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A NOTE ON HARMONIC MEAN GRAPHS

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ABSTRACT

A Graph G = (V, E) with p vertices and q edges is said to be a Harmonic mean graph if is possible to label the vertices $x \in V$ with distinct labels f(x) from 1,2…..q+1 is such a way that when each edge e=uv is labeled with f(e=uv) = $\left\lceil\frac{2f(u)f(v)}{f(u)+f(v)}\right\rceil$ (or) $\left\lfloor\frac{2f(u)f(v)}{f(u)+f(v)}\right\rfloor$, then the resulting edge labels are distinct. In this case f is called Harmonic mean labeling of G. In this paper we prove that Triple Triangular, Alternate Triple Triangular, Triple Quadrilateral and Alternate Triple Quadrilateral snakes are Harmonic mean graphs.

Keywords: *Graph, Harmonic mean graph. Triple Triangular snake, Alternate Triple Triangular snake, Triple Quadrilateral snake, Alternate Triple Quadrilateral snake graphs.*

1. INTRODUCTION

Throughout this paper we consider only finite, undirected and simple graphs. Let G= (V, E) be a graph with p vertices and q edges. For all terminologies and notations we follow Harary [1]. There are several types of labeling and detailed survey can be found in [2]. The concept of mean labeling has been introduced in [3] and the Harmonic mean labeling was introduced in [4]. The concept of Double Triangular snake and Double Quadrilateral snake has been proved in [5] and [6]. In this paper we prove that the Harmonic mean labeling behaviour of Triple Triangular snake, Triple Quadrilateral snake graphs. Also we wish to investigate Alternate Triple Triangular snake and Alternate Triple Quadrilateral snake graphs are Harmonic mean graphs. The following definitions are necessary for our present investigation.

Definition 1.1: A Graph G= (V, E) with p vertices and q edges is said to be a Harmonic mean graph if it is possible to label the vertices $x \in$V with distinct labels f(x) from 1, 2…q+1 in such way that when each edge e=uv is labeled with f(e=uv) = $\left\lceil\frac{2f(u)f(v)}{f(u)+f(v)}\right\rceil$ (or) $\left\lfloor\frac{2f(u)f(v)}{f(u)+f(v)}\right\rfloor$, then the resulting edge labels are distinct. In this case f is called Harmonic mean labeling of G.

Definition 1.2: A Triangular snake T_n is obtained from a path $u_1u_2\ldots.u_n$ by joining u_i and u_{i+1} to a new vertex v_i for $1 \le i \le n-1$. That is every edge of a path is replaced by a triangles C_3.

Definition 1.3: An Alternate Triangular snake $A(T_n)$ is obtained from a path $u_1,u_2\ldots u_n$ by joining u_{i+1} (Alternatively) to new vertex v_i. That is, every alternate edge of a path is replaced by C_3.

Definition 1.4: A Double Triangular snake $D(T_n)$ consists of two Triangular snakes that have a common path.

Definition 1.5: An Alternate Double Triangular snake $A[D(T_n)]$ consists of two Alternate Triangular snake that have a common path.

Definition 1.6: A Triple Triangular snake $T(T_n)$ consists of three Triangular snakes that have a common path.

Definition 1.7: An Alternate Triple Triangular snake $A(T(T_n))$ consists of three Alternate Triangular snakes that have a common path.

Definition 1.8: A Quadrilateral snake Q_n is obtained from a path $u_1u_2..u_n$ by joining u_i and u_{i+1} to new vertices v_i, w_i respectively and then joining v_i and w_i, $1 \le i \le n-1$. That is, every edge of a path is replaced by cycle C_4.

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Definition 1.9: An Alternate Quadrilateral snake $A(Q_n)$ is obtained from a path $u_1u_2\ldots\ldots u_n$ by joining u_i and u_{i+1} (Alternatively) to new vertices v_i and w_i respectively and then joining v_i and w_i. That is, every alternate edge of path is replaced by a cycle C_4.

Definition 1.10: A Double Quadrilateral snake $D(Q_n)$ consists of two Quadrilateral snakes that have a common path.

Definition 1.11: An Alternate Double Quadrilateral snake $A[D(Q_n)]$ consists of two Alternate Quadrilateral snakes that have a common path.

Definition 1.12: An Triple Quadrilateral snakes $T(Q_n)$ consists of three Quadrilateral snakes that have a common path.

Definition 1.13: An Alternate Triple Quadrilateral snake $A[T(Q_n)$ consists of three Alternate Quadrilateral snakes that have a common path.

2. MAIN RESULTS

Theorem 2.1: Triple Triangular snakes are Harmonic mean graphs.

Proof: Let G be the graph obtained from a path $u_1u_2\ldots..u_n$ by joining u_i and u_{i+1} to three new vertices v_i, w_i and t_i, $1 \le i \le n-1$.

Define a function $f:V(G)\to\{1,2\ldots..q+1\}$ by
$f(u_1)=3$
$f(u_i)=7i-6,\quad 1\le i\le n$
$f(v_1)=1$
$f(v_i)=7i-5,\quad 1\le i\le n-1$
$f(w_i)=7i-3,\quad 1\le i\le n-1$
$f(t_i) = 7i,\quad 1\le i\le n-1.$

From the above labeling pattern, we get distinct edge labels.

Thus f provides a Harmonic mean labeling for G.

Example 2.2: The labeling pattern of Triple Triangular snake obtained from six vertices is given below

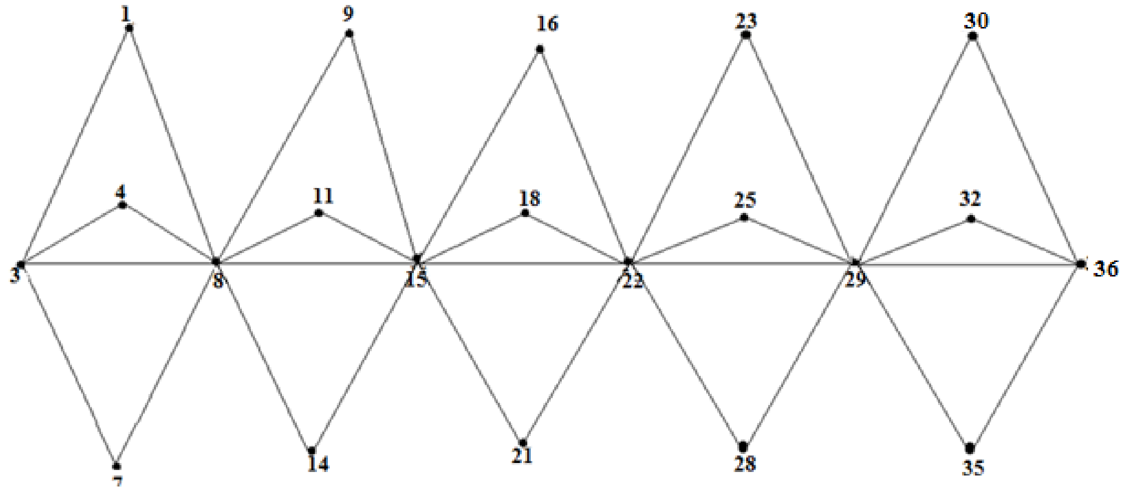

Figure: 1

Theorem 2.3: Alternate Triple Triangular snakes are Harmonic mean graphs.

Proof: Let $G = A[T(T_n)]$ be the Alternate Triple Triangular snake graph and its vertices be v_i, w_i and t_i, $1 \le i \le n-1$.
Here we consider two cases.

Case (i): If the triangle starts from u_1 then

We need to considered two subcases.

Subcase (i) (a): If n is odd then

Define a function f: $V(G) \rightarrow \{1,2\ldots q+1\}$ by
$f(u_1) = 3$

$$f(u_i) = \begin{cases} 4i-3, & \forall\ i = 3,5,7\ldots\ldots n \\ 4i, & \forall\ i = 2,4,6\ldots.n-1. \end{cases}$$

$f(v_1)=1$
$f(v_i) = 8i-6, \quad \forall\ i = 2,3,4,\ldots..\frac{n-1}{2}$
$f(w_i) = 8i-4, \quad \forall\ i = 1,2,3,\ldots..\frac{n-1}{2}$
$f(t_i) = 8i-2, \quad \forall\ i = 1,2,3,\ldots..\frac{n-1}{2}$

From the above labeling pattern, we get the edge labels are all distinct.

Thus f provide a Harmonic mean labeling for G. Harmonic mean labeling of G obtained from seven vertices is given below.

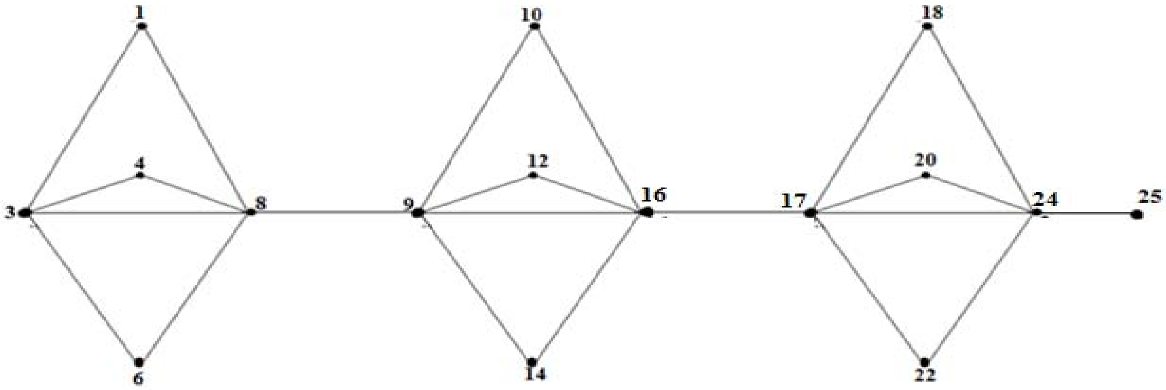

Figure: 2

Subcase (i) (b): If n is even then

Define a function f: $V(G) \rightarrow \{1,2\ldots.q+1\}$by
$f(u_1) = 3$

$$f(u_i) = \begin{cases} 4i, & \forall\ i = 2,4,6\ldots\ldots n \\ 4i-3 & \forall\ i = 3, 5, 7\ldots\ldots n-1 \end{cases}$$

$f(v_i) = 8i-6, \quad \forall\ i = 2,3,4\ldots.\frac{n}{2}$
$f(w_i) = 8i-4, \quad \forall\ i = 1,2,3\ldots.\frac{n}{2}$
$f(t_i) = 8i-2, \quad \forall\ i = 1,2,3\ldots.\frac{n}{2}$

From the above labeling pattern, we get the edges labels are all distinct.

Thus f provide a Harmonic mean labeling for G the labeling pattern is shown in the following figure.

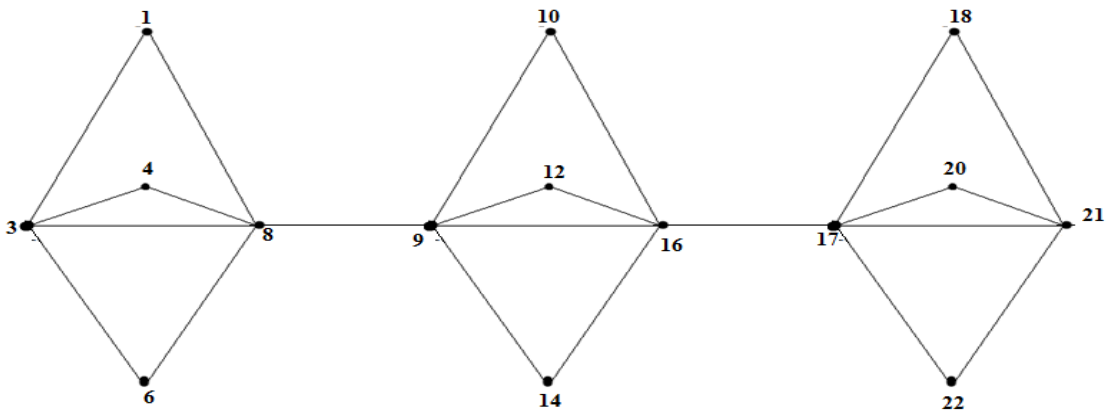

Figure: 3

Case (ii): If the triangular starts from u_2 then also we consider two subcases

Subcase (ii) (a): If n is odd then

Define a function f:V(G)→{1,2…..q+1} by
$f(u_1) = 1$
$f(u_2) = 4$

$$f(u_i) = \begin{cases} 4i-3, & \forall\ i=3,5\ldots\ldots.n. \\ 4i-6, & \forall\ i=4,\ 6\ldots\ldots.n-1. \end{cases}$$

$f(v_1) = 2$
$f(v_i) = 8i-5, \quad \forall\ i= 2,3\ldots\ldots\frac{n-1}{2}$
$f(w_i) = 8i-3, \quad \forall\ i=1,2,3,\ldots\ldots \frac{n-1}{2}.$
$f(t_i) = 8i, \forall\ i=2,4,6,\ldots\ldots \frac{n-1}{2}.$

From above labeling pattern, we get distinct edge labels.

Thus f is Harmonic mean labeling of G and its labeling pattern of seven vertices is shown below.

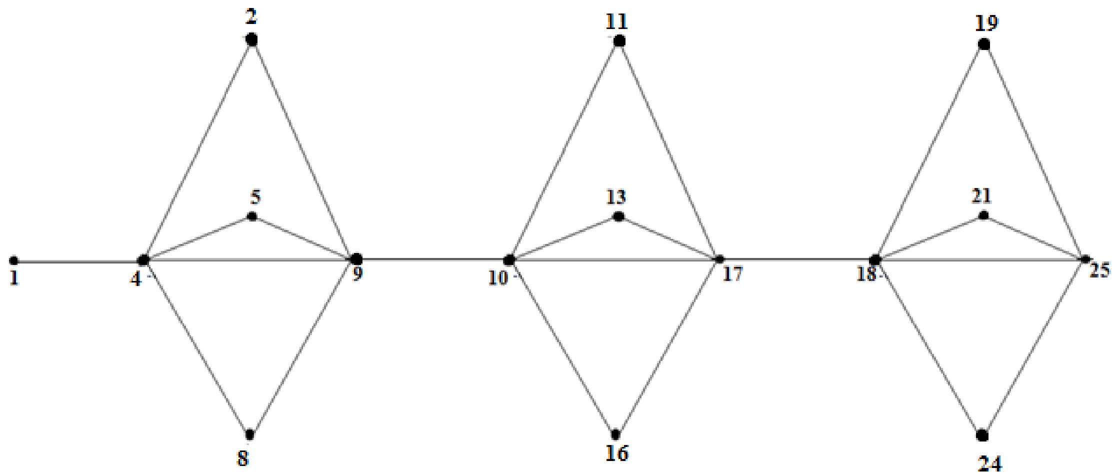

Figure: 4

Sub case (ii) (b): If n is even then

Define a function f: V(G) → {1,2….q+1} by
$f(u_1) = 1$
$f(u_2) = 4$

$$f(u_i) = \begin{cases} 4i-3, & \forall\ i=3,5\ldots\ldots.n. \\ 4i-6, & \forall\ i=4,6\ldots\ldots.n-1. \end{cases}$$

$f(v_1) = 2$
$f(v_i) = 8i-5, \quad \forall\ i= 2,3\ldots\ldots\frac{n-2}{2}$
$f(w_i) = 8i-3, \quad \forall\ i=1,2,3,\ldots\ldots \frac{n-2}{2}.$
$f(t_i) = 8i, \forall\ i=2,4,6,\ldots\ldots \frac{n-2}{2}.$

Then we get distinct edge labels.

Thus f provide a Harmonic mean labeling of G and its labeling pattern of G is obtained eight vertices is shown below.

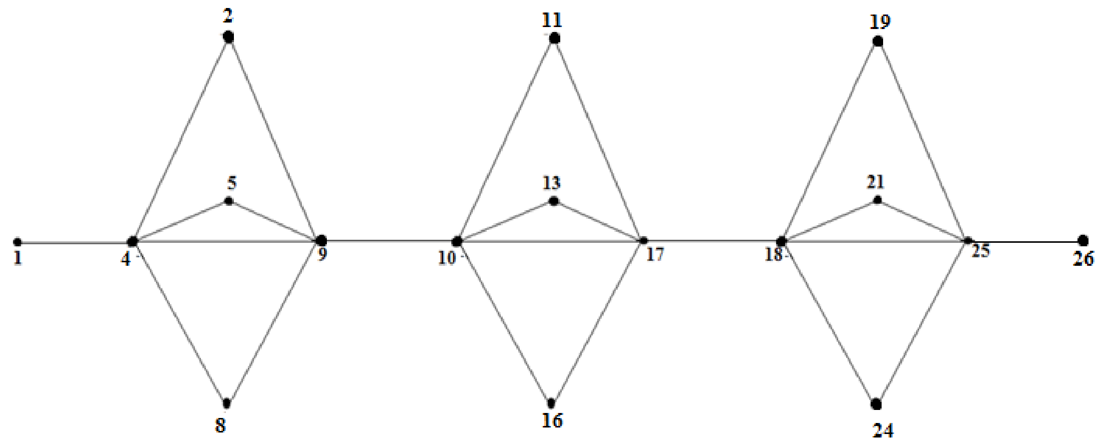

Figure: 5

From above all the cases, we conclude that Alternate Triple Triangular snakes are Harmonic mean graphs.

Theorem 2.4: Triple Quadrilateral snakes are Harmonic mean graphs.

Proof: Let G be the graph obtained from a path $u_1u_2.....u_n$ by joining u_i and u_{i+1} (alternatively) to new vertices v_i, w_i x_i, y_i, z_i and t_i respectively and then joining v_i w_i, x_iy_i and z_i, t_i $1 \le i \le n-1$ and its labeling pattern is shown below

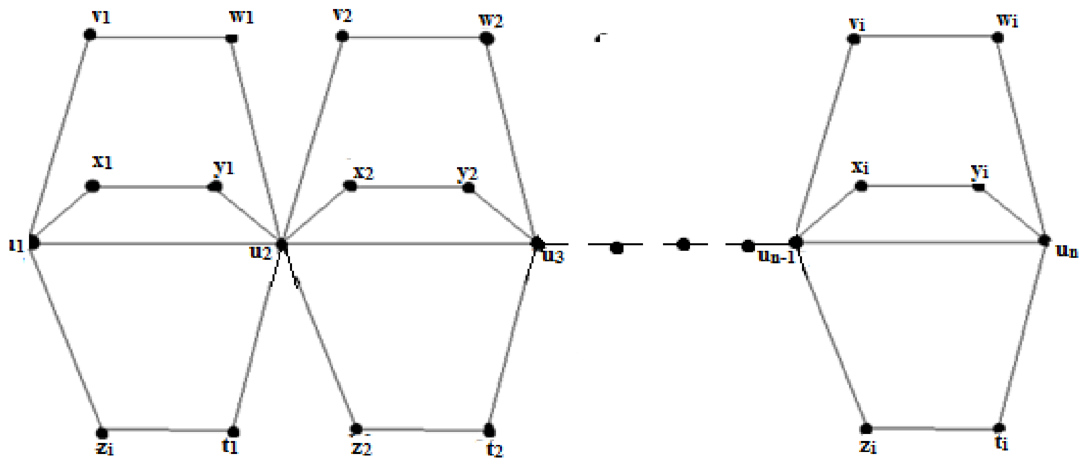

Figure: 6

Define a function $f:V(G) \to \{1,2.......q+1\}$ by

$f(u_1) = 4$

$f(u_i) = 10i-9$, $2 \le i \le n$.

$f(v_1)=1$

$f(v_i) = 10i-8$, $1 \le i \le n-1$

$f(w_i) = 10i-3$, $1 \le i \le n-1$.

$f(x_i) = 10i-7$, $1 \le i \le n-1$.

$f(y_i) = 10i-1$ $1 \le i \le n-1$.

$f(z_i) = 10i-5$, $1 \le i \le n-1$.

$f(t_i) = 10i$ $1 \le i \le n-1$.

From above labeling pattern, we get the edge labels are all distinct.

Thus f provide a Harmonic mean labeling for G.

Example 2.5: Harmonic mean labeling of G is obtained from five vertices is given below.

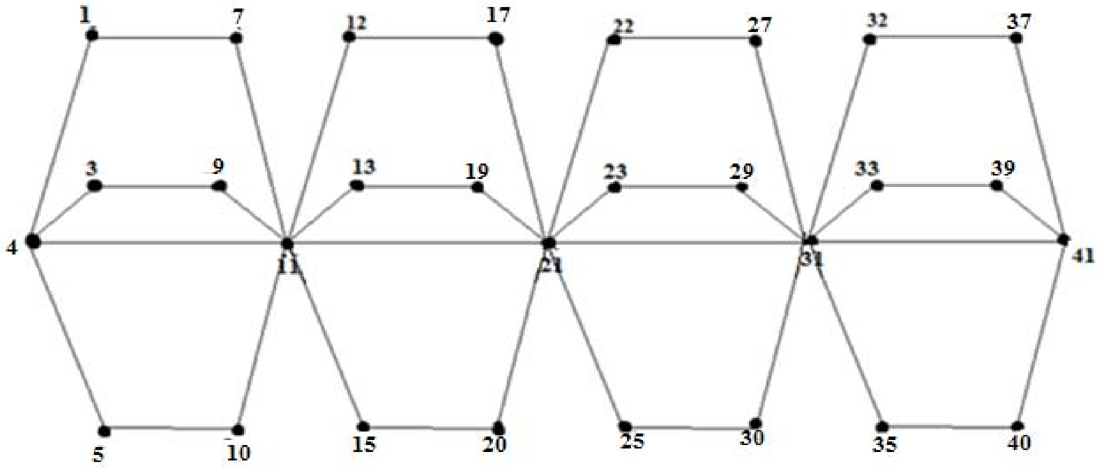

Figure: 7

Theorem 2.6: Alternate Triple Quadrilateral snakes are Harmonic mean graphs

Proof: Let G be an Alternate Triple Quadrilateral snake and its vertices be v_i, w_i, x_i, y_i, z_i and t_i ($1 \le i \le n-1$).

Case (i): If the Quadrilateral starts from u_1 then

We have two cases.

Sub case (i)(a): If n is odd then

Define a function f: V(G)→{1,2…..q+1} by

Define a function f: V(G)→{1,2…..q+1} by

$f(u_1) = 3$

$$f(u_i) = \begin{cases} \frac{11\,i}{2}, & \forall\ i = 2,4,6\ldots n. \\ \frac{11(i-1)}{2}+1, & \forall\ i = 3, 5, 7\ldots.n-1. \end{cases}$$

$f(v_1) = 1$
$f(v_i) = 11i-9, \quad \forall\ i = 2,3,4\ldots.\frac{n-1}{2}$
$f(w_i) = 11i-5, \quad \forall\ i = 1,2,3\ldots.\frac{n-1}{2}$
$f(x_i) = 11i-7, \quad \forall\ i = 1,2,3\ldots.\frac{n-1}{2}$
$f(y_1) = 10$
$f(y_i) = 11i-2, \quad \forall\ i = 2,3,4\ldots.\frac{n-1}{2}$
$f(z_1) = 6$
$f(z_i) = 11i-5, \quad \forall\ i = 1,2,3\ldots.\frac{n-1}{2}$
$f(t_1) = 8$
$f(t_i) = 11i-2, \quad \forall\ i = 2,3,4\ldots.\frac{n-1}{2}$

From the above labeling pattern we get the edge labels are all distinct. G is a Harmonic mean graph.

Labeling pattern of G is obtained from seven vertices is shown in the following figure.

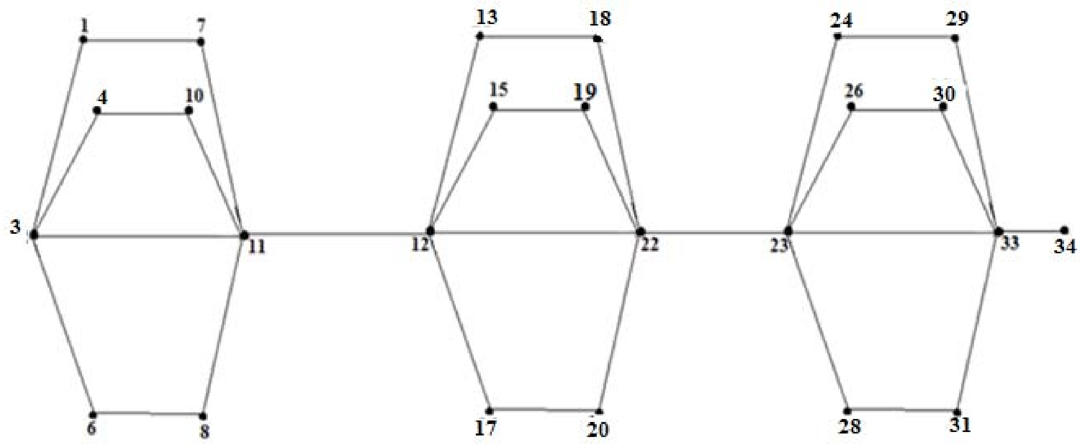

Figure: 8

Subcase (i) (b): If n is even then

Define a function f: V(G)→{1,2…..q+1} by
$f(u_1) = 3$

$$f(u_i) = \begin{cases} \frac{11\,i}{2}, & \forall\ i = 2,4,6\ldots n. \\ \frac{11(i-1)}{2}+1, & \forall\ i = 3, 5, 7\ldots.n-1. \end{cases}$$

$f(v_1) = 1$
$f(v_i) = 11i-9$, $\forall\ i = 2,3,4\ldots.\frac{n}{2}$
$f(w_i) = 11i-5$, $\forall\ i = 1,2,3\ldots.\frac{n}{2}$
$f(x_i) = 11i-7$, $\forall\ i = 1,2,3\ldots.\frac{n}{2}$
$f(y_1) = 10$
$f(y_i) = 11i-2$, $\forall\ i = 2,3,4\ldots.\frac{n}{2}$
$f(z_1) = 6$
$f(z_i) = 11i-5$, $\forall\ i = 1,2,3\ldots.\frac{n}{2}$
$f(t_1) = 8$
$f(t_i) = 11i-2$, $\forall\ i = 2,3,4\ldots.\frac{n}{2}$

From the above labeling pattern we get the edge labels are all distinct. G is a Harmonic mean graph.

Labeling pattern of G is obtained from seven vertices is shown in the following figure.

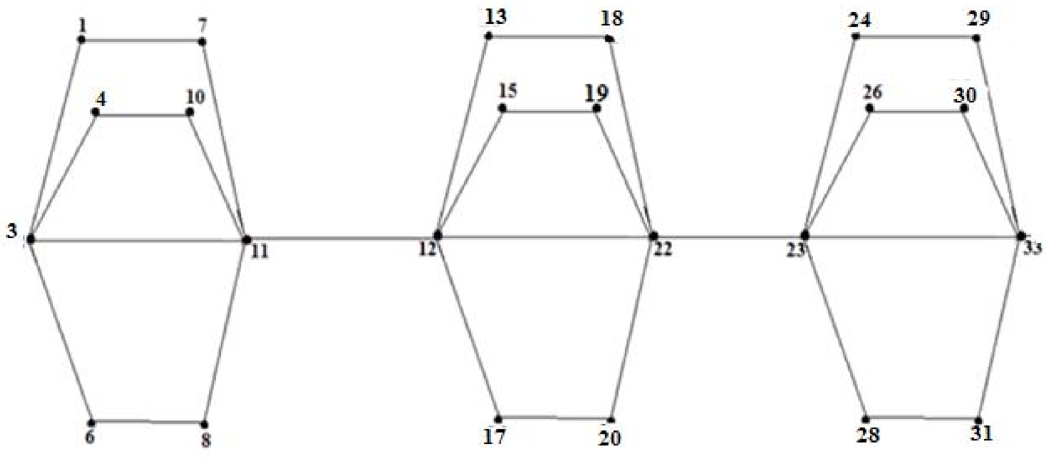

Figure: 9

Case (ii): If the Quadrilateral starts from u_2.

Here we consider two subcases

Subcase(ii)(a): If n is odd then

Define a function f: $V(G)\rightarrow\{1,2\ldots,q+1\}$ by
$f(u_2) = 4$
$f(u_i) = \frac{11(i-1)}{2}+1, \ \forall i=1,3,5\ldots.n.$
$f(u_i) = \frac{11i}{2}-9 \quad \forall\ i = 4,6\ldots n-1.$
$f(v_1) = 2$
$f(v_i) = 11i-8, \quad \forall i = 2,3,4\ldots.\frac{n-1}{2}$
$f(w_i) = 11i-3, \quad \forall i = 1,2,3\ldots.\frac{n-1}{2}$
$f(x_i) = 11i-6, \quad \forall\ i = 1,2,3\ldots.\frac{n-1}{2}$
$f(y_i) = 11i-1, \quad \forall i = 1,2,3\ldots.\frac{n-1}{2}$
$f(z_i) = 11i-4, \quad \forall i = 1,2,3\ldots.\frac{n-1}{2}$
$f(t_i) = 11i, \quad \forall i = 1,2,3\ldots.\frac{n-1}{2}$

From the above labeling pattern we get distinct edge labels.

Thus f provides a Harmonic mean labeling for G and it's labeling pattern obtained from seven vertices is given below.

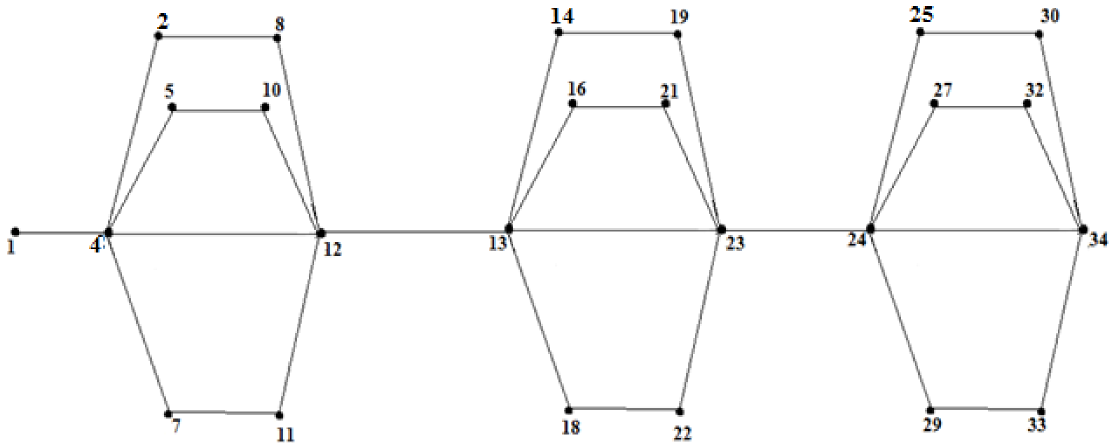

Figure: 10

Subcase (ii) (b): If n is even then

Define a function f: $V(G)\rightarrow\{1,2\ldots..q+1\}$ by

$f(u_2) = 4$
$f(u_i) = \frac{11(i-1)}{2}+1, \ \forall_i=1,3,5\ldots.n.$
$f(u_i) = \frac{11i}{2}-9 \quad \forall\ i = 4,6\ldots n-1.$
$f(v_1) = 2$
$f(v_i) = 11i-8, \quad \forall i = 2,3,4\ldots.\frac{n-2}{2}$
$f(w_i) = 11i-3, \quad \forall i = 1,2,3\ldots.\frac{n-2}{2}$
$f(x_i) = 11i-6, \quad \forall\ i = 1,2,3\ldots.\frac{n-2}{2}$
$f(y_i) = 11i-1, \quad \forall i = 1,2,3\ldots.\frac{n-2}{2}$
$f(z_i) = 11i-4, \quad \forall i = 1,2,3\ldots.\frac{n-2}{2}$
$f(t_i) = 11i, \quad \forall i = 1,2,3\ldots.\frac{n-2}{2}$

From the above labeling pattern we get distinct edge labels.

Hence G is a Harmonic mean graph.

The labeling pattern of G is obtained from eight vertices is shown in the following figure

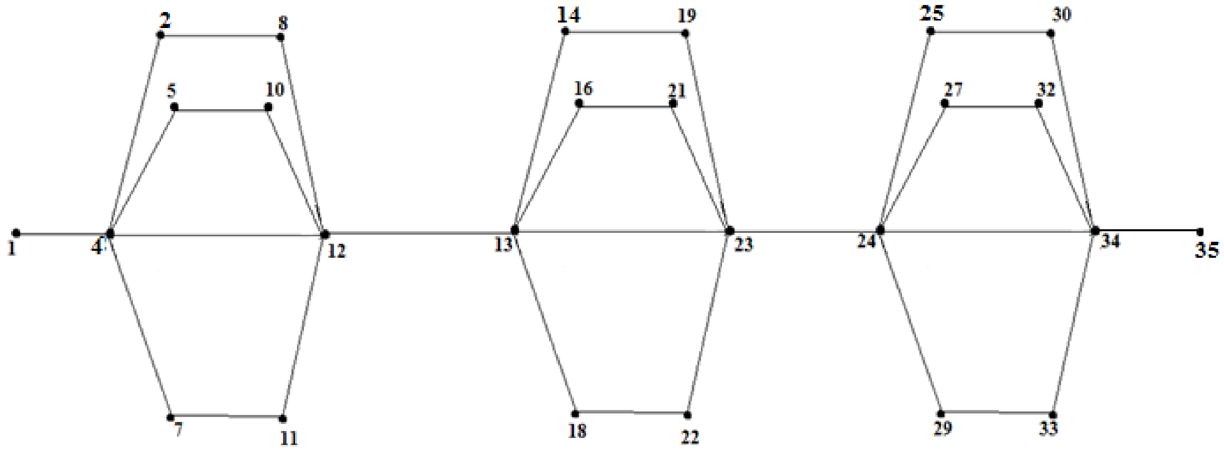

Figure: 11

From all the above cases it is clear that Alternate Triple Quadrilateral snake graphs are Harmonic mean graph.

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