta}\right)^{-1/2}\right] \quad (3)$$

Or

$$T = \beta\,\{1 + 2X^2 + 2X\,(1+ X^2)^{1/2}\} \quad (4)$$

then from (1), it readily follows that X is distributed as normal with mean zero and variance $(\alpha^2/4)$.

1.2 SHAPE OF THE HAZARD

To examine the shape of the hazard function, let us assume that the scale parameter $\beta = 1$, without loss of any generality. Let us consider the function

$$\epsilon(t) = t^{1/2} - t^{-1/2} \quad (5)$$

For which

$$\epsilon'(t) = \frac{1}{2}\,(t^{-1/2} + t^{-3/2})$$
$$= \frac{1}{2t}\,(t^{1/2} + t^{-1/2}) \quad (6)$$

$$\epsilon''(t) = -\frac{1}{4t^2}\,(t^{1/2} + 3t^{-1/2}) \quad (7)$$

And also

$$\epsilon^2(t) = t + \frac{1}{t} - 2 \quad (8)$$

The density function of the BS distribution in (2) (for $\beta = 1$) is then

$$f(t;\alpha) = \frac{1}{\alpha\sqrt{2\pi}}\,\epsilon'(t)\,\exp -\frac{1}{2\alpha^2}\,\epsilon^2(t) \quad (9)$$

This, in conjunction with the expression of the distribution function in (1), gives the hazard function as

$$h(t;\alpha) = \frac{f(t;\alpha)}{1 - F(t;\alpha,1)} = \frac{\frac{1}{\alpha\sqrt{2\pi}}\,\epsilon'(t)\,\exp -\frac{1}{2\alpha^2}\,\epsilon 2(t)}{\phi\left(-\frac{\epsilon(t)}{\alpha}\right)} \quad (10)$$

Result 1: The hazard function of Birnbaum-Saunders distribution is either an upside down or a decreasing function of $t > 0$, for all values of the shape parameter α.

Result 2: For $\alpha > 0$, the hazard function of the B-S distribution is indeed an upside down function.

Thus, from the above results we get the following theorem.

Theorem 1: The hazard function of the B-S distribution is an upside down function for all values of the shape parameter α.

2. APPLICATION

Because of the ubiquitous HPA axis has been proposed as a major reason to link psychological stress to its negative consequences on somatic and psychological wellbeing because of its ubiquitous effects on physiological systems. There is accumulating evidence supporting causal relation between increased activity and reactivity of HPA axis hormone and negative health outcomes [7]. For e.g.; the upper respiratory tract infection, associated with natural environment has been found when the HPA axis responses acute the laboratory stressors [6]. With respect to long term effects in men, increased incidence of cardiovascular-related events and type 2 diabetes was identified because of abnormal cortisol pattern. In women, long term detrimental effects of elevated Cortisol level in the elderly, with higher baseline urinary free cortisol levels being related to declines in memory performance were identified from the 'MacArthur studies of successful aging' or higher incidence of fractures in men and women[10,13,15].

In addition Cortisol is found a primary mediator of allostatic load (representing the cumulative physiological burden exacted on the body through attempts to adapt to life's demands) which has been associated with significantly increased risk for 7- year mortality as well as declines in cognitive & physical functioning in the elderly. Also, when Cortisol is administered before retrieval leads to a decrease in women performance. The experience or anticipation of stress has been shown related to the release of Cortisol. Even though, modulating the perception & appraisal as well as improving stress- related coping strategies are possible. Modulation in Cortisol levels leads a consequence of stressful experiences. Two weeks later a short group based cognitive-behavioral management (CBSM) training, reportedly attenuated Cortisol responses was observed to a standardized psychological stress test in healthy male subjects[8].

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2.1 METHODS

Three hundred and fifteen subjects were invited by letter for this study. This population was restricted from recruitment, to reduce interindividual differences in the frequencies 7extent of external academic stressors, online enrolment opportunity had given to interested subjects. Then they received a screening questionnaire contain exclusion criteria (like, acute or chronic somatic or psychiatric disorder). Out of the hundred and two online enrolled subjects, 83 were included in this study. After completing all screening questionnaire and informed consent and also fulfilling all the selection criteria all subjects were randomly assigned into 8 groups. Group size was restricted to maximum 12, because to allow CBSM group sessions. Then all the subjects were assigned to form one treatment group and one control group because, the treatment groups 1-4 & waiting control group 5-8 did not differ significantly by the demographic variables (age, gender, habitual smoking, use of oral contraceptives and body mass index). All subjects underwent TSST, 4 months after the CBSM. TSST was performed within a two week period by the subjects in the treatment group of CBSM; whereas control subjects in 4 week period. To produce major differences of external factors like season and external stressors, all subjects except some dropouts perform the TSST in the same 4 weeks. Refusal, inability, to keep the TSST appointment within the given period was found the reasons for the dropouts. But the dropouts did not differ from the TSST performances in the base line variables like age, gender, smoking etc., After TSST all controls had given CBSM training [4].

TSST

TSST has reportedly performed to induce profound endocrine responses in 70-80% of tested subjects. They were given a brief 2 min introduction to the TSST, then taken into a different room, where they given 10 min to prepare & complete the questionnaire designed to estimate cognitive appraisal processes (PASA) of the anticipated stress situation [9]. Then they were returned to TSST room, where they underwent a job interview (5 min) followed by a mental arithmetic task 5min in front of a committee of two people. Before TSST and every 10, 20, 30, 45, and 60 min time difference after the TSST saliva samples were taken[12, 14]. Also TSST was performed 14.00 to 18.00 hr to manage the variations of Cortisol levels over the circadian rhythm.

CBSM

Group –based cognitive –behavioral –stress management training was given to all subjects. Groups met on two alternate Saturdays, or Sundays respectively. A total of 10 hr CBSM based standardized training manual received by a groups of 11 subjects. Four cognitive-behavioral stress-reducing techniques (cognitive restructuring, problem-solving, self-instruction, progressive muscle relaxation) were focused in the CBSM [4]. All subjects had given a training manual containing a summary of theoretical models and stress-reducing techniques also with flash cards which briefly described each CBSM technique. Exercises and role-plays used in the group therapy were different from TSST.

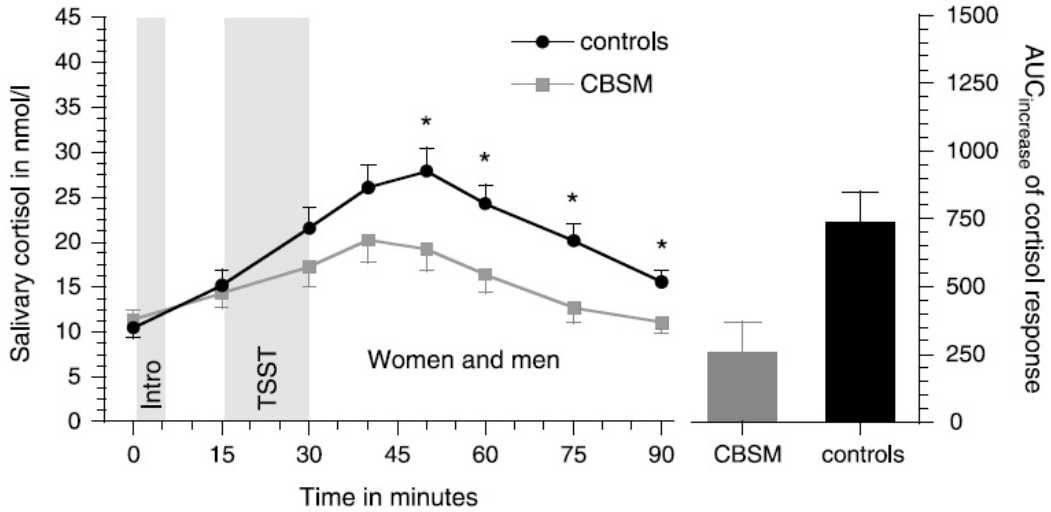


Figure 1: Cortisol responses to stress in all subjects (left-hand graph: repeated cortisol samples, right-hand graph: area under the response curve with regard to increase). *Indicate significance differences in pair wise comparison on the respective time point.

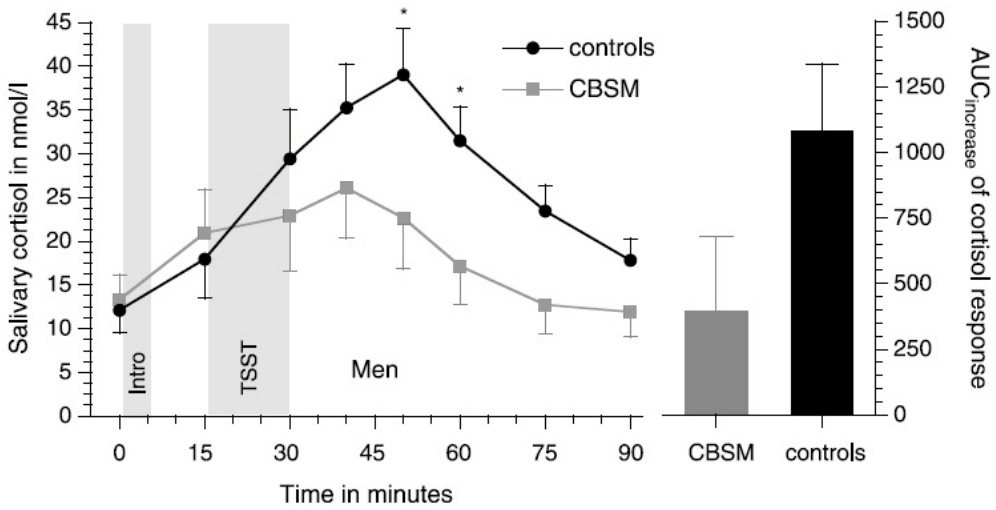


Figure 2: Cortisol responses to stress in men (left-hand graph: repeated Cortisol samples, right-hand graph: area under the response curve with regard to increase). *Indicate significance differences in pair wise comparison on the respective time point.

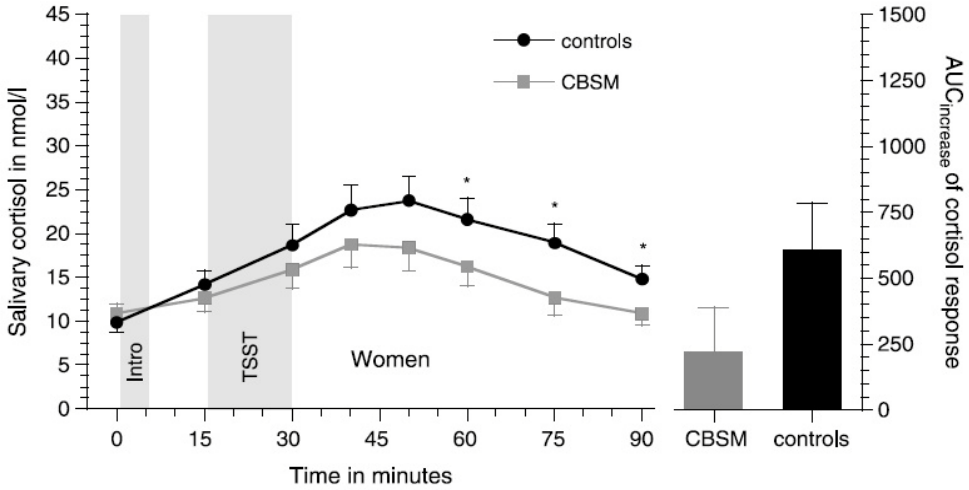
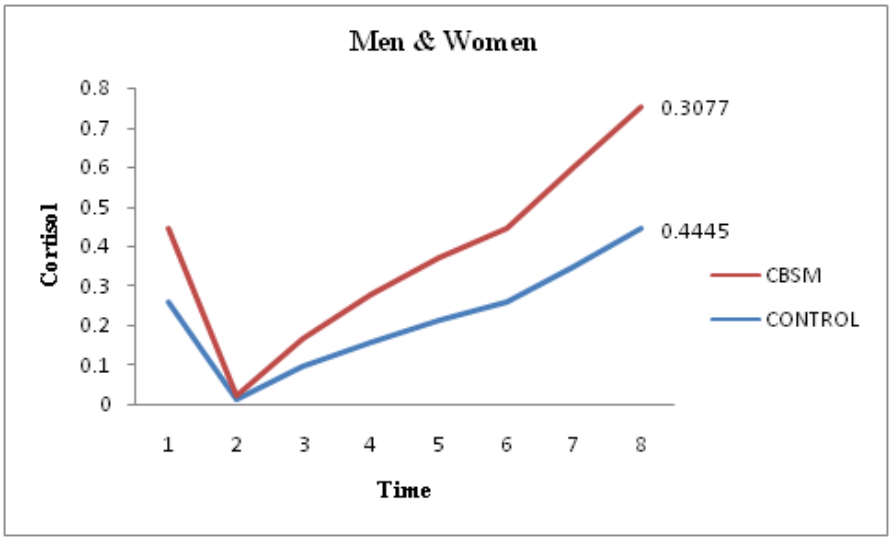
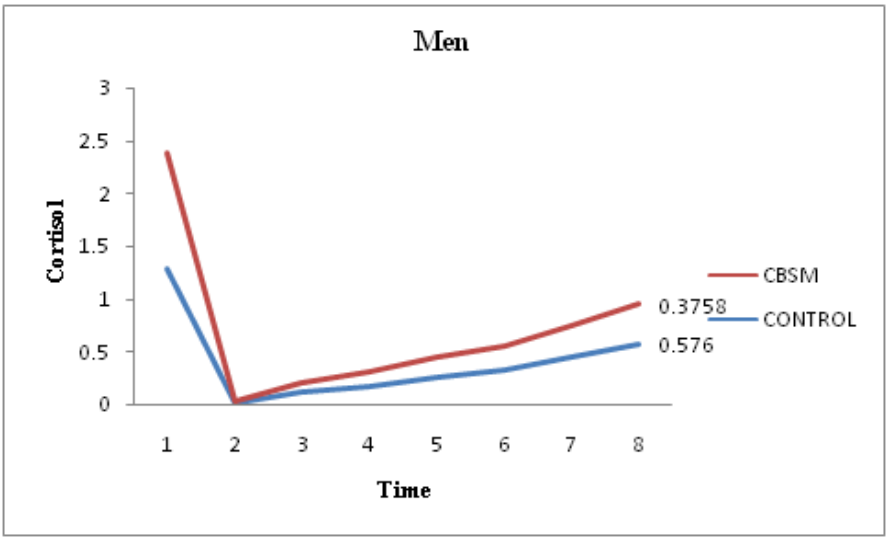
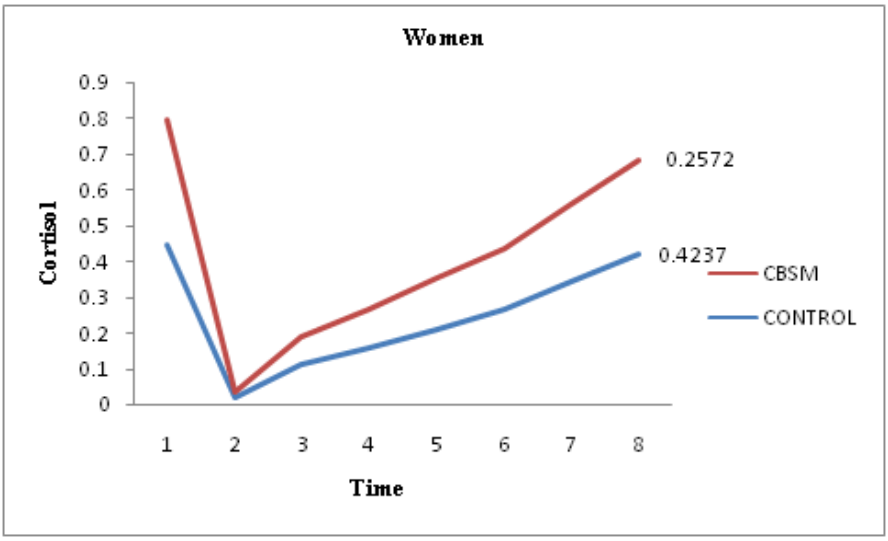


Figure 3: Cortisol responses to stress in women (left-hand graph: repeated Cortisol samples, right-hand graph: area under the response curve with regard to increase). *Indicate significance differences in pair wise comparison on the respective time point.

3. MATHEMATICAL RESULTS







4. CONCLUTION

Release of Cortisol caused by psychosocial stress. Also exaggerated secretion of Cortisol has been identified for the negative effects on somatic health and cognitive functioning. The long term effects of CBSM on Cortisol stress responses examined in healthy men and women. Results showed the significantly reduced Cortisol stress responses for the subjects in CBSM group particularly in women. When this result is applied to the hazard rate of B-S distribution, we found that the women subjects had the notably reduced hazard rate level when compared to men.

5. REFERENCES

1. Balakrishnan,N., Leiva,V., Lopez, J. (2007), "Acceptance sampling plans from truncated life tests based on the generalized Birnbaum-Saunders distribution", Communications in Statistics – Simulation and Computation, vol. 36, 643-656.
2. Bennett, S. (1983).”Logistic regression model for survival data", Applied Statistics, vol. 32, 165-171.
3. Birnbaum, Z. W. and Saunders, S. C. (1969). “A new family of life distribution", Journal of Applied Probability, vol. 6, 319-327.
4. Cruess,D.G.,Antoni,M.H.,McGregor,B.A.,Kilbourn,K.M.,Boyers,A.E.,Alferi,S.M.,Carver,C.S.,Kumar,M.,2000.Cognitive-behavioral stress management reduces serum cortisol by enhancing benefit finding among women being treated for early stage breast cancer. Psychosom. Med.62, 304–308.
5. Desmond, A. F. (1985). \Stochastic models of failure in random environments", Canadian Journal of Statistics, vol. 13, 171-183.
6. Dickerson, S.S., Kemeny, M.E., 2004.Acute stressors and cortisol responses: a theoretical integration and synthesis of laboratoryresearch.Psychol.Bull.130, 355–391.
7. Ehlert, U., Gaab, J., Heinrichs, M., 2001.Psychoneuroendocrinological contributions to the etiology of depression, posttraumatic stress disorder, and stress-related bodily disorders: the role of the hypothalamus-pituitary-adrenal axis.Biol.Psychol.57, 141–152.

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8. Gaab,J., Blattler,N.,Menzi,T.,Pabst,B.,Stoyer,S.,Ehlert,U.,2003.Randomized controlled Evaluation of the effects of cognitive-behavioral stress management on cortisol responses to acute stress in healthy subjects. Psychoneuroendocrinology 28,767–779.
9. Gaab, J., Rohleder, N., Nater, U.M., Ehlert, U., 2005.Psychological determinants of the cortisol stress response: the role of anticipatory cognitive appraisal.Psychoneuroendocrinology30, 599–610.
10. Greendale,G.A.,Unger,J.B.,Rowe,J.W.,Seeman,T.E.,1999.Therelation between cortisol excretion and fractures in healthyolderpeople: results from the MacArthurstudiesMac.J.Am.Geriatr.Soc.47,799–803.
11. Glaser, R. A. (1980). “Bathtub and related failure rate characterization", Journal of the American Statistical Association, vol. 75, 667-672.
12. Kirschbaum, C., Hellhammer, D.H., 1994.Salivary cortisol in psychoneuroendocrine research: recent developments and applications.Psychoneuroendocrinology19,313–333.
13. Kirschbaum,C.,Wust,S.,Hellhammer,D.,1992.Consistent sex differences in cortisol responses to psychological stress.Psychosom.Med.54,648–657.
14. Kirschbaum,C.,Pirke,K.M.,Hellhammer,D.H.,1993.The’Trier Social Stress Test’–a tool for investigating psychobiological stress responses in a laboratory setting. Neuropsychobiology 28,76–81
15. Seeman,T.E.,McEwen,B.S.,Singer,B.H.,Albert,M.S.,Rowe,J.W.,1997.Increase in urinary cortisol excretion and memory declines: Mac Arthur studies of successful aging. J. Clin.Endocrinol.Metab.82, 2458–2465.

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