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EIGEN VALUE AND STATIONARY DISTRIBUTION OF DOUBLY STOCHASTIC MATRIX

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ABSTRACT

Different types of symmetric doubly stochastic matrix are formed to using the respective eigen values. The basic concepts and theorems of symmetric doubly stochastic matrices using eigen values are introduced with examples. A simple graph theoretic formula for finding the stationary distribution value to the well-known flow graph formulae. In case of Markov chains arc "weights" correspond to the transition probabilities P_{ij}. Using this method to draw the transition graph and find the stationary distribution.

Key Words: *Symmetric doubly stochastic matrix [5], Stationary distribution of the symmetric doubly stochastic matrix [11] and, eigen value of the symmetric doubly stochastic matrix [6].*

AMS Classifications: *15A51, 15B99, 15A18.*

INTRODUCTION

In this paper Λ becomes a list of 2m real numbers or 2m+1 real numbers for any positive integer m, and the results gives sufficient conditions for a list of 2m+1 real numbers to be realizable by a symmetric doubly stochastic matrix.

J.J. Solberg in 1975, using the graph theoretic formula to find the steady state distribution of finite Markov Processes [10]. Using the same formula to find the stationary distribution V_K.

Type: 1 If a list of real fractional numbers
$\Lambda = \{\lambda_1, \lambda_2, \ldots\ldots.\lambda_{2m+1}\} = \Lambda_1 \cup \Lambda_2 \cup \ldots.. \Lambda_m \cup \{\lambda_{m+1}\}$
$\Lambda_k = \{\lambda_k, \lambda_{2m+2-k}\}$, k = 1, 2, …. m satisfies
$1 = \lambda_1 > 0 \geq \lambda_2 \geq \ldots\ldots\ldots \geq \lambda_m \geq \lambda_{m+1} \geq \ldots\ldots. \geq \lambda_{2m+1} \geq -1$
$S = \lambda_1 + \lambda_2 + \ldots\ldots\ldots + \lambda_{2m+1} > \lambda_{m+1}$ and $\lambda_{2m+2-k} < \lambda_k$, k = 2, 3, ….m. Then Λ is realizable by the following doubly stochastic matrix

$$M = \begin{pmatrix} M_{11} & M_{12} & \cdots & M_{1m} & M_{1,m+1} \\ M_{21} & M_{22} & \cdots & M_{2m} & M_{2,m+1} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ M_{m1} & M_{m2} & \cdots & M_{mm} & M_{m,m+1} \\ M_{m+1,1} & M_{m+1,2} & \cdots & M_{m+1,m} & S \end{pmatrix}$$

where $M_{kk} = \begin{pmatrix} 0 & -\lambda_{2m+2-k} \\ -\lambda_{2m+2-k} & 0 \end{pmatrix}$ for k = 1, 2, ….m

$M_{k,m+1} = \begin{pmatrix} S - \lambda_{m+1} \\ S - \lambda_{m+1} \end{pmatrix}$ for k = 1, 2, ….m

and $M_{m+1,k} = (-\lambda_{2m+1} - S \quad -\lambda_{2m+1} - S)$ for k = 1, 2, ….m

$M_{k1} = \begin{pmatrix} -\lambda_k & -\lambda_{2m+1} \\ -\lambda_{2m+1} & -\lambda_k \end{pmatrix}$ for k = 2, 3, ….m

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$$M_{kj} = \begin{pmatrix} -\lambda_j & -\lambda_{2m+2-j} \\ -\lambda_{2m+2-j} & -\lambda_j \end{pmatrix} \text{ for } j = 2, 3, \ldots m$$

Example: 1
$\Lambda = \{\lambda_1, \lambda_2, \lambda_3\} = \{1, -1/2, -1/2\}$ satisfies $1 = \lambda_1 > 0 > \lambda_2 \geq \lambda_3 \geq -1$ with m = 1,
$\lambda_1 = 1, \lambda_2 = -0.5, \lambda_3 = -0.5$ and $S = \lambda_1 + \lambda_2 + \ldots + \lambda_{2m+1} > \lambda_{m+1}$ and $\lambda_{2m+2-k} < \lambda_k$, k = 2, 3, ….m
$\Lambda_1 = \{\lambda_1, \lambda_3\}$ for k = 1. Therefore $\Lambda = \Lambda_1 \cup \{\lambda_2\} = \{\lambda_1, \lambda_2, \lambda_3\} = \{1, -1/2, -1/2\}$, Then Λ is realizable by the doubly stochastic matrix

$$M = \begin{pmatrix} M_{11} & M_{1,2} \\ M_{2,1} & S \end{pmatrix} \text{ for m = 1}$$

where $M_{11} = \begin{pmatrix} 0 & -\lambda_3 \\ -\lambda_3 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1/2 \\ 1/2 & 0 \end{pmatrix}$

$$M_{1,2} = \begin{pmatrix} S - \lambda_2 \\ S - \lambda_2 \end{pmatrix} = \begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix}$$

and $M_{2,1} = (-\lambda_3 - S \quad -\lambda_3 - S) = (1/2 \quad 1/2)$

Then the doubly stochastic matrix is

$$M = \begin{pmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{pmatrix}$$

Example: 2
$\Lambda = \{\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5\}$, $\Lambda_k = \{\lambda_k, \lambda_{2m+2-k}\}$ for m = 2.

Now $\Lambda_1 = \{\lambda_1, \lambda_5\}$ and
$\Lambda_2 = \{\lambda_2, \lambda_4\}$ then $\Lambda = \{\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5\} = \{1, -1/4, -1/4, -1/4, -1/4\}$ satisfies $S \geq \lambda_3$ and $S = 0 > \lambda_3$ with m = 2, $\lambda_1 = 1$, $\lambda_2 = -1/4$, $\lambda_3 = -1/4$, $\lambda_4 = -1/4$, $\lambda_5 = -1/4$

$$M_{11} = \begin{pmatrix} 0 & -\lambda_{2m+2-1} \\ -\lambda_{2m+2-1} & 0 \end{pmatrix} = \begin{pmatrix} 0 & -\lambda_5 \\ -\lambda_5 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1/4 \\ 1/4 & 0 \end{pmatrix}$$

$$M_{22} = \begin{pmatrix} 0 & -\lambda_{2m+2-2} \\ -\lambda_{2m+2-2} & 0 \end{pmatrix} = \begin{pmatrix} 0 & -\lambda_4 \\ -\lambda_4 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1/4 \\ 1/4 & 0 \end{pmatrix}$$

$$M_{k,m+1} = \begin{pmatrix} S - \lambda_{m+1} \\ S - \lambda_{m+1} \end{pmatrix} = \begin{pmatrix} -\lambda_3 \\ -\lambda_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \text{ for k = 1, 2}$$

$$M_{m+1,k} = (-\lambda_{2m+1} - S \quad -\lambda_{2m+1} - S) = (-\lambda_5 \quad -\lambda_5) = (1/4 \quad 1/4) \text{ for k = 1, 2}$$

$$M_{k1} = \begin{pmatrix} -\lambda_k & -\lambda_{2m+1} \\ -\lambda_{2m+1} & -\lambda_k \end{pmatrix} \text{ for k = 2 then } M_{21} = \begin{pmatrix} -\lambda_2 & -\lambda_5 \\ -\lambda_5 & -\lambda_2 \end{pmatrix} = \begin{pmatrix} 1/4 & 1/4 \\ 1/4 & 1/4 \end{pmatrix}$$

$$M_{kj} = \begin{pmatrix} -\lambda_j & -\lambda_{2m+2-j} \\ -\lambda_{2m+2-j} & -\lambda_j \end{pmatrix} = \begin{pmatrix} -\lambda_2 & -\lambda_4 \\ -\lambda_4 & -\lambda_2 \end{pmatrix} = \begin{pmatrix} 1/4 & 1/4 \\ 1/4 & 1/4 \end{pmatrix}.$$ Then the doubly stochastic matrix is

$$M = \begin{pmatrix} 0 & 1/4 & 1/4 & 1/4 & 1/4 \\ 1/4 & 0 & 1/4 & 1/4 & 1/4 \\ 1/4 & 1/4 & 0 & 1/4 & 1/4 \\ 1/4 & 1/4 & 1/4 & 0 & 1/4 \\ 1/4 & 1/4 & 1/4 & 1/4 & 0 \end{pmatrix}$$

Type: 2 If a list of real fractional numbers
$\Lambda = \{\lambda_1, \lambda_2, \ldots\ldots\lambda_{2m}\} = \Lambda_1 \cup \Lambda_2 \cup \ldots\ldots \Lambda_m$
$\Lambda_k = \{\lambda_k, \lambda_{2m+1-k}\}$, k = 1, 2, …. m satisfies
$1 = \lambda_1 > 0 > \lambda_2 \geq \ldots\ldots\ldots \geq \lambda_{m+1} \geq \lambda_{m+2} \geq \ldots\ldots\ldots \geq \lambda_{2m} \geq -1$
$S = \lambda_1 + \lambda_2 + \ldots\ldots\ldots + \lambda_{2m} > \lambda_{m+1}$ and $\lambda_{2m+1-k} < \lambda_k$, k = 2, 3, ….m. Then Λ is realizable by the following doubly stochastic matrix

$$M = \begin{pmatrix} M_{11} & M_{12} & \cdots & M_{1m} \\ M_{21} & M_{22} & \cdots & M_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ M_{m1} & M_{m2} & \cdots & M_{mm} \end{pmatrix}$$

where $M_{kk} = \begin{pmatrix} 0 & -\lambda_{2m+1-k} \\ -\lambda_{2m+1-k} & 0 \end{pmatrix}$ for k =1, 2, 3, ….m

and $M_{k1} = \begin{pmatrix} -\lambda_k & -\lambda_{2m} \\ -\lambda_{2m} & -\lambda_k \end{pmatrix} =$ for k = 2, 3, ….m

$M_{kj} = \begin{pmatrix} -\lambda_j & -\lambda_{2m+1-j} \\ -\lambda_{2m+1-j} & -\lambda_j \end{pmatrix}$ for j = 2, 3, ….m

Example: 3

$\Lambda = \{\lambda_1, \lambda_2, \lambda_3, \lambda_4\}$, $\Lambda_k = \{\lambda_k, \lambda_{2m+1-k}\}$ for m = 2. Now $\Lambda_1 = \{\lambda_1, \lambda_4\}$ and

$\Lambda_2 = \{\lambda_2, \lambda_3\}$ then $\Lambda = \{\lambda_1, \lambda_2, \lambda_3, \lambda_4\} = \{1, -1/3, -1/3, -1/3\}$ satisfies

$S = 0 > \lambda_3$ with m = 2, $\lambda_1 = 1$, $\lambda_2 = -1/3$, $\lambda_3 = -1/3$, $\lambda_4 = -1/3$

$$M_{11} = \begin{pmatrix} 0 & -\lambda_{2m} \\ -\lambda_{2m} & 0 \end{pmatrix} = \begin{pmatrix} 0 & -\lambda_4 \\ -\lambda_4 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1/3 \\ 1/3 & 0 \end{pmatrix}$$

$$M_{22} = \begin{pmatrix} 0 & -\lambda_{2m-1} \\ -\lambda_{2m-1} & 0 \end{pmatrix} = \begin{pmatrix} 0 & -\lambda_3 \\ -\lambda_3 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1/3 \\ 1/3 & 0 \end{pmatrix}$$

$$M_{12} = \begin{pmatrix} -\lambda_j & -\lambda_{5-j} \\ -\lambda_{5-j} & -\lambda_j \end{pmatrix} = \begin{pmatrix} -\lambda_2 & -\lambda_3 \\ -\lambda_3 & -\lambda_2 \end{pmatrix} = \begin{pmatrix} 1/3 & 1/3 \\ 1/3 & 1/3 \end{pmatrix}$$

$M_{21} = \begin{pmatrix} -\lambda_2 & -\lambda_4 \\ -\lambda_4 & -\lambda_2 \end{pmatrix} = \begin{pmatrix} 1/3 & 1/3 \\ 1/3 & 1/3 \end{pmatrix}$. Then the doubly stochastic matrix is

$$M = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} = \begin{pmatrix} 0 & 1/3 & 1/3 & 1/3 \\ 1/3 & 0 & 1/3 & 1/3 \\ 1/3 & 1/3 & 0 & 1/3 \\ 1/3 & 1/3 & 1/3 & 0 \end{pmatrix}$$

Type: 3 If a list of real unit numbers

$\Lambda = \{\lambda_1, \lambda_2, \ldots\ldots\lambda_{2m+1}\} = \Lambda_1 \cup \Lambda_2 \cup \ldots\ldots \Lambda_m \cup \{\lambda_{m+1}\}$

$\Lambda_k = \{\lambda_k, \lambda_{2m+2-k}\}$, k = 1, 2, …. m satisfies

$1 = \lambda_1 \geq \lambda_2 \geq \ldots\ldots\ldots \geq \lambda_{m+1} > 0 > \lambda_{m+2} \geq \ldots\ldots\geq \lambda_{2m+1} \geq -1$

$S = \lambda_1 + \lambda_2 + \ldots\ldots\ldots + \lambda_{2m+1} \geq \lambda_{m+1}$ and $\lambda_{2m+2-k} < \lambda_k$, k = 2, 3, ….m. Then Λ is realizable by the following doubly stochastic matrix

$$M = \begin{pmatrix} M_{11} & M_{12} & \cdots & M_{1m} & M_{1,m+1} \\ M_{21} & M_{22} & \cdots & M_{2m} & M_{2,m+1} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ M_{m1} & M_{m2} & \cdots & M_{mm} & M_{m,m+1} \\ M_{m+1,1} & M_{m+1,2} & \cdots & M_{m+1,m} & S \end{pmatrix}$$

where $M_{kk} = \begin{pmatrix} 0 & -\lambda_{2m+2-k} \\ -\lambda_{2m+2-k} & 0 \end{pmatrix}$ for k = 1, 2, ….m

$M_{k,m+1} = \begin{pmatrix} S - \lambda_{m+1} \\ S - \lambda_{m+1} \end{pmatrix}$ for k = 1, 2, ….m

and $M_{m+1,k} = \begin{pmatrix} -\lambda_{2m+1} - S & -\lambda_{2m+1} - S \end{pmatrix}$ for k = 1, 2, ….m

$M_{k1} = \begin{pmatrix} 0 & -\lambda_{2m+1} - \lambda_k \\ -\lambda_{2m+1} - \lambda_k & 0 \end{pmatrix}$ for k = 2, 3 ….m

$M_{kj} = \begin{pmatrix} 0 & -\lambda_{2m+2-j} - \lambda_j \\ -\lambda_{2m+2-j} - \lambda_j & 0 \end{pmatrix}$ for j = 2, 3 ….m

Example: 4

$\Lambda = \{\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5\}$, $\Lambda_k = \{\lambda_k, \lambda_{2m+2-k}\}$ for m = 2.

Now $\Lambda_1 = \{\lambda_1, \lambda_5\}$ and $\Lambda_2 = \{\lambda_2, \lambda_4\}$ then $\Lambda = \{\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5\} = \{1, 1, 1, -1, -1\}$ satisfies $S \geq \lambda_3$ and $S = 1 = \lambda_3$ with m = 2, $\lambda_1 = 1$, $\lambda_2 = 1$, $\lambda_3 = 1$, $\lambda_4 = -1$, $\lambda_5 = -1$

$$M_{11} = \begin{pmatrix} 0 & -\lambda_5 \\ -\lambda_5 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$
$$M_{22} = \begin{pmatrix} 0 & -\lambda_4 \\ -\lambda_4 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$
$$M_{k,,3} = \begin{pmatrix} S-\lambda_3 \\ S-\lambda_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \text{ for } k = 1, 2$$
$$M_{3,k} = \begin{pmatrix} -\lambda_3 - S & -\lambda_3 - S \end{pmatrix} = \begin{pmatrix} 0 & 0 \end{pmatrix} \text{ for } k = 1, 2$$

$$M_{12} = \begin{pmatrix} 0 & -\lambda_{6-j} - \lambda_j \\ -\lambda_{6-j} - \lambda_j & 0 \end{pmatrix} = \begin{pmatrix} 0 & -\lambda_4 - \lambda_2 \\ -\lambda_4 - \lambda_2 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$M_{21} = \begin{pmatrix} 0 & -\lambda_5 - \lambda_2 \\ -\lambda_5 - \lambda_2 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$. Then the doubly stochastic matrix is

$$M = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Type: 4 If a list of real unit numbers

$\Lambda = \{\lambda_1, \lambda_2, \ldots\ldots\lambda_{2m}\} = \Lambda_1 \cup \Lambda_2 \cup \ldots\ldots \Lambda_m$

$\Lambda_k = \{\lambda_k, \lambda_{2m+1-k}\}$, k = 1, 2, …. m satisfies

$1 = \lambda_1 \geq \lambda_2 \geq \ldots\ldots\ldots \geq \lambda_{m+1} > 0 > \lambda_{m+2} \geq \ldots\ldots\ldots \geq \lambda_{2m+1} \geq -1$

$S = \lambda_1 + \lambda_2 + \ldots\ldots\ldots + \lambda_{2m} \geq \lambda_{m+1}$ and $\lambda_{2m+1-k} < \lambda_k$, k = 2, 3, ….m. Then Λ is realizable by the following doubly stochastic matrix

$$M = \begin{pmatrix} M_{11} & M_{12} & \cdots & M_{1m} \\ M_{21} & M_{22} & \cdots & M_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ M_{m1} & M_{m2} & \cdots & M_{mm} \end{pmatrix}$$

where $M_{kk} = \begin{pmatrix} 0 & -\lambda_{2m+1-k} \\ -\lambda_{2m+1-k} & 0 \end{pmatrix}$ for k =1, 2, 3, ….m

and $M_{k1} = \begin{pmatrix} 0 & -\lambda_{2m} - \lambda_k \\ -\lambda_{2m} - \lambda_k & 0 \end{pmatrix}$ for k = 2, 3, ….m

$M_{kj} = \begin{pmatrix} 0 & -\lambda_{2m+1-j} - \lambda_j \\ -\lambda_{2m+1-j} - \lambda_j & 0 \end{pmatrix}$ for j = 2, 3, ….m

Example: 5

$\Lambda = \{\lambda_1, \lambda_2, \lambda_3, \lambda_4\}$, $\Lambda_k = \{\lambda_k, \lambda_{2m+1-k}\}$ for m = 2. Now $\Lambda_1 = \{\lambda_1, \lambda_4\}$ and $\Lambda_2 = \{\lambda_2, \lambda_3\}$ then

$\Lambda = \{\lambda_1, \lambda_2, \lambda_3, \lambda_4\} = \{1, 1, -1, -1\}$ satisfies $S = 0 > \lambda_3$ with m = 2, $\lambda_1 = 1$, $\lambda_2 = 1$, $\lambda_3 = -1$, $\lambda_4 = -1$

$$M_{11} = \begin{pmatrix} 0 & -\lambda_4 \\ -\lambda_4 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$
$$M_{22} = \begin{pmatrix} 0 & -\lambda_3 \\ -\lambda_3 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$M_{12} = \begin{pmatrix} 0 & -\lambda_{5-j} - \lambda_j \\ -\lambda_{5-j} - \lambda_j & 0 \end{pmatrix} = \begin{pmatrix} 0 & -\lambda_3 - \lambda_2 \\ -\lambda_3 - \lambda_2 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$M_{21} = \begin{pmatrix} 0 & -\lambda_4 - \lambda_2 \\ -\lambda_4 - \lambda_2 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$. Then the doubly stochastic matrix is

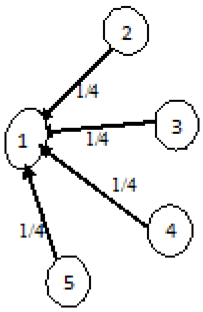
$$M = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

STATIONARY DISTRIBUTION OF THE SYMMETRIC DOUBLY STOCHASTIC MATRIX

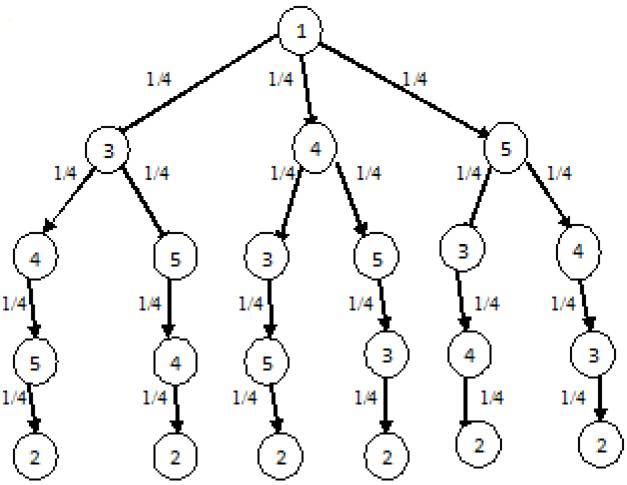
In generally the doubly stochastic matrices of whose entries are 1/ (n-1) in all except the main diagonal. But the main diagonal entries are zero. For example

$$\text{Let } A = \begin{pmatrix} 0 & 1/4 & 1/4 & 1/4 & 1/4 \\ 1/4 & 0 & 1/4 & 1/4 & 1/4 \\ 1/4 & 1/4 & 0 & 1/4 & 1/4 \\ 1/4 & 1/4 & 1/4 & 0 & 1/4 \\ 1/4 & 1/4 & 1/4 & 1/4 & 0 \end{pmatrix}$$

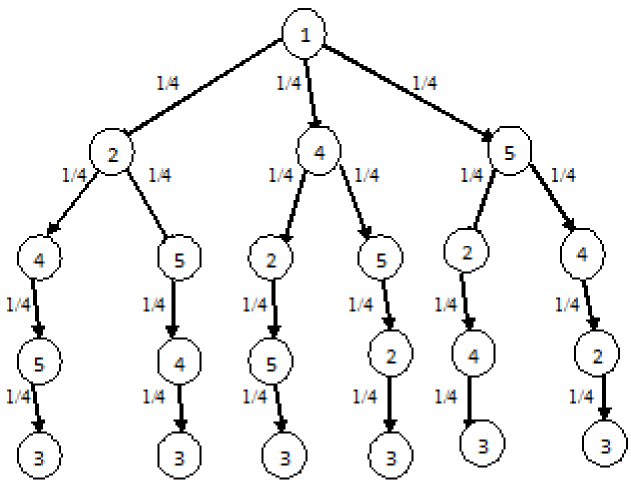
$\mathbf{T_1(W_1) = \frac{1}{256}}$



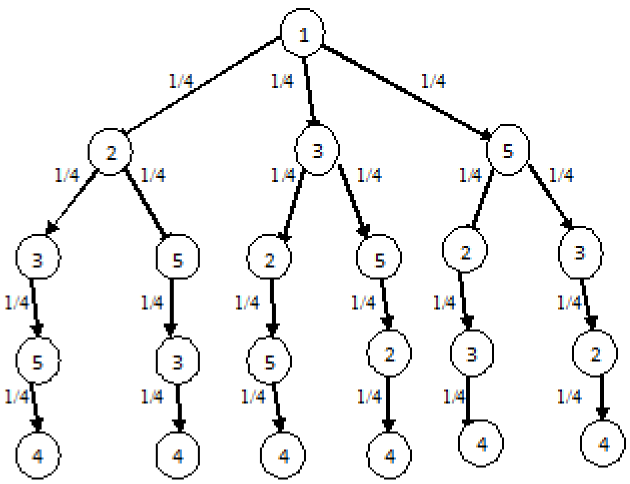
$\mathbf{T_1(W_2) = \frac{6}{256}}$



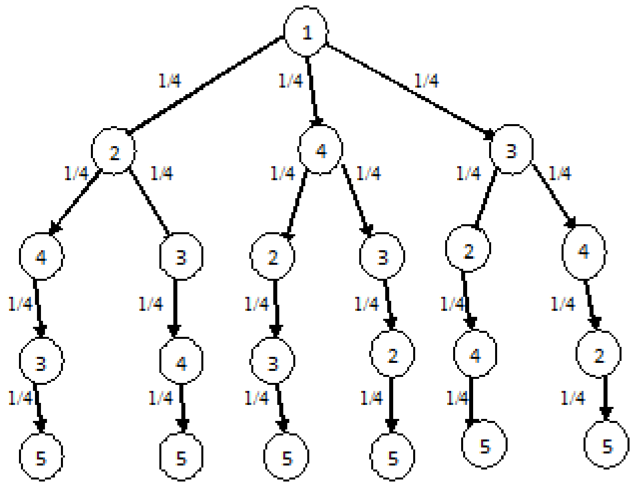
$\mathbf{T_1(W_3) = \frac{6}{256}}$



$\mathbf{T_1(W_4)} = \frac{\mathbf{6}}{\mathbf{256}}$



$\mathbf{T_1(W_5)} = \frac{\mathbf{6}}{\mathbf{256}}$



$C_1 = \sum_{i=1}^{5} T_1 W_i = T_1W_1 + T_1W_2 + T_1W_3 + T_1W_4 + T_1W_5$
$= \frac{1}{256} + \frac{6}{256} + \frac{6}{256} + \frac{6}{256} + \frac{\mathbf{6}}{\mathbf{256}} = \frac{\mathbf{25}}{\mathbf{256}}$

Similarly we can draw the intree for all nodes, we have

$C_2 = \sum_{i=1}^{5} T_2 W_i = T_2W_1 + T_2W_2 + T_2W_3 + T_2W_4 + T_2W_5$
$= \frac{6}{256} + \frac{1}{256} + \frac{6}{256} + \frac{6}{256} + \frac{\mathbf{6}}{\mathbf{256}} = \frac{\mathbf{25}}{\mathbf{256}}$

$C_3 = \sum_{i=1}^{5} T_3 W_i = T_3W_1 + T_3W_2 + T_3W_3 + T_3W_4 + T_3W_5$
$= \frac{6}{256} + \frac{6}{256} + \frac{1}{256} + \frac{6}{256} + \frac{\mathbf{6}}{\mathbf{256}} = \frac{\mathbf{25}}{\mathbf{256}}$

$C_4 = \sum_{i=1}^{5} T_4 W_i = T_4W_1 + T_4W_2 + T_4W_3 + T_4W_4 + T_4W_5$
$= \frac{6}{256} + \frac{6}{256} + \frac{6}{256} + \frac{1}{256} + \frac{\mathbf{6}}{\mathbf{256}} = \frac{\mathbf{25}}{\mathbf{256}}$

$C_5 = \sum_{i=1}^{5} T_5 W_i = T_5W_1 + T_5W_2 + T_5W_3 + T_5W_4 + T_5W_5$
$= \frac{6}{256} + \frac{6}{256} + \frac{6}{256} + \frac{6}{256} + \frac{\mathbf{1}}{\mathbf{256}} = \frac{\mathbf{25}}{\mathbf{256}}$

Stationary distribution for the doubly stochastic matrix is $V_j = \frac{C_j}{\sum_{i=1}^{5} C_i}$

And $\sum_{i=1}^{5} C_i = C_1 + C_2 + C_3 + C_4 + C_5 = \frac{\mathbf{125}}{\mathbf{256}}$

$V_1 = \frac{C_1}{\sum_{i=1}^{5} C_i} = \frac{\mathbf{25}}{\mathbf{256}} / \frac{\mathbf{125}}{\mathbf{256}} = \frac{\mathbf{1}}{\mathbf{5}}$

$V_2 = \frac{C_2}{\sum_{i=1}^{5} C_i} = \frac{\mathbf{25}}{\mathbf{256}} / \frac{\mathbf{125}}{\mathbf{256}} = \frac{\mathbf{1}}{\mathbf{5}}$

$V_3 = \frac{C_3}{\sum_{i=1}^{5} C_i} = \frac{\mathbf{25}}{\mathbf{256}} / \frac{\mathbf{125}}{\mathbf{256}} = \frac{\mathbf{1}}{\mathbf{5}}$

$V_4 = \frac{C_4}{\sum_{i=1}^{5} C_i} = \frac{\mathbf{25}}{\mathbf{256}} / \frac{\mathbf{125}}{\mathbf{256}} = \frac{\mathbf{1}}{\mathbf{5}}$

$V_5 = \frac{C_5}{\sum_{i=1}^{5} C_i} = \frac{\mathbf{25}}{\mathbf{256}} / \frac{\mathbf{125}}{\mathbf{256}} = \frac{\mathbf{1}}{\mathbf{5}}$

Finally the stationary distribution values of the doubly stochastic matrix are all equal to 1/n, where n is the order of the doubly stochastic matrix and the main diagonal values are all entry in zero.

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