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SOME QUADRUPLE FIXED POINT THEOREMS IN QUASI- PARTIAL METRIC SPACES

Manoj Ughade*[1] and R. D. Daheriya[2]

[1]Department of Mathematics,
Sarvepalli Radhakrishnan University, Bhopal, P.O. Box 462026, Madhya Pradesh, India.

[2]Department of Mathematics,
Government J.H. Post Graduate College, Betul, P.O. Box 460001, Madhya Pradesh, India.

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ABSTRACT

Recently, many results on quadruple fixed point theory exist in the literature, for more details, one can see in [6, 8, 5,4,10, and 34]. In this manuscript, we introduce the concept of w-compatible of mappings $F: X^4 \to X$ and $g: X \to X$ and based on this notion, we obtain some new common quadruple fixed point results for mappings defined on a set equipped with two quasi-partial metrics that unify, generalize, enrich, improve, extend and complement several existing results in the previous works: Aydi [14], Karapinar et al. [9], Shatanawi and Pitea [29], Feng Gu et al. [11] and Feng Gu [12] to the quadruple fixed point theory. Our result is also substantiated with the aid of an appropriate example.

Keywords and Phrases: *Common quadruple fixed point; quadruple coincidence point; w-compatible pairs, quasi-partial metric space.*

AMS Subject Classification (2010): *47H10, 54H25.*

1. INTRODUCTION AND PRELIMINARIES

In 1994, Matthews [23] introduced the notion of partial metric spaces as follows.

Definition 1.1: (see [23]) A partial metric on a nonempty set X is a function $p: X \times X \to \mathbb{R}^+$ such that for all $x, y, z \in X$:
(p1) $x = y \Leftrightarrow p(x,x) = p(x,y) = p(y,y)$,
(p2) $p(x,x) \leq p(x,y)$,
(p3) $p(x,y) = p(x,y)$,
(p4) $p(x,y) \leq p(x,z) + p(z,y) - p(z,z)$.

A partial metric space is a pair (X,p) such that X is a nonempty set and p is a partial metric on X.

In [23], Matthews extended the Banach contraction principle from metric spaces to partial metric spaces. Karapinar *et al.* [9] introduced the concept of quasi-partial metric spaces and studied some fixed-point problems on quasi-partial metric spaces. The notion of a quasi-partial metric space is defined as follows.

Definition 1.2: (see [9]) A quasi-partial metric on a nonempty set X is a function $q: X \times X \to \mathbb{R}^+$ which satisfies:
(QPM1) If $q(x,x) = q(x,y) = q(y,y)$, then $x = y$,
(QPM2) $q(x,x) \leq q(x,y)$,
(QPM3) $q(x,x) \leq q(y,x)$,
(QPM4) $q(x,y) + q(z,z) \leq q(x,z) +, q(z,y)$.
for all $x, y, z \in X$.

A quasi-partial metric space is a pair (X,q) such that X is a nonempty set and q is a quasi-partial metric on X.

Let q is a quasi-partial metric on set X. Then

$$d_q(x,y) = q(x,y) + q(y,x) - q(y,y) \tag{1.1}$$

is a metric on X.

Corresponding Author: Manoj Ughade[1]*

Definition 1.3: (see [9]) Let (X, q) be a quasi-partial metric space. Then

1) A sequence (x_n) converges to a point $x \in X$ if and only if
$$q(x,x) = lim_{n\to\infty} q(x,x_n) = lim_{n\to\infty} q(x_n,x)\,;$$
2) A sequence (x_n) is called a Cauchy sequence if
$$lim_{n\to\infty} q(x_n,x_m) \text{ and } lim_{n\to\infty} q(x_m,x_n)$$
exist (and are finite);
3) The quasi-partial metric space (X, q) is said to be complete if every Cauchy sequence (x_n) in X converges, with respect to τ_q, to a point $x \in X$ such that
$$q(x,x) = lim_{n\to\infty} q(x_n,x_m) = lim_{n\to\infty} q(x_m,x_n).$$

The following Lemma is crucial in our work.

Lemma 1.4: (see [9]) Let (X, q) be a quasi-partial metric space. Then the following statements hold true:
(A) If $q(x,y) = 0$, then $x = y$.
(B) If $x \neq y$, then $q(x,y) > 0$ and $q(y,x) > 0$.

Bhaskar and Lakshmikantham in [25] introduced the concept of coupled fixed point and investigated the existence and uniqueness of a coupled fixed point theorem in partially ordered complete metric spaces. Later, Lakshmikantham and Ciric in [28] defined mixed g-monotone property and coupled coincidence point in partially ordered metric spaces. For some work on a coupled fixed point, we refer the reader to [1, 2, 7, 14, 17, 18, 20, 24, 28-33, 35, 36]. Recently, Feng Gun and Lin Wang [11] obtained some common coupled fixed-point results for a pair of mappings in two quasi-partial metric spaces.

In 2011, Samet and Vetro [33] introduced a fixed point of order N ≥ 3. Note that, Berinde and Borcut [18] defined differently the notion of a tripled fixed point in the case of ordered sets in order to keep true the mixed monotone property. For more details, see [18]. For some work on a tripled fixed point, we refer the reader to [13, 15, 16, 19, 26, 27, and 35]. Abbas *et al.* [20] introduced the concept of w-compatible mapping and based on this notion, established some common coupled fixed point theorems in cone metric spaces. Following this trend, H. Aydi *et al.* [13] introduced the concept of w-compatible mapping and established some common tripled fixed point theorems in partial metric spaces. Very recently, Feng Gu [12], established some new common tripled fixed point theorems for mappings defined on a set equipped with two quasi-partial metrics.

Theorem 1.5: (see [12]) Let q_1 and q_2 be two quasi-metrics on X such that $q_2(x,y) \leq q_1(x,y)$, for all x, y ∈ X, and let $F : X^3 \to X,\ g : X \to X$ be two mappings. Suppose that there exist k_1, k_2, k_3, k_4, k_5 in [0,1) with
$$k_1 + k_2 + k_3 + 2k_4 + k_5 < 1 \tag{1.2}$$
such that the condition
$$\begin{aligned} q_1\big(F(x,y,z),F(u,v,w)\big) + q_1\big(F(y,x,y),F(v,u,v)\big) + q_1\big(F(z,y,x),F(w,v,u)\big) \\ \leq k_1[q_2(gx,gu) + q_2(gy,gv) + q_2(gz,gw)] \\ +k_2\big[q_2\big(gx,F(x,z,y)\big) + q_2\big(gy,F(y,x,y)\big) + q_2\big(gz,F(z,y,x)\big)\big] \\ +k_3\big[q_2\big(gu,F(u,v,w)\big) + q_2\big(gv,F(v,u,v)\big) + q_2\big(gw,F(w,v,u)\big)\big] \\ +k_4\big[q_2\big(gx,F(u,v,w)\big) + q_2\big(gy,F(v,u,v)\big) + q_2\big(gz,F(w,v,u)\big)\big] \\ +k_5\big[q_2\big(gu,F(x,y,z)\big) + q_2\big(gv,F(y,x,y)\big) + q_2\big(gw,F(z,y,x)\big)\big] \end{aligned} \tag{1.3}$$
holds for all $x, y, z, u, v, w \in X$. Also, suppose we have the following hypotheses:
(i) $F(X^3) \subset g(X)$.
(ii) $g(X)$ is a complete subspace of X with respect to the quasi-partial metric q_1.

Then the mappings F and g have a coincidence point (x, y, z) satisfying $gx = F(x,y,z) = gy = F(y,x,y) = gz = F(z,y,x)$. Moreover, if F and g are w-compatible, then F and g have a unique common tripled fixed point of the form (u, u, u).

Karapinar [6] used the notion of quadruple fixed point and obtained some quadruple fixed point theorems in partially ordered metric spaces. Later, various results on quadruple fixed point have been obtained; see, for example, [4-6, 8, 10, and 34].

Definition 1.6: (see [4-6, 8, 10, and 34]) An element $(x, y, z, w) \in X^4$ is called

1) a quadruple fixed point of the mapping $F: X^4 \to X$ if
$F(x,y,z,w) = x, F(y,z,w,x) = y, F(z,w,x,y) = z, F(w,x,y,z) = w$;
2) a quadruple coincidence point of the mapping $F : X^4 \to X$ and $g : X \to X$ if $F(x,y,z,w) = gx$, $F(y,z,w,x) = gy$, $F(z,w,x,y) = gz$, $F(w,x,y,z) = gw$; in this case (gx, gy, gz, gw) is called point of coincidence of mappings F and g;

3) a common quadruple fixed point of the mapping $F : X^4 \to X$ and $g : X \to X$ if
$$F(x,y,z,w) = gx = x, F(y,z,w,x) = gy = y,$$
$$F(z,w,x,y) = gz = z, F(w,x,y,z) = gw = w;$$

In this paper, we introduce the concept of w-compatible of mappings $F: X^4 \to X$ and $g: X \to X$ and based on this notion, we obtain some new common quadruple fixed point results for mappings defined on a set equipped with two quasi-partial metrics. This paper can be considered as a continuation of the remarkable works of Karapinar *et al.* [Math. Comput. Model. 57:2442-2448, 2013], Shatanawi and Pitea [Fixed Point Theory and Applications 2013, 2013:153], Feng Gu *et al.* [Fixed Point Theory Appl. 2014, 2014:19] and Feng Gu [Fixed Point Theory and Applications 2014, 2014:71]. We also state some examples to illustrate the effectiveness of our results.

2. MAIN RESULTS

We start this section with the following definition.

Definition 2.1: We say that the mappings $F: X^4 \to X$ and $g: X \to X$ are w-compatible if
$$gF(x,y,z,w) = F(gx,gy,gz,gw)$$
whenever
$F(x,y,z,w) = gx, F(y,z,w,x) = gy, F(z,w,x,y) = gz, F(w,x,y,z) = gw.$

Denote
$F_x = F(x,y,z,w), \quad F_y = F(y,z,w,y), \quad F_x = F(z,w,x,y), \quad F_x = F(w,x,y,z).$

Now we shall prove our main results as follows.

Theorem 2.2: Let q_1 and q_2 be two quasi-metrics on X such that $q_2(x,y) \le q_1(x,y)$, for all $x,y \in X$, and let $F: X^4 \to X$, $g : X \to X$ be two mappings. Suppose that there exist k_1, k_2, k_3, k_4, k_5 in [0,1) with
$$k_1 + k_2 + k_3 + 2k_4 + k_5 < 1 \tag{2.1}$$
such that the condition

$$\begin{aligned} q_1(F_x,F_a) + q_1(F_y,F_b) + q_1(F_z,F_c) + q_1(F_w,F_d) \le\ & k_1[q_2(gx,ga) + q_2(gy,gb) + q_2(gz,gc) + q_2(gw,gd)] \\ & +k_2[q_2(gx,F_x) + q_2(gy,F_y) + q_2(gz,F_z) + q_2(gw,F_w)] \\ & +k_3[q_2(ga,F_a) + q_2(gb,F_b) + q_2(gc,F_c) + q_2(gd,F_d)] \\ & +k_4[q_2(gx,F_a) + q_2(gy,F_b) + q_2(gz,F_c) + q_2(gw,F_d)] \\ & +k_5[q_2(ga,F_x) + q_2(gb,F_y) + q_2(gc,F_z) + q_2(gd,F_w)] \end{aligned} \tag{2.2}$$
holds for all $x,y,z,w,a,b,c,d \in X$. Also, suppose we have the following hypotheses:
(iii) $F(X^4) \subset g(X)$.
(iv) $g(X)$ is a complete subspace of X with respect to the quasi-partial metric q_1.

Then the mappings F and g have a quadruple coincidence point (x,y,z,w) satisfying $gx = F_x = gy = F_y = gz = F_z = gw = F_w$. Moreover, if F and g are w-compatible, then F and g have a unique common quadruple fixed point of the form (u,u,u,u).

Proof: Let $x_0, y_0, z_0, w_0 \in X$. Since $F(X^4) \subset g(X)$, we can choose $x_1, y_1, z_1, w_1 \in X$ such that $gx_1 = F_{x_0}$, $gy_1 = F_{y_0}, gz_1 = F_{z_0}$ and $gw_1 = F_{w_0}$. Similarly, we can choose x_2, y_2, z_2, $w_2 \in X$ such that $gx_2 = F_{x_1}$, $gy_2 = F_{y_1}, gz_2 = F_{z_1}$ and $gw_2 = F_{w_1}$. Repeating this process, we can construct three sequences $\{x_n\}, \{y_n\}, \{z_n\}$and $\{w_n\}$ in X such that

$$gx_{n+1} = F_{x_n},\ gy_{n+1} = F_{y_n},\ gz_{n+1} = F_{z_n}, gw_{n+1} = F_{w_n}\ \forall\, n \ge 0. \tag{2.3}$$

Now, for $n \in \mathbb{N}$, we denote
$Q^1_{x_n} = q_1(gx_n, gx_{n+1}), \qquad Q^1_{y_n} = q_1(gy_n, gy_{n+1}),$
$Q^1_{z_n} = q_1(gz_n, gz_{n+1}), \qquad Q^1_{w_n} = q_1(gw_n, gw_{n+1}),$
$Q^2_{x_n} = q_2(gx_n, gx_{n+1}), \qquad Q^2_{y_n} = q_2(gy_n, gy_{n+1}),$
$Q^2_{z_n} = q_2(gz_n, gz_{n+1}), \qquad Q^1_{w_n} = q_2(gw_n, gw_{n+1}),$

It follows from (2.2) and (QPM4) that
$$Q^1_{x_n} + Q^1_{y_n} + Q^1_{z_n} + Q^1_{w_n} = q_1(gx_n, gx_{n+1}) + q_1(gy_n, gy_{n+1}) + q_1(gz_n, gz_{n+1}) + q_1(gw_n, gw_{n+1})$$
$$= q_1(F_{x_{n-1}}, F_{x_n}) + q_1(F_{y_{n-1}}, F_{y_n}) + q_1(F_{z_{n-1}}, F_{z_n}) + q_1(F_{w_{n-1}}, F_{w_n})$$

$$\begin{aligned}
&\leq k_1[q_2(gx_{n-1},gx_n)+q_2(gy_{n-1},gy_n)+q_2(gz_{n-1},gz_n)+q_2(gw_{n-1},gw_n)]\\
&\quad +k_2\left[q_2\left(gx_{n-1},F_{x_{n-1}}\right)+q_2\left(gy_{n-1},F_{y_{n-1}}\right)+q_2\left(gz_{n-1},F_{z_{n-1}}\right)+q_2\left(gw_{n-1},F_{w_{n-1}}\right)\right]\\
&\quad +k_3\left[q_2\left(gx_n,F_{x_n}\right)+q_2\left(gy_n,F_{y_n}\right)+q_2\left(gz_n,F_{z_n}\right)+q_2\left(gw_n,F_{w_n}\right)\right]\\
&\quad +k_4\left[q_2\left(gx_{n-1},F_{x_n}\right)+q_2\left(gy_{n-1},F_{y_n}\right)++q_2\left(gz_{n-1},F_{z_n}\right)+q_2\left(gw_{n-1},F_{w_n}\right)\right]\\
&\quad +k_5\left[q_2\left(gx_n,F_{x_{n-1}}\right)+q_2\left(gy_n,F_{y_{n-1}}\right)+q_2\left(gz_n,F_{z_{n-1}}\right)+q_2\left(gw_n,F_{w_{n-1}}\right)\right]\\
&= (k_1+k_2)\begin{bmatrix} q_2(gx_{n-1},gx_n)+q_2(gy_{n-1},gy_n)+q_2(gz_{n-1},gz_n)\\ +q_2(gw_{n-1},gw_n)\end{bmatrix}\\
&\quad +k_3[q_2(gx_n,gx_{n+1})+q_2(gy_n,gy_{n+1})+q_2(gz_n,gz_{n+1})+q_2(gw_n,gw_{n+1})]\\
&\quad +k_4[q_2(gx_{n-1},gx_{n+1})+q_2(gy_{n-1},gy_{n+1})+q_2(gz_{n-1},gz_{n+1})]\\
&\quad +k_5[q_2(gx_n,gx_n)+q_2(gy_n,gy_n)+q_2(gz_n,gz_n)]\\
&\leq (k_1+k_2)\left[Q^2_{x_{n-1}}+Q^2_{y_{n-1}}+Q^2_{z_{n-1}}+Q^2_{w_{n-1}}\right]+k_3\left[Q^2_{x_n}+Q^2_{y_n}+Q^2_{z_n}+Q^2_{w_n}\right]\\
&\quad +k_4\begin{bmatrix} Q^2_{x_{n-1}}+Q^2_{x_n}-q_2(gx_n,gx_n)+Q^2_{y_{n-1}}+Q^2_{y_n}-q_2(gy_n,gy_n)\\ +Q^2_{z_{n-1}}+Q^2_{z_n}-q_2(gz_n,gz_n)+Q^2_{w_{n-1}}+Q^2_{w_n}-q_2(gw_n,gw_n)\end{bmatrix}\\
&\quad +k_5\left[Q^2_{x_n}+Q^2_{y_n}+Q^2_{z_n}+Q^2_{w_n}\right]\\
&\leq (k_1+k_2+k_4)\left[Q^2_{x_{n-1}}+Q^2_{y_{n-1}}+Q^2_{z_{n-1}}+Q^2_{w_{n-1}}\right]\\
&\quad +(k_3+k_4+k_5)\left[Q^2_{x_n}+Q^2_{y_n}+Q^2_{z_n}+Q^2_{w_n}\right]\\
&\leq (k_1+k_2+k_4)\left[Q^1_{x_{n-1}}+Q^1_{y_{n-1}}+Q^1_{z_{n-1}}+Q^1_{w_{n-1}}\right]\\
&\quad +(k_3+k_4+k_5)\left[Q^1_{x_n}+Q^1_{y_n}+Q^1_{z_n}+Q^1_{w_n}\right].
\end{aligned}$$

This implies that

$$Q^1_{x_n}+Q^1_{y_n}+Q^1_{z_n}+Q^1_{w_n}\leq \frac{k_1+k_2+k_4}{1-k_3+k_4+k_5}\left[Q^1_{x_{n-1}}+Q^1_{y_{n-1}}+Q^1_{z_{n-1}}+Q^1_{w_{n-1}}\right] \tag{2.4}$$

Put $k=\frac{k_1+k_2+k_4}{1-k_3+k_4+k_5}$. Obviously, $0\leq k<1$. By repetition of the above inequality (2.4) n-times, we get

$$Q^1_{x_n}+Q^1_{y_n}+Q^1_{z_n}+Q^1_{w_n}\leq k^n\left[Q^1_{x_{n-1}}+Q^1_{y_{n-1}}+Q^1_{z_{n-1}}+Q^1_{w_{n-1}}\right] \tag{2.5}$$

Next, we shall prove that $\{gx_n\},\{gy_n\},\{gz_n\}$ and $\{gw_n\}$ are Cauchy sequences in $g(X)$. In fact, for each $n,m\in\mathbb{N}, m>n$, from(QPM$_4$) and (2.5) we have

$$\begin{aligned}
q_1(gx_n,gx_m)+q_1(gy_n,gy_m)+q_1(gz_n,gz_m)+q_1(gw_n,gw_m) &\leq \textstyle\sum_{i=n}^{m-1}\left[Q^1_{x_i}+Q^1_{y_i}+Q^1_{z_i}+Q^1_{w_i}\right]\\
&\leq \textstyle\sum_{i=n}^{m-1}k^i\left[Q^1_{x_0}+Q^1_{y_0}+Q^1_{z_0}+Q^1_{w_0}\right]\\
&\leq \frac{k^n}{1-k}\left[Q^1_{x_0}+Q^1_{y_0}+Q^1_{z_0}+Q^1_{w_0}\right]
\end{aligned} \tag{2.6}$$

This implies that

$lim_{n,m\to\infty}[q_1(gx_n,gx_m)+q_1(gy_n,gy_m)+q_1(gz_n,gz_m)+q_1(gw_n,gw_m)]=0,$
and so

$$\begin{aligned}
&lim_{n,m\to\infty}\, q_1(gx_n,gx_m)=0,\ lim_{n,m\to\infty}\, q_1(gy_n,gy_m)=0,\\
&lim_{n,m\to\infty}\, q_1(gz_n,gz_m)=0,\ \ lim_{n,m\to\infty}\, q_1(gw_n,gw_m)=0.
\end{aligned} \tag{2.7}$$

By similar arguments as above, we can show that

$$\begin{aligned}
&lim_{n,m\to\infty}\, q_1(gx_m,gx_n)=0, lim_{n,m\to\infty}\, q_1(gy_m,gy_n)=0,\\
&lim_{n,m\to\infty}\, q_1(gz_m,gz_n)=0, lim_{n,m\to\infty}\, q_1(gw_m,gw_n)=0.
\end{aligned} \tag{2.8}$$

Hence $\{gx_n\},\{gy_n\},\{gz_n\}$ and $\{gw_n\}$ are Cauchy sequences in $(g(X),q_1)$. Since $(g(X),q_1)$ is complete, there exist $gx,gy,gz\in g(X)$ such that $\{gx_n\},\{gy_n\},\{gz_n\}$ and $\{gw_n\}$converge to gx,gy,gz and gw with respect to τ_{q_1}, that is,

$$\begin{aligned}
q_1(gx,gx) &= \lim_{n,m\to\infty} q_1(gx_n,gx)=\lim_{n,m\to\infty} q_1(gx,gx_n)\\
&= lim_{n,m\to\infty}\, q_1(gx_n,gx_m)=lim_{n,m\to\infty}\, q_1(gx_m,gx_n),
\end{aligned} \tag{2.9}$$

$$\begin{aligned}
q_1(gy,gy) &= \lim_{n,m\to\infty} q_1(gy_n,gy)=\lim_{n,m\to\infty} q_1(gy,gy_n)\\
&= lim_{n,m\to\infty}\, q_1(gy_n,gy_m)=lim_{n,m\to\infty}\, q_1(gy_m,gy_n),
\end{aligned} \tag{2.10}$$

$$\begin{aligned}
q_1(gz,gz) &= \lim_{n,m\to\infty} q_1(gz_n,gz)=\lim_{n,m\to\infty} q_1(gz,gz_n)\\
&= lim_{n,m\to\infty}\, q_1(gz_n,gz_m)=lim_{n,m\to\infty}\, q_1(gz_m,gz_n).
\end{aligned} \tag{2.11}$$

and

$$q_1(gw,gw) = \lim_{n,m\to\infty} q_1(gw_n,gw) = \lim_{n,m\to\infty} q_1(gw,gw_n)$$
$$= \lim_{n,m\to\infty} q_1(gw_n,gw_m) = \lim_{n,m\to\infty} q_1(gw_m,gw_n). \tag{2.12}$$

Combining (2.7)-(2.12), we have

$$q_1(gx,gx) = \lim_{n,m\to\infty} q_1(gx_n,gx) = \lim_{n,m\to\infty} q_1(gx,gx_n)$$
$$= \lim_{n,m\to\infty} q_1(gx_n,gx_m) = \lim_{n,m\to\infty} q_1(gx_m,gx_n) = 0, \tag{2.13}$$

$$q_1(gy,gy) = \lim_{n,m\to\infty} q_1(gy_n,gy) = \lim_{n,m\to\infty} q_1(gy,gy_n)$$
$$= \lim_{n,m\to\infty} q_1(gy_n,gy_m) = \lim_{n,m\to\infty} q_1(gy_m,gy_n) = 0, \tag{2.14}$$

$$q_1(gz,gz) = \lim_{n,m\to\infty} q_1(gz_n,gz) = \lim_{n,m\to\infty} q_1(gz,gz_n)$$
$$= \lim_{n,m\to\infty} q_1(gz_n,gz_m) = \lim_{n,m\to\infty} q_1(gz_m,gz_n) = 0. \tag{2.15}$$

and

$$q_1(gw,gw) = \lim_{n,m\to\infty} q_1(gw_n,gw) = \lim_{n,m\to\infty} q_1(gw,gw_n)$$
$$= \lim_{n,m\to\infty} q_1(gw_n,gw_m) = \lim_{n,m\to\infty} q_1(gw_m,gw_n) = 0. \tag{2.16}$$

By (QPM_4), we obtain

$$\begin{aligned} q_1(gx_{n+1},F_x) &\le q_1(gx_{n+1},gx) + q_1(gx,F_x) - q_1(gx,gx) \\ &\le q_1(gx_{n+1},gx) + q_1(gx,F_x) \\ &\le q_1(gx_{n+1},gx) + q_1(gx,gx_{n+1}) + q_1(gx_{n+1},F_x) - q_1(gx_{n+1},gx_{n+1}) \\ &\le q_1(gx_{n+1},gx) + q_1(gx,gx_{n+1}) + q_1(gx_{n+1},F_x) \end{aligned}$$

Letting $n \to \infty$ in the above inequalities and using (2.13), we have

$$\lim_{n\to\infty} q_1(gx_{n+1},F_x) \le q_1(gx,F_x) \le \lim_{n\to\infty} q_1(gx_{n+1},F_x).$$

That is,

$$\lim_{n\to\infty} q_1(gx_{n+1},F_x) = q_1(gx,F_x) \tag{2.17}$$

Similarly, using (2.14), (2.15) and (2.14), respectively we have

$$\lim_{n\to\infty} q_1(gy_{n+1},F_y) = q_1(gy,F_y) \tag{2.18}$$

$$\lim_{n\to\infty} q_1(gz_{n+1},F_z) = q_1(gz,F_z) \tag{2.19}$$

and

$$\lim_{n\to\infty} q_1(gw_{n+1},F_w) = q_1(gw,F_w) \tag{2.20}$$

Now, we prove that $gx = F_x, gy = F_y$, gz $= F_z$ and $gw = F_w$. In fact, it follows from (2.2) and (2.3) that

$$\begin{aligned} & q_1(gx_{n+1},F_x) + q_1(gy_{n+1},F_y) + q_1(gz_{n+1},F_z) + q_1(gw_{n+1},F_w) \\ &= q_1(F_{x_n},F_x) + q_1(F_{y_n},F_y) + q_1(F_{z_n},F_z) + q_1(F_{w_n},F_w) \\ &\le k_1[q_2(gx_n,gx) + q_2(gy_n,gy) + q_2(gz_n,gz) + q_2(gw_n,gw)] \\ &\quad + k_2[q_2(gx_n,F_{x_n}) + q_2(gy_n,F_{y_n}) + q_2(gz_n,F_{z_n}) + q_2(gw_n,F_{w_n})] \\ &\quad + k_3[q_2(gx,F_x) + q_2(gy,F_y) + q_2(gz,F_z) + q_2(gw,F_w)] \\ &\quad + k_4[q_2(gx_n,F_x) + q_2(gy_n,F_y) + q_2(gz_n,F_z) + q_2(gw_n,F_w)] \\ &\quad + k_5[q_2(gx,F_{x_n}) + q_2(gy,F_{y_n}) + q_2(gz,F_{z_n}) + q_2(gw,F_{w_n})] \\ &= k_1[q_2(gx_n,gx) + q_2(gy_n,gy) + q_2(gz_n,gz) + q_2(gw_n,gw)] \\ &\quad + k_2[Q_{x_n}^2 + Q_{y_n}^2 + Q_{z_n}^2 + Q_{w_n}^2] \\ &\quad + k_3[q_2(gx,F_x) + q_2(gy,F_y) + q_2(gz,F_z) + q_2(gw,F_w)] \\ &\quad + k_4[q_2(gx_n,F_x) + q_2(gy_n,F_y) + q_2(gz_n,F_z) + q_2(gw_n,F_w)] \\ &\quad + k_5[q_2(gx,gx_{n+1}) + q_2(gy,gy_{n+1}) + q_2(gz,gz_{n+1}) + q_2(gw,gw_{n+1})] \\ &\le k_1[q_1(gx_n,gx) + q_1(gy_n,gy) + q_1(gz_n,gz) + q_1(gw_n,gw)] \\ &\quad + k_2[Q_{x_n}^1 + Q_{y_n}^1 + Q_{z_n}^1 + Q_{w_n}^1] \\ &\quad + k_3[q_1(gx,F_x) + q_1(gy,F_y) + q_1(gz,F_z) + q_1(gw,F_w)] \\ &\quad + k_4[q_1(gx_n,F_x) + q_1(gy_n,F_y) + q_1(gz_n,F_z) + q_1(gw_n,F_w)] \\ &\quad + k_5[q_1(gx,gx_{n+1}) + q_1(gy,gy_{n+1}) + q_1(gz,gz_{n+1}) + q_1(gw,gw_{n+1})] \end{aligned}$$

Letting $n \to \infty$ in the above inequality, using (2.13)-(2.20), we obtain

$$q_1(gx,F_x)+q_1(gy,F_y)+q_1(gz,F_z)++q_1(gw,F_w) \le (k_3+k_4)\left[q_1(gx,F_x)+q_1(gy,F_y)+q_1(gz,F_z)++q_1(gw,F_w)\right] \tag{2.21}$$

By (2.1), we have $k_3+k_4<1$. Hence, it follows from (2.21) that $q_1(gx,F_x)=q_1(gy,F_y)=q_1(gz,F_z)=q_1(gw,F_w)=0$. By Lemma 1.4, we get, $gx=F_x, gy=F_y, gz=F_z$ and $gw=F_w$. Hence, (gx,gy,gz,gw) is a quadruple point of coincidence of mappings F and g. Next, we will show that the tripled point of coincidence is unique. Suppose that $(x^*,y^*,z^*,w^*)\in X^4$ with $F_{x^*}=gx^*, F_{y^*}=gy^*, F_{z^*}=gz^*$ and $F_{w^*}=gw^*$. Now, from (2.2), we have

$$\begin{aligned}
q_1(gx,gx^*)+q_1(gy,gy^*)+q_1(gz,gz^*)+q_1(gw,gw^*) &= q_1(F_x,F_{x^*})+q_1(F_y,F_{y^*})+q_1(F_z,F_{z^*})+q_1(F_w,F_{w^*})\\
&\le k_1[q_2(gx,gx^*)+q_2(gy,gy^*)+q_2(gz,gz^*)+q_2(gw,gw^*)]\\
&\quad +k_2\left[q_2(gx,F_x)+q_2(gy,F_y)+q_2(gz,F_z)+q_2(gw,F_w)\right]\\
&\quad +k_3\left[q_2(gx^*,F_{x^*})+q_2(gy^*,F_{y^*})+q_2(gz^*,F_{z^*})+q_2(gw^*,F_{w^*})\right]\\
&\quad +k_4\left[q_2(gx,F_{x^*})+q_2(gy,F_{y^*})+q_2(gz,F_{z^*})+q_2(gw,F_{w^*})\right]\\
&\quad +k_5\left[q_2(gx^*,F_x)+q_2(gy^*,F_y)+q_2(gz^*,F_z)+q_2(gw^*,F_w)\right]\\
&= k_1[q_2(gx,gx^*)+q_2(gy,gy^*)+q_2(gz,gz^*)+q_2(gw,gw^*)]\\
&\quad +k_2[q_2(gx,gx)+q_2(gy,gy)+q_2(gz,gz)+q_2(gw,gw)]\\
&\quad +k_3[q_2(gx^*,gx^*)+q_2(gy^*,gy^*)+q_2(gz^*,gz^*)+q_2(gw^*,gw^*)]\\
&\quad +k_4[q_2(gx,gx^*)+q_2(gy,gy^*)+q_2(gz,gz^*)+q_2(gw,gw^*)]\\
&\quad +k_5[q_2(gx^*,gx)+q_2(gy^*,gy)+q_2(gz^*,gz)+q_2(gw^*,gw)]\\
&\le (k_1+k_4)[q_2(gx,gx^*)+q_2(gy,gy^*)+q_2(gz,gz^*)+q_2(gw,gw^*)]\\
&\quad +k_2[q_2(gx,gx)+q_2(gy,gy)+q_2(gz,gz)+q_2(gw,gw)]\\
&\quad +k_3[q_2(gx^*,gx^*)+q_2(gy^*,gy^*)+q_2(gz^*,gz^*)+q_2(gw^*,gw^*)]\\
&\quad +k_5[q_2(gx^*,gx)+q_2(gy^*,gy)+q_2(gz^*,gz)+q_2(gw^*,gw)]
\end{aligned} \tag{2.22}$$

Using (2.13) - (2.16), we obtain

$$\begin{aligned}
q_1(gx,gx^*)+q_1(gy,gy^*)+q_1(gz,gz^*)+q_1(gw,gw^*) &\le (k_1+k_4)[q_1(gx,gx^*)+q_1(gy,gy^*)+q_1(gz,gz^*)+q_1(gw,gw^*)]\\
&\quad +k_3[q_1(gx,gx^*)+q_1(gy,gy^*)+q_1(gz,gz^*)+q_1(gw,gw^*)]\\
&\quad +k_5[q_1(gx^*,gx)+q_1(gy^*,gy)+q_1(gz^*,gz)+q_1(gw^*,gw)]\\
&\le (k_1+k_4+k_3)[q_1(gx,gx^*)+q_1(gy,gy^*)+q_1(gz,gz^*)+q_1(gw,gw^*)]\\
&\quad +k_5[q_1(gx^*,gx)+q_1(gy^*,gy)+q_1(gz^*,gz)+q_1(gw^*,gw)].
\end{aligned}$$

This implies that

$$q_1(gx,gx^*)+q_1(gy,gy^*)+q_1(gz,gz^*)+q_1(gw,gw^*) \le \frac{k_5}{1-k_1-k_4-k_3}[q_1(gx^*,gx)+q_1(gy^*,gy)+q_1(gz^*,gz)+q_1(gw^*,gw)]. \tag{2.23}$$

Similarly, we have

$$q_1(gx^*,gx)+q_1(gy^*,gy)+q_1(gz^*,gz)+q_1(gw^*,gw) \le \frac{k_5}{1-k_1-k_4-k_3}[q_1(gx,gx^*)+q_1(gy,gy^*)+q_1(gz,gz^*)+q_1(gw,gw^*)]. \tag{2.24}$$

Substituting (2.24) into (2.23), we obtain

$$q_1(gx,gx^*)+q_1(gy,gy^*)+q_1(gz,gz^*)+q_1(gw,gw^*) \le \left(\frac{k_5}{1-k_1-k_4-k_3}\right)^2[q_1(gx^*,gx)+q_1(gy^*,gy)+q_1(gz^*,gz)+q_1(gw,gw^*)]. \tag{2.25}$$

Since $\frac{k_5}{1-k_1-k_4-k_3}<1$, from (2.25), must have $q_1(gx,gx^*)=q_1(gy,gy^*)=q_1(gz,gz^*)=q_1(gw,gw^*)=0$. By Lemma 1.4(A), we get $gx=gx^*, gy=gy^*, gz=gz^*$ and $gw=gw^*$, which implies that the uniqueness of the quadruple point of coincidence of F and g. Next, we will show that $gx=gy=gz=gw$. In fact, from (2.2), (2.13), (2.14), (2.15) and (2.16) we have

$$\begin{aligned}
q_1(gx,gy)+q_1(gy,gz)+q_1(gz,gw)+q_1(gw,gx) &= q_1(F_x,F_y)+q_1(F_y,F_z)+q_1(F_z,F_w)+q_1(F_w,F_x)
\end{aligned}$$

$$\begin{aligned}
&\leq k_1[q_2(gx,gy)+q_2(gy,gz)+q_2(gz,gw)+q_2(gw,gx)]\\
&\quad +k_2\big[q_2(gx,F_x)+q_2\big(gy,F_y\big)+q_2(gz,F_z)+q_2(gw,F_w)\big]\\
&\quad +k_3\big[q_2\big(gy,F_y\big)+q_2(gz,F_z)+q_2(gw,F_w)+q_2(gx,F_x)\big]\\
&\quad +k_4\big[q_2\big(gx,F_y\big)+q_2(gy,F_z)+q_2(gz,F_w)+q_2(gw,F_x)\big]\\
&\quad +k_5\big[q_2(gy,F_x)+q_2\big(gz,F_y\big)+q_2(gw,F_z)+q_2(gx,F_w)\big]\\
&= (k_1+k_4)[q_2(gx,gy)+q_2(gy,gz)+q_2(gz,gw)+q_2(gw,gx)]\\
&\quad +(k_2+k_3)[q_2(gx,gx)+q_2(gy,gy)+q_2(gz,gz)+q_2(gw,gw)]\\
&\quad +k_5[q_2(gy,gx)+q_2(gz,gy)+q_2(gw,gz)+q_2(gx,gw)]\\
&\leq (k_1+k_4)[q_1(gx,gy)+q_1(gy,gz)+q_1(gz,gw)+q_1(gw,gx)]\\
&\quad +(k_2+k_3)[q_1(gx,gx)+q_1(gy,gy)+q_1(gz,gz)+q_1(gw,gw)]\\
&\quad +k_5[q_1(gy,gx)+q_1(gz,gy)+q_1(gw,gz)+q_1(gx,gw)]\\
&= (k_1+k_4)[q_2(gx,gy)+q_2(gy,gz)+q_2(gz,gw)+q_2(gw,gx)]\\
&\quad +k_5[q_1(gy,gx)+q_1(gz,gy)+q_1(gw,gz)+q_1(gx,gw)]
\end{aligned}$$

This implies that

$$q_1(gx,gy)+q_1(gy,gz)+q_1(gz,gw)+q_1(gw,gx) \leq \frac{k_5}{1-k_1-k_4}[q_1(gy,gx)+q_1(gz,gy)+q_1(gw,gz)+q_1(gx,gw)] \tag{2.26}$$

Similarly, we have

$$[q_1(gy,gx)+q_1(gz,gy)+q_1(gw,gz)+q_1(gx,gw)] \leq \frac{k_5}{1-k_1-k_4}[q_1(gx,gy)+q_1(gy,gz)+q_1(gz,gw)+q_1(gw,gx)] \tag{2.27}$$

Substituting (2.27) into (2.26), we obtain

$$q_1(gx,gy)+q_1(gy,gz)+q_1(gz,gw)+q_1(gw,gx) \leq \left(\frac{k_5}{1-k_1-k_4}\right)^2[q_1(gy,gx)+q_1(gz,gy)+q_1(gw,gz)+q_1(gx,gw)] \tag{2.28}$$

Since $\frac{k_5}{1-k_1-k_4}<1$, from (2.28), we must have $q_1(gx,gy)=q_1(gy,gz)=q_1(gz,gw)=q_1(gw,gx)=0$. By Lemma 1.4(A), we get $gx=gy=gz=gw$. Finally, assume that F and g are w-compatible. Let $u=gx$, then $u=gx=F_x=gy=F_y=gz=F_z=gw=F_w$, so that

$$gu=ggx=g(F_x)=gF(x,y,z,w)=F(gx,gy,gz,gw)=F(u,u,u,u). \tag{2.29}$$

Consequently, (u,u,u,u) is a quadruple coincidence point of F and g, and therefore (gu,gu,gu,gu) is a quadruple point of coincidence of F and g, and by its uniqueness, we get $gu=gx$. Thus, we obtain $F(u,u,u,u)=gu=u$. Therefore (u,u,u,u) is the unique common quadruple fixed point of F and g. This completes the proof of the Theorem 2.2. □

Remark 2.3: Theorem 2.2 improves and extends Theorem 2.1 of Shatanawi and Pitea [29], Theorem 2.1 of Feng Gu *et al.* [11] and Theorem 2.1 of Feng Gu [12] in the following aspects:

1) The single quasi-partial metric extends to two quasi-partial metrics.
2) The coupled fixed point [11, 29] and the tripled fixed point [12] extend to a quadruple fixed point.
3) The contractive condition (2.1) of [11, 29] and contractive condition (2.1) of [12] is replaced by the new contractive condition defined by (2.1). .

As consequences of Theorem 2.2, we give the following corollaries.

Corollary 2.4: Let q_1 and q_2 be two quasi-metrics on X such that $q_2(\mathrm{x},\mathrm{y})\leq q_1(\mathrm{x},\mathrm{y})$, for all $x,y\in X$, and let $F: X^4\to \mathrm{X}$, $g: X\to X$ be two mappings. Suppose that there exist k in $[0,1)$ with such that the condition

$$q_1(F_x,F_a)+q_1\big(F_y,F_b\big)+q_1(F_z,F_c)+q_1(F_w,F_d)\leq k[q_2(gx,ga)+q_2(gy,gb)+q_2(gz,gc)+q_2(gw,gd)] \tag{2.30}$$

holds for all $x,y,z,w,a,b,c,d\in X$. Also, suppose we have the following hypotheses:

(i) $F(X^4)\subset g(X)$.
(ii) $g(X)$ is a complete subspace of X with respect to the quasi-partial metric q_1.

Then the mappings F and g have a quadruple coincidence point (x,y,z,w) satisfying $gx=F_x=gy=F_y=gz=F_z=gw=F_w$. Moreover, if F and g are w-compatible, then F and g have a unique common quadruple fixed point of the form (u,u,u,u).

Corollary 2.5: Let q_1 and q_2 be two quasi-metrics on X such that $q_2(\mathrm{x},\mathrm{y}) \le q_1(\mathrm{x},\mathrm{y})$, for all $x, y \in X$, and let $F: X^4 \to \mathrm{X}$, $g : X \to X$ be two mappings. Suppose that there exist k in [0,1) such that the condition

$$q_1(F_x, F_a) + q_1(F_y, F_b) + q_1(F_z, F_c) + q_1(F_w, F_d) \le k[q_2(gx, F_x) + q_2(gy, F_y) + q_2(gz, F_z) + q_2(gw, F_w)] \quad (2.31)$$

holds for all $x, y, z, w, a, b, c, d \in X$. Also, suppose we have the following hypotheses:

(i) $F(X^4) \subset g(X)$.
(ii) $g(X)$ is a complete subspace of X with respect to the quasi-partial metric q_1.

Then the mappings F and g have a quadruple coincidence point (x, y, z, w) satisfying $gx = F_x = gy = F_y = gz = F_z = gw = F_w$. Moreover, if F and g are w-compatible, then F and g have a unique common quadruple fixed point of the form (u, u, u, u).

Corollary 2.6: Let q_1 and q_2 be two quasi-metrics on X such that $q_2(\mathrm{x},\mathrm{y}) \le q_1(\mathrm{x},\mathrm{y})$, for all $x, y \in X$, and let $F: X^4 \to \mathrm{X}$, $g : X \to X$ be two mappings. Suppose that there exist k in [0,1) such that the condition

$$q_1(F_x, F_a) + q_1(F_y, F_b) + q_1(F_z, F_c) + q_1(F_w, F_d) \le k[q_2(gx, ga) + q_2(gy, gb) + q_2(gz, gc) + q_2(gw, gd)] \quad (2.32)$$

holds for all $x, y, z, w, a, b, c, d \in X$. Also, suppose we have the following hypotheses:

(i) $F(X^4) \subset g(X)$.
(ii) $g(X)$ is a complete subspace of X with respect to the quasi-partial metric q_1.

Then the mappings F and g have a quadruple coincidence point (x, y, z, w) satisfying $gx = F_x = gy = F_y = gz = F_z = gw = F_w$. Moreover, if F and g are w-compatible, then F and g have a unique common quadruple fixed point of the form (u, u, u, u).

Corollary 2.7: Let q_1 and q_2 be two quasi-metrics on X such that $q_2(\mathrm{x},\mathrm{y}) \le q_1(\mathrm{x},\mathrm{y})$, for all $x, y \in X$, and let $F: X^4 \to \mathrm{X}$, $g : X \to X$ be two mappings. Suppose that there exist k in [0,1) such that the condition

$$q_1(F_x, F_a) + q_1(F_y, F_b) + q_1(F_z, F_c) + q_1(F_w, F_d) \le k[q_2(gx, F_a) + q_2(gy, F_b) + q_2(gz, F_c) + q_2(gw, F_d)] \quad (2.33)$$

holds for all $x, y, z, w, a, b, c, d \in X$. Also, suppose we have the following hypotheses:

(i) $F(X^4) \subset g(X)$.
(ii) $g(X)$ is a complete subspace of X with respect to the quasi-partial metric q_1.

Then the mappings F and g have a quadruple coincidence point (x, y, z, w) satisfying $gx = F_x = gy = F_y = gz = F_z = gw = F_w$. Moreover, if F and g are w-compatible, then F and g have a unique common quadruple fixed point of the form (u, u, u, u).

In Theorem 2.2, if we take $\mathrm{q}_2(\mathrm{x},\mathrm{y}) = \mathrm{q}_1(\mathrm{x},\mathrm{y})$ for all $\mathrm{x},\mathrm{y} \in \mathrm{X}$, then we get the following.

Corollary 2.8 Let $\mathrm{q}(\mathrm{x},\mathrm{y})$ be a quasi-metric on X and let $F: X^4 \to \mathrm{X}$, $g : X \to X$ be two mappings. Suppose that there exist k_1, k_2, k_3, k_4, k_5 in [0,1) with $k_1 + k_2 + k_3 + 2k_4 + k_5 < 1$ such that the condition

$$\begin{aligned} q(F_x, F_a) + q(F_y, F_b) + q(F_z, F_c) + q(F_w, F_d) \le\ & k_1[q(gx, ga) + q(gy, gb) + q(gz, gc) + q(gw, gd)] \\ & + k_2[q(gx, F_x) + q(gy, F_y) + q(gz, F_z) + q(gw, F_w)] \\ & + k_3[q(ga, F_a) + q(gb, F_b) + q(gc, F_c) + q(gd, F_d)] \\ & + k_4[q(gx, F_a) + q(gy, F_b) + q(gz, F_c) + q(gw, F_d)] \\ & + k_5[q(ga, F_x) + q(gb, F_y) + q(gc, F_z) + q(gd, F_w)] \end{aligned} \quad (2.34)$$

holds for all $x, y, z, w, a, b, c, d \in X$. Also, suppose we have the following hypotheses:

(i) $F(X^4) \subset g(X)$.
(ii) $g(X)$ is a complete subspace of X .

Then the mappings F and g have a quadruple coincidence point (x, y, z, w) satisfying $gx = F_x = gy = F_y = gz = F_z = gw = F_w$. Moreover, if F and g are w-compatible, then F and g have a unique common quadruple fixed point of the form (u, u, u, u).

Remark 2.9 Corollary 2.8 improves and extends Corollary 2.2 of Aydi and Abbas [19] to quasi-partial metric spaces and to a quadruple fixed point.

Corollary 2.10 Let $\mathrm{q}(\mathrm{x},\mathrm{y})$ be a quasi-metric on X and let $F: X^4 \to \mathrm{X}$, $g : X \to X$ be two mappings. Suppose that there exist k in [0,1) such that the condition

$$q(F_x, F_a) + q(F_y, F_b) + q(F_z, F_c) + q(F_w, F_d) \le k[q(gx, ga) + q(gy, gb) + q(gz, gc) + q(gw, gd)] \quad (2.35)$$

holds for all $x, y, z, w, a, b, c, d \in X$. Also, suppose we have the following hypotheses:

(i) $F(X^4) \subset g(X)$.
(ii) $g(X)$ is a complete subspace of X .

Then the mappings F and g have a quadruple coincidence point (x, y, z, w) satisfying $gx = F_x = gy = F_y = gz = F_z = gw = F_w$. Moreover, if F and g are w-compatible, then F and g have a unique common quadruple fixed point of the form (u, u, u, u).

Corollary 2.11: Let q(x, y) be a quasi-metric on X and let $F: X^4 \to$ X, $g : X \to X$ be two mappings. Suppose that there exist k in [0,1) such that the condition

$$q(F_x, F_a) + q(F_y, F_b) + q(F_z, F_c) + q(F_w, F_d) \leq k\left[q(gx, F_x) + q(gy, F_y) + q(gz, F_z) + q(gw, F_w)\right] \tag{2.36}$$

holds for all $x, y, z, w, a, b, c, d \in X$. Also, suppose we have the following hypotheses:

(i) $F(X^4) \subset g(X)$.
(ii) $g(X)$ is a complete subspace of X .

Then the mappings F and g have a quadruple coincidence point (x, y, z, w) satisfying $gx = F_x = gy = F_y = gz = F_z = gw = F_w$. Moreover, if F and g are w-compatible, then F and g have a unique common quadruple fixed point of the form (u, u, u, u).

Corollary 2.12: Let q(x, y) be a quasi-metric on X and let $F: X^4 \to$ X, $g : X \to X$ be two mappings. Suppose that there exist k in [0,1) such that the condition

$$q(F_x, F_a) + q(F_y, F_b) + q(F_z, F_c) + q(F_w, F_d) \leq k[q(ga, F_a) + q(gb, F_b) + q(gc, F_c) + q(gd, F_d)] \tag{2.37}$$

holds for all $x, y, z, w, a, b, c, d \in X$. Also, suppose we have the following hypotheses:

(i) $F(X^4) \subset g(X)$.
(ii) $g(X)$ is a complete subspace of X .

Then the mappings F and g have a quadruple coincidence point (x, y, z, w) satisfying $gx = F_x = gy = F_y = gz = F_z = gw = F_w$. Moreover, if F and g are w-compatible, then F and g have a unique common quadruple fixed point of the form (u, u, u, u).

Corollary 2.13: Let q(x, y) be a quasi-metric on X and let $F: X^4 \to$ X, $g : X \to X$ be two mappings. Suppose that there exist k in [0,1) such that the condition

$$q(F_x, F_a) + q(F_y, F_b) + q(F_z, F_c) + q(F_w, F_d) \leq k[q(gx, F_a) + q(gy, F_b) + q(gz, F_c) + q(gw, F_d)] \tag{2.38}$$

holds for all $x, y, z, w, a, b, c, d \in X$. Also, suppose we have the following hypotheses:

(i) $F(X^4) \subset g(X)$.
(ii) $g(X)$ is a complete subspace of X .

Then the mappings F and g have a quadruple coincidence point (x, y, z, w) satisfying $gx = F_x = gy = F_y = gz = F_z = gw = F_w$. Moreover, if F and g are w-compatible, then F and g have a unique common quadruple fixed point of the form (u, u, u, u).

Corollary 2.14: Let q_1 and q_2 be two quasi-metrics on X such that $q_2(\mathrm{x}, \mathrm{y}) \leq q_1(\mathrm{x}, \mathrm{y})$, for all $x, y \in X$, and let $F: X^4 \to$ X, $g : X \to X$ be two mappings. Suppose that there exist $a_i, i \in \{1,2,3, \ldots\ldots, 20\}$ in [0,1) with

$$\sum_{i=1}^{20} a_i + \sum_{i=13}^{16} a_i < 1 \tag{2.39}$$

such that the condition

$$\begin{aligned} q_1(F_x, F_a) \leq\ & a_1 q_2(gx, ga) + a_2 q_2(gy, gb) + a_3 q_2(gz, gc) + a_4 q_2(gw, gd) \\ & + a_5 q_2(gx, F_x) + a_6 q_2(gy, F_y) + a_7 q_2(gz, F_z) + a_8 q_2(gw, F_w) \\ & + a_9 q_2(ga, F_a) + a_{10} q_2(gb, F_b) + a_{11} q_2(gc, F_c) + a_{12} q_2(gd, F_d) \\ & + a_{13} q_2(gx, F_a) + a_{14} q_2(gy, F_b) + a_{15} q_2(gz, F_c) + a_{16} q_2(gw, F_d) \\ & + a_{17} q_2(ga, F_x) + a_{18} q_2(gb, F_y) + a_{19} q_2(gc, F_z) + a_{20} q_2(gd, F_w) \end{aligned} \tag{2.40}$$

holds for all $x, y, z, w, a, b, c, d \in X$. Also, suppose we have the following hypotheses:

(i) $F(X^4) \subset g(X)$.
(ii) $g(X)$ is a complete subspace of X with respect to the quasi-partial metric q_1.

Then the mappings F and g have a quadruple coincidence point (x, y, z, w) satisfying $gx = F_x = gy = F_y = gz = F_z = gw = F_w$. Moreover, if F and g are w-compatible, then F and g have a unique common quadruple fixed point of the form (u, u, u, u).

Proof: Given $x, y, z, w, a, b, c, d \in X$. It follows from (2.40) that

$$\begin{aligned} q_1(F_x, F_a) \leq\ & a_1 q_2(gx, ga) + a_2 q_2(gy, gb) + a_3 q_2(gz, gc) + a_4 q_2(gw, gd) \\ & + a_5 q_2(gx, F_x) + a_6 q_2(gy, F_y) + a_7 q_2(gz, F_z) + a_8 q_2(gw, F_w) \\ & + a_9 q_2(ga, F_a) + a_{10} q_2(gb, F_b) + a_{11} q_2(gc, F_c) + a_{12} q_2(gd, F_d) \\ & + a_{13} q_2(gx, F_a) + a_{14} q_2(gy, F_b) + a_{15} q_2(gz, F_c) + a_{16} q_2(gw, F_d) \\ & + a_{17} q_2(ga, F_x) + a_{18} q_2(gb, F_y) + a_{19} q_2(gc, F_z) + a_{20} q_2(gd, F_w) \end{aligned} \tag{2.41}$$

$$\begin{aligned} q_1(F_y,F_b) \le\ & a_1q_2(gy,gb)+a_2q_2(gz,gc)+a_3q_2(gw,gd)+a_4q_2(gx,ga)\\ &+a_5q_2(gy,F_y)+a_6q_2(gz,F_z)+a_7q_2(gw,F_w)+a_8q_2(gx,F_x)\\ &+a_9q_2(gb,F_b)+a_{10}q_2(gc,F_c)+a_{11}q_2(gd,F_d)+a_{12}q_2(ga,F_a)\\ &+a_{13}q_2(gy,F_b)+a_{14}q_2(gz,F_c)+a_{15}q_2(gw,F_d)+a_{16}q_2(gx,F_a)\\ &+a_{17}q_2(gb,F_y)+a_{18}q_2(gc,F_z)+a_{19}q_2(gd,F_w)+a_{20}q_2(ga,F_x) \end{aligned} \tag{2.42}$$

$$\begin{aligned} q_1(F_z,F_c) \le\ & a_1q_2(gz,gc)+a_2q_2(gw,gd)+a_3q_2(gx,ga)+a_4q_2(gy,gb)\\ &+a_5q_2(gz,F_z)+a_6q_2(gw,F_w)+a_7q_2(gx,F_x)+a_8q_2(gy,F_y)\\ &+a_9q_2(gc,F_c)+a_{10}q_2(gd,F_d)+a_{11}q_2(ga,F_a)+a_{12}q_2(gb,F_b)\\ &+a_{13}q_2(gz,F_c)+a_{14}q_2(gw,F_d)+a_{15}q_2(gx,F_a)+a_{16}q_2(gy,F_b)\\ &+a_{17}q_2(gc,F_z)+a_{18}q_2(gd,F_w)+a_{19}q_2(ga,F_x)+a_{20}q_2(gb,F_y) \end{aligned} \tag{2.43}$$

and

$$\begin{aligned} q_1(F_w,F_d) \le\ & a_1q_2(gw,gd)+a_2q_2(gx,ga)+a_3q_2(gy,gb)+a_4q_2(gz,gc)\\ &+a_5q_2(gw,F_w)+a_6q_2(gx,F_x)+a_7q_2(gy,F_y)+a_8q_2(gz,F_z)\\ &+a_9q_2(gd,F_d)+a_{10}q_2(ga,F_a)+a_{11}q_2(gb,F_b)+a_{12}q_2(gc,F_c)\\ &+a_{13}q_2(gw,F_d)+a_{14}q_2(gx,F_a)+a_{15}q_2(gy,F_b)+a_{16}q_2(gz,F_c)\\ &+a_{17}q_2(gd,F_w)+a_{18}q_2(ga,F_x)+a_{19}q_2(gb,F_y)+a_{20}q_2(gc,F_z) \end{aligned} \tag{2.44}$$

Adding inequalities from (2.41) to (2.42), we get

$$\begin{aligned} & q_1(F_x,F_a)+q_1(F_y,F_b)+q_1(F_z,F_c)+q_1(F_w,F_d)\\ &\le (a_1+a_2+a_3+a_4)[q_2(gx,ga)+q_2(gy,gb)+q_2(gz,gc)+q_2(gw,gd)]\\ &\quad+(a_5+a_6+a_7+a_8)[q_2(gx,F_x)+q_2(gy,F_y)+q_2(gz,F_z)+q_2(gw,F_w)]\\ &\quad+(a_9+a_{10}+a_{11}+a_{12})[q_2(ga,F_a)+q_2(gb,F_b)+q_2(gc,F_c)+q_2(gd,F_d)]\\ &\quad+(a_{13}+a_{14}+a_{15}+a_{16})[q_2(gx,F_a)+q_2(gy,F_b)+q_2(gz,F_c)+q_2(gw,F_d)]\\ &\quad+(a_{17}+a_{18}+a_{19}+a_{20})[q_2(ga,F_x)+q_2(gb,F_y)+q_2(gc,F_z)+q_2(gd,F_w)] \end{aligned} \tag{2.45}$$

Therefore, the result follows from Theorem 2.2.

Remark 2.15:

(1). If we take $q_1(\mathrm{x},\mathrm{y}) = q_2(\mathrm{x},\mathrm{y})$, for all $x, y \in X$ and $a_i(i = 13,14,\ldots.20) = 0$, then Corollary 2.14 improves and extends Corollary 2.6 of Shatanawi and Pitea [29] to the case of quadruple fixed point.

(2). If we take $q_1(\mathrm{x},\mathrm{y}) = q_2(\mathrm{x},\mathrm{y}) = p(\mathrm{x},\mathrm{y})$, for all $x, y \in X$ where $p(\mathrm{x},\mathrm{y})$is a partial metric space, then Theorem 2.2 improves and extends Theorem 2.1 of Aydi et al. [13] to the case of quadruple fixed point on the class of quasi-partial metric spaces.

(3). If we take $q_1(\mathrm{x},\mathrm{y}) = q_2(\mathrm{x},\mathrm{y})$, for all $x, y \in X$ and $a_i(i = 5,6,\ldots.20) = 0$, then Corollary 2.14 improves and extends Theorem 2.1 of Aydi [14] to the case of quadruple fixed point on the class of quasi-partial metric spaces.

(4). If we take $q_1(\mathrm{x},\mathrm{y}) = q_2(\mathrm{x},\mathrm{y})$, for all $x, y \in X$, $a_1 = a_2 = a_3 = a_4$and $a_i(i = 5,6,\ldots.20) = 0$, then Corollary 2.14 improves and extends Corollary 2.2 of Aydi [14] to the case of quadruple fixed point on the class of quasi-partial metric spaces.

(5). If we take $q_1(\mathrm{x},\mathrm{y}) = q_2(\mathrm{x},\mathrm{y})$, for all $x, y \in X$ and

$$a_i\ (i = 1,2,3,4,9,10,11,12,17,18,19,20) = 0,$$

then Corollary 2.14 improves and extends Theorem 2.4 of Aydi [14] to the case of quadruple fixed point on the class of quasi-partial metric spaces.

(6). Corollary 2.14 improves and extends Corollary 2.2 of Feng et al. [11] and Corollary 2.2 of Feng Gu [12] in the following aspects:

a) The single quasi-partial metric extends to two quasi-partial metrics.
b) The coupled fixed point [11] and the tripled fixed point [12] extend to a quadruple fixed point.

Let $g = I_X$ (the identity mapping) in Theorem 2.2 and Corollary 2.4-2.8, 2.10-2.14 then we have the following results.

Corollary 2.16 Let q_1 and q_2 be two quasi-metrics on X such that $q_2(\mathrm{x},\mathrm{y}) \le q_1(\mathrm{x},\mathrm{y})$, for all $x, y \in X$, and let $F: X^4 \to \mathrm{X}$ be a mapping. Suppose that there exist k_1, k_2, k_3, k_4, k_5 in [0,1) with $k_1 + k_2 + k_3 + 2k_4 + k_5 < 1$ such that the condition

$$\begin{aligned} q_1(F_x,F_a)+q_1(F_y,F_b)+q_1(F_z,F_c)+q_1(F_w,F_d) \le\ & k_1[q_2(x,a)+q_2(y,b)+q_2(z,c)+q_2(w,d)]\\ &+k_2[q_2(x,F_x)+q_2(y,F_y)+q_2(z,F_z)+q_2(w,F_w)]\\ &+k_3[q_2(a,F_a)+q_2(b,F_b)+q_2(c,F_c)+q_2(d,F_d)]\\ &+k_4[q_2(x,F_a)+q_2(y,F_b)+q_2(z,F_c)+q_2(w,F_d)]\\ &+k_5[q_2(a,F_x)+q_2(b,F_y)+q_2(c,F_z)+q_2(d,F_w)] \end{aligned} \tag{2.46}$$

holds for all $x, y, z, w, a, b, c, d \in X$. Also, suppose (X, q_1) is a complete quasi-partial metric space. Then the mapping F has a unique common quadruple fixed point of the form (u, u, u, u).

Corollary 2.17: Let q_1 and q_2 be two quasi-metrics on X such that $q_2(\mathrm{x},\mathrm{y}) \leq q_1(\mathrm{x},\mathrm{y})$, for all $x, y \in X$, and let $F: X^4 \to$ X be a mapping. Suppose that there exist k in [0,1) such that the condition

$$q_1(F_x, F_a) + q_1(F_y, F_b) + q_1(F_z, F_c) + q_1(F_w, F_d) \leq k[q_2(x, a) + q_2(y, b) + q_2(z, c) + q_2(w, d)] \tag{2.47}$$

holds for all $x, y, z, w, a, b, c, d \in X$. Also, suppose (X, q_1) is a complete quasi-partial metric space. Then the mapping F has a unique common quadruple fixed point of the form (u, u, u, u).

Corollary 2.18: Let q_1 and q_2 be two quasi-metrics on X such that $q_2(\mathrm{x},\mathrm{y}) \leq q_1(\mathrm{x},\mathrm{y})$, for all $x, y \in X$, and let $F: X^4 \to$ X be a mapping. Suppose that there exist k in [0,1) such that the condition

$$q_1(F_x, F_a) + q_1(F_y, F_b) + q_1(F_z, F_c) + q_1(F_w, F_d) \leq k\left[q_2(x, F_x) + q_2(y, F_y) + q_2(z, F_z) + q_2(w, F_w)\right] \tag{2.48}$$

holds for all $x, y, z, w, a, b, c, d \in X$. Also, suppose (X, q_1) is a complete quasi-partial metric space. Then the mapping F has a unique common quadruple fixed point of the form (u, u, u, u).

Corollary 2.19: Let q_1 and q_2 be two quasi-metrics on X such that $q_2(\mathrm{x},\mathrm{y}) \leq q_1(\mathrm{x},\mathrm{y})$, for all $x, y \in X$, and let $F: X^4 \to$ X be a mapping. Suppose that there exist k in [0,1) such that the condition

$$q_1(F_x, F_a) + q_1(F_y, F_b) + q_1(F_z, F_c) + q_1(F_w, F_d) \leq k[q_2(a, F_a) + q_2(b, F_b) + q_2(c, F_c) + q_2(d, F_d)] \tag{2.49}$$

holds for all $x, y, z, w, a, b, c, d \in X$. Also, suppose (X, q_1) is a complete quasi-partial metric space. Then the mapping F has a unique common quadruple fixed point of the form (u, u, u, u).

Corollary 2.20: Let q_1 and q_2 be two quasi-metrics on X such that $q_2(\mathrm{x},\mathrm{y}) \leq q_1(\mathrm{x},\mathrm{y})$, for all $x, y \in X$, and let $F: X^4 \to$ X be a mapping. Suppose that there exist k in [0,1) such that the condition

$$q_1(F_x, F_a) + q_1(F_y, F_b) + q_1(F_z, F_c) + q_1(F_w, F_d) \leq k[q_2(x, F_a) + q_2(y, F_b) + q_2(z, F_c) + q_2(w, F_d)] \tag{2.50}$$

holds for all $x, y, z, w, a, b, c, d \in X$. Also, suppose (X, q_1) is a complete quasi-partial metric space. Then the mapping F has a unique common quadruple fixed point of the form (u, u, u, u).

Corollary 2.21: Let $q(\mathrm{x},\mathrm{y})$ be a quasi-metric on X and let $F: X^4 \to$ X be a mapping. Suppose that there exist k_1, k_2, k_3, k_4, k_5 in [0,1) with $k_1 + k_2 + k_3 + 2k_4 + k_5 < 1$ such that the condition

$$\begin{aligned} q(F_x, F_a) + q(F_y, F_b) + q(F_z, F_c) + q(F_w, F_d) \leq\ & k_1[q(x, a) + q(y, b) + q(z, c) + q(w, d)] \\ & + k_2\left[q(x, F_x) + q(y, F_y) + q(z, F_z) + q(w, F_w)\right] \\ & + k_3[q(a, F_a) + q(b, F_b) + q(c, F_c) + q(d, F_d)] \\ & + k_4[q(x, F_a) + q(y, F_b) + q(z, F_c) + q(w, F_d)] \\ & + k_5\left[q(a, F_x) + q(b, F_y) + q(c, F_z) + q(d, F_w)\right] \end{aligned} \tag{2.51}$$

holds for all $x, y, z, w, a, b, c, d \in X$. Also, suppose (X, q) is a complete quasi-partial metric space. Then the mapping F has a unique common quadruple fixed point of the form (u, u, u, u).

Corollary 2.22: Let $q(\mathrm{x},\mathrm{y})$ be a quasi-metric on X and let $F: X^4 \to$ X be a mapping. Suppose that there exist k in [0,1) such that the condition

$$q(F_x, F_a) + q(F_y, F_b) + q(F_z, F_c) + q(F_w, F_d) \leq k[q(x, a) + q(y, b) + q(z, c) + q(w, d)] \tag{2.52}$$

holds for all $x, y, z, w, a, b, c, d \in X$. Also, suppose (X, q) is a complete quasi-partial metric space. Then the mapping F has a unique common quadruple fixed point of the form (u, u, u, u).

Corollary 2.23: Let $q(\mathrm{x},\mathrm{y})$ be a quasi-metric on X and let $F: X^4 \to$ X be a mapping. Suppose that there exist k in [0,1) such that the condition

$$q(F_x, F_a) + q(F_y, F_b) + q(F_z, F_c) + q(F_w, F_d) \leq k\left[q(x, F_x) + q(y, F_y) + q(z, F_z) + q(w, F_w)\right] \tag{2.53}$$

holds for all $x, y, z, w, a, b, c, d \in X$. Also, suppose (X, q) is a complete quasi-partial metric space. Then the mapping F has a unique common quadruple fixed point of the form (u, u, u, u).

Corollary 2.24: Let $q(\mathrm{x},\mathrm{y})$ be a quasi-metric on X and let $F: X^4 \to$ X be a mapping. Suppose that there exist k in [0,1) such that the condition

$$q(F_x, F_a) + q(F_y, F_b) + q(F_z, F_c) + q(F_w, F_d) \leq k[q(a, F_a) + q(b, F_b) + q(c, F_c) + q(d, F_d)] \tag{2.54}$$

holds for all $x, y, z, w, a, b, c, d \in X$. Also, suppose (X, q) is a complete quasi-partial metric space. Then the mapping F has a unique common quadruple fixed point of the form (u, u, u, u).

Corollary 2.25: Let $q(\mathrm{x}, \mathrm{y})$ be a quasi-metric on X and let $F: X^4 \to$ X be a mapping. Suppose that there exist k in [0,1) such that the condition

$$q(F_x, F_a) + q(F_y, F_b) + q(F_z, F_c) + q(F_w, F_d) \leq k[q(x, F_a) + q(y, F_b) + q(z, F_c) + q(w, F_d)] \tag{2.55}$$

holds for all $x, y, z, w, a, b, c, d \in X$. Also, suppose (X, q) is a complete quasi-partial metric space. Then the mapping F has a unique common quadruple fixed point of the form (u, u, u, u).

Corollary 2.26: Let (X, q) is a complete quasi-metrics on X and let $F: X^4 \to$ X be a mapping. Suppose that there exist $a_i, i \in \{1,2,3, \ldots\ldots, 20\}$ in [0,1) with

$$\sum_{i=1}^{20} a_i + \sum_{i=13}^{16} a_i < 1 \tag{2.56}$$

such that the condition

$$\begin{aligned} q(F_x, F_a) \leq\ & a_1 q(x, a) + a_2 q(y, b) + a_3 q(z, c) + a_4 q(w, d) \\ & + a_5 q(x, F_x) + a_6 q(y, F_y) + a_7 q(z, F_z) + a_8 q(w, F_w) \\ & + a_9 q(a, F_a) + a_{10} q(b, F_b) + a_{11} q(c, F_c) + a_{12} q(d, F_d) \\ & + a_{13} q(x, F_a) + a_{14} q(y, F_b) + a_{15} q(z, F_c) + a_{16} q(w, F_d) \\ & + a_{17} q(a, F_x) + a_{18} q(b, F_y) + a_{19} q(c, F_z) + a_{20} q(d, F_w) \end{aligned}$$

holds for all $x, y, z, w, a, b, c, d \in X$. Then the mapping F has a unique quadruple fixed point of the form (u, u, u, u).

3. EXAMPLE

Example 3.1: Let $X = [0,1]$, and two quasi-partial metrics q_1, q_2 on X be given as

$$q_1(x, y) = |x - y| + x \text{ and } q_2(x, y) = \frac{1}{2}(|x - y| + x)$$

for all $x, y \in X$. Also, define $F: X^4 \to X$ and $g: X \to X$ as

$$F(x, y, z, w) = \frac{x + y + z + w}{128} \text{ and } gx = \frac{1}{4}x$$

for all $x, y \in X$. Then

(1). (X, q_1) is a complete quasi-partial metric space
(2). $F(X^4) \subset g(X)$.
(3). F and g is w-compatible.
(4). For any $x, y, z, w, a, b, c, d \in X$., we have

$$q_1(F_x, F_a) + q_1(F_y, F_b) + q_1(F_z, F_c) + q_1(F_w, F_d) \leq \frac{1}{4}\left[q_2(gx, F_x) + q_2(gy, F_y) + q_2(gz, F_z) + q_2(gw, F_w)\right]$$

Proof: The proofs of (1), (2), and (3) are clear. Next we show that (4). In fact, $x, y, z, w, a, b, c, d \in X$, we have

$$\begin{aligned} q_1(F_x, F_a) + q_1(F_y, F_b) + q_1(F_z, F_c) + q_1(F_w, F_d) &= \frac{1}{8}\left(\left|\frac{1}{4}(x + y + z + w) - \frac{1}{4}(a + b + c + d)\right| + \frac{1}{4}(x + y + z + w)\right) \\ &= \frac{1}{8}\left(\left|\frac{1}{4}x - \frac{1}{4}a\right| + \frac{1}{4}x\right) + \frac{1}{8}\left(\left|\frac{1}{4}y - \frac{1}{4}b\right| + \frac{1}{4}y\right) + \frac{1}{8}\left(\left|\frac{1}{4}z - \frac{1}{4}c\right| + \frac{1}{4}z\right) \\ &\quad + \frac{1}{8}\left(\left|\frac{1}{4}w - \frac{1}{4}d\right| + \frac{1}{4}w\right). \end{aligned}$$

That is,

$$q_1(F_x, F_a) + q_1(F_y, F_b) + q_1(F_z, F_c) + q_1(F_w, F_d) \leq \frac{1}{4}\left[q_2(gx, F_x) + q_2(gy, F_y) + q_2(gz, F_z) + q_2(gw, F_w)\right]$$

Thus, F and g satisfy all the hypotheses of Corollary 2.3. So, F and g have a unique common quadruple fixed point. Here $(0,0,0,0)$ is the common quadruple fixed point of F and g. (2.57)

4. CONCLUSION

Recently, Shatanawi and Pitea [29] and Feng Gu *et al.* [11] presented some coupled fixed point theorems in quasi-partial metric spaces and in two quasi-partial metric spaces, respectively. Further a result on tripled fixed point theorems in two quasi-partial metric spaces is obtained by Feng Gu [12]. Whether, we introduced the concept of w-compatible of mappings $F: X^4 \to X$ and $g: X \to X$ and based on this notion, we can obtained some new common quadruple fixed point results for mappings defined on a set equipped with two quasi-partial metrics. In our main results (Theorems 2.2) the considered contractions are of linear type contractions. Also, presented examples are showing that our main results are real generalization of known ones in quadruple fixed point theory. Our main results are generalized, improved, enriched and extended several existing results in the literature: Aydi [Int. J. Math. Sci., Article

ID 647091(2011)], Karapinar *et al.* [Math. Comput. Model. 57:2442-2448, 2013], Shatanawi and Pitea [Fixed Point Theory and Applications 2013:153, 2013], Feng Gu et al. [Fixed Point Theory Appl. 2014, 19] and Feng Gu [Fixed Point Theory and Applications 2014, 2014:71]. Our results may be the motivation to other authors for extending and improving these results to be suitable tools for their applications.

COMPETING INTERESTS

The authors declare that there is no conflict of interests regarding the publication of this paper.

AUTHORS' CONTRIBUTIONS

Both authors contributed equally and significantly in writing this article. Both authors read and approved the final manuscript.

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