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SINGLE SERVER RETRAIL QUEUE STARTING FAILURE, SUBJECT TO BREAK DOWN WITH MULTIPLE VACATIONS

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ABSTRACT

A single server infinite capacity queueing system with Poisson arrival and exponential service time, where the server is subject to starting failure, interrupt server and go for multiple vacation .We consider that server starting failure cannot offer service to the customer; this type of a customer is called as primary customer. The primary customer used to go to orbit. Where the regular server is subject to breakdown. This type of an involved customer is called as secondary customer. Secondary customer used to go to orbit. We assume that random variable relevant to server failures and repairs are exponential distribution. The server become idle, it takes a vacation, and after finishing its vacation it returns back to the system when it found at least one customer in the system to serve. Otherwise he take another vacation, the vacation time is also an exponential distribution. The steady state probability generating function of the system size is obtained by using supplementary variable method. Also find the average number of customers in the queue.

Keywords: *retrial queue, primary and secondary customer, idle state, multiple vacations.*

Mathematical subject classification: *60K25, 60K30, 90B22.*

1. INTRODUCTION

The retrial queues are characterized by waiting customers who, unlike ordinary queues, can’t be in continuous contact with server, but can only call into test the state of the server. Recently there have been significant contributions to retrial queueing system in which arriving customer who finds the server busy or down will join the orbit, such a queueing model are widely used in telecommunication and telephone switching systems. As we can see in many books devoted to the queueing theory, such as the books [6] and [8], in most common queueing models we often neglect the fact that a server is subject to failures.

As regards papers written last 20 years many papers can be found. Retrial queues considered by researchers so far have the characteristics that each service is preceded and followed by an idle period. Artalejo, J.R *et al.* [1] have considered a retrial queue in which immediately after completion the server’s searchers for customer from the orbit or remain idle. Such systems with starting failures have been studied as queueing models by Yang and Li [3], Krishnakumar *et al.* [2], Mokaddis *et al.* [4], Medhi [5].

A queueing system might suddenly break down and hence the server will not able to continue providing service unless the system is repaired. Aissani and Artalejo, J.R [6], Takine and sengupta [7], Federgruen and So [8], Vinck and Bruneel [9] have studied different queueing system subject to random breakdowns. Kulkarni and Choi [10] and wang *et al.* [11] have studied retrial queues with system breakdowns and repair.

The rest of the article is as follows, the model under consideration is described in section 2.In section 3 we analyzed the definitions and equations governing the system. steady state distribution governing the model obtained in section 4.In section 5 the steady state results of the given model.

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2. MATHEMATICAL MODEL

2.1 Model description

We consider a single-server retrial queue with the server being subject to starting failure, breakdown and multiple vacations. New customer arrive from outside, the server which offers service to the customer, according to a Poisson processes with rate λ. If the server is available the arrival customer enter the service get service immediately, otherwise if has join the orbit, named as Primary customer. Join orbit directly with probability p and test the availability of the server with probability 1-p.The server may encounter random breakdown, the breakdown times follows the Poisson distribution with mean breakdown rate $\alpha > 0$, we assume that when the server breakdown the customer whose service is interrupted comes to the orbit otherwise leave the system, is named as secondary customary. Once the system breakdown, it enters a repair process immediately, the repair times are exponentially distributed with mean repair rate $\beta > 0$.

Various stochastic processes involved in the system are assumed to the independent of each other. The system is free, he call upon the primary and secondary customer respectively (by FCFS). As soon as the system becomes idle, then the server will take vacation of random length. The server takes a vacation with probability η, and with probability 1-η server finds at least one customer in the system, otherwise he goes for another vacation. From the result we obtain the joint probability generating function of the orbit size. To our knowledge this paper is the some of the special case, we discussed in terms of starting failure, breakdown with multiple vacation.

Table(i):

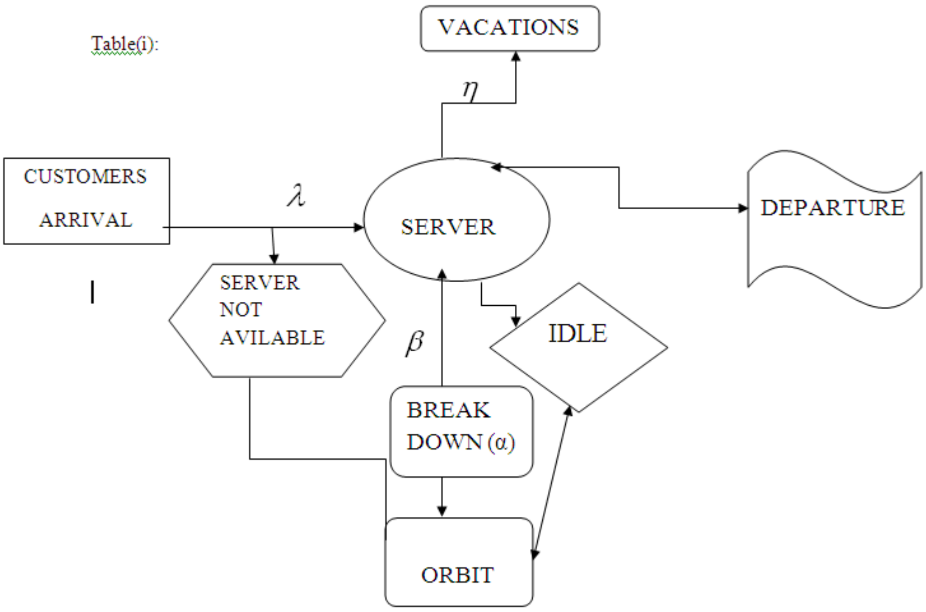

Diagram-1: Flow chart for single server retrial queue with breakdown

3. DEFINITIONS AND EQUATIONS GOVERNING THE SYSTEM

The non - availability of the server time for both primary and secondary customer have the same distribution function R_k the density function $r_k(x)$ and the first two moments R_{k_1}, R_{k_2}.

Let $r_k(x) = a_k(x)(1 - A_k(x))^{-1}$ by the instantaneous service intensity of customers $\tilde{A}_k(s) = \int_0^\infty e^{-sx} dA_k(x)$ by the Laplace-Stieltjes transform of A(x).

The service time for both primary and secondary customer have the same distribution function G_k the density function $g_k(x)$ and the first two moments G_{k_1}, G_{k_2}.

Let $\mu_k(x) = g_k(x)(1 - G_k(x))^{-1}$ by the instantaneous service intensity of customers $\tilde{G}_k(s) = \int_0^\infty e^{-sx} dG_k(x)$ by the Laplace-Stieltjes transform of G(x).

Once the server fails it is immediately sent for the repair. The required time is a random variable with probability distribution function G(y) density function g(y) and the two moments γ_1, γ_2 .We denote by $\beta(y) = b(y)(1 - B(y))^{-1}$ the instantaneous repair intensity given that the elapsed repair time is y_1 $\tilde{B}_k(s) = \int_0^\infty e^{-sy} dB_k(s)$ by the Laplace-Stieltjes transform of B(y).

Vacation times are assumed to be independent and exponentially distributed according to a common probability distribution function $V_0(y)$ of finite, first and second moments η_1, η_2 .We denote by $g_0(y)$ the density function $\eta_0(y) = v_0(y)(1 - V_0(y))^{-1}$ the instantaneous vacation intensity given that the elapsed vacation time is

y , $\tilde{V}_0(s) = \int_0^\infty e^{-sy} dV_0(y)$ by the Laplace-Stieltjes transform V(y).

The state of the system at time t can be described by the Markov process

$$\{N(t), t \geq 0\} = \{C(t), X(t), \xi_0(t), \xi_1(t), \xi_2(t), \xi_3(t) t \geq 0\}$$

Where C(t) denotes the server state{0,1,2,3} according to the server being not available, busy, breakdown vacation respectively and X(t) denotes to the number of customer in the orbit ‘t’.

The server state is dented by

$$C(t) = \begin{cases} 0, X(t) \succ \xi_0(t) \quad \textit{represents}\ \text{non-availability}\ \textit{for the server elapsed retrial time,} \\ 1, \textit{then}\ \xi_1(t)\ \textit{corresponds to the elapsed retrial of the customer being served,} \\ 2, \textit{then}\ \xi_2(t)\ \textit{elapsed breakdown time at time}\ 't'. \\ 3, \textit{then}\ \xi_3(t)\ \textit{elapsed vacation time at time}\ 't'. \end{cases}$$

The function $r(x), \mu(x), \beta(x), \eta(x)$ are the conditional completion rates (at time x) for non-availability for the server, for service, repair and vacation respond.

i.e., $r(x) = \dfrac{a(x)}{1 - A(x)}$, $\mu(x) = \dfrac{g(x)}{1 - G(x)}$, $\beta(x) = \dfrac{b(x)}{1 - B(x)}$, $\eta(x) = \dfrac{v(x)}{1 - V(x)}$.

4. STEADY STATE DISTRIBUTION GOVERNING THE MODEL

The steady state probabilities for the following:

(i) $P_{0,v}(n)$ is the probability that there are n (n=0,1,2,3….no customers in the orbit and the server is on vacation.

(ii) $P_{0,s}(n)$ is the probability that there are n (n=0,1,2,…….customers in the orbit and the customer in the service is in phase i(i=1,2,……k) when the server is before going to vacation is in busy state.

(iii) $P_{0,r}(n)$ is the probability that there are n (n=0,1,2,…….customers in the orbit (regular customer) and the customer in the repeated attempts.

(iv) $P_{0,b}(n)$ is the probability that there are n (n=0,1,2,…….customers in queue at the time t, the server is inactive due to breakdown and the system is under repair.

(v) $P_{0,n}(n)$ is the probability that server non-availability.

Now we define

$$I_0(t) = P\{C(t) = 0, X(t) = 0\}$$

$$I_n(x,t)dx = P\{C(t) = 0, X(t) = n, x \leq \xi_0(t) < x + dx\}; t \geq 0, x \geq 0, n \geq 1$$

$$W_n(x,t)dx = P\{C(t) = 1, X(t) = n, x \leq \xi_1(t) < x + dx\}; t \geq 0, x \geq 0, n \geq 0$$

$$R_n(x,t)dx = P\{C(t) = 2, X(t) = n, x \leq \xi_2(t) < x + dx\}; t \geq 0, x \geq 0, n \geq 0$$

$$V_n(x,t)dx = P\{C(t) = 3, X(t) = n, x \leq \xi_0(t) < x + dx\}; t \geq 0, x \geq 0, n \geq 0$$

We assume that the stability condition is fulfilled, so we can set for
$t \geq 0, x \geq 0, n \geq 0$
$I_0 = \lim_{t\to\infty} I_0(t), I_n(x) = \lim_{t\to\infty} I_n(x,t), W_n(x) = \lim_{t\to\infty} W_n(x,t), R_n(x) = \lim_{t\to\infty} R_n(x,t), V_n(x) = \lim_{t\to\infty} V_n(x,t).$

By the method of supplementary variable technique, we obtain the following system of governing equations,

$$(\lambda+\eta)I_0(x) = \int_0^\infty W_0(x,t)\mu(x)dx + \int_0^\infty R_0(x,t)\beta(x)dx + \int_0^\infty V_0(x,t)\eta(x)dx \tag{1}$$

$$\frac{\partial}{\partial x} I_n(x,t) = -(\lambda + r(x))I_n(x,t), n \geq 1 \tag{2}$$

$$\frac{\partial}{\partial x} W_0(x,t) = -(\lambda + \mu(x))W_0(x,t) \tag{3}$$

$$\frac{\partial}{\partial x} W_n(x,t) = -(\lambda + \mu(x))W_n(x,t) + \lambda W_{n-1}(x,t), n \geq 1 \tag{4}$$

$$\frac{\partial}{\partial x} R_1(x,t) = -(\lambda + \beta(x))R_1(x,t) \tag{5}$$

$$\frac{\partial}{\partial x} R_n(x,t) = -(\lambda + \beta(x))R_n(x,t) + \lambda R_{n-1}(x,t), n \geq 2 \tag{6}$$

$$\frac{\partial}{\partial x} V_0(x,t) = -(\lambda + \eta(x))V_0(x,t) \tag{7}$$

$$\frac{\partial}{\partial x} V_n(x,t) = -(\lambda + \eta(x))V_n(x,t) + \lambda V_{n-1}(x,t), n \geq 1 \tag{8}$$

With boundary conditions

$$I_0(0) = (1-\eta)\int_0^\infty W_n(x,t)\mu(x)dx + (1-\beta)\int_0^\infty R_0(x)\beta(x)dx + \int_0^\infty V_0(x)\eta(x)dx, n \geq 1 \tag{9}$$

$$W_0(0) = \alpha\lambda I_0 + \lambda\int_0^\infty I_n(x,t)r(x)dx + \int_0^\infty V_1(x,t)\eta(x)dx \tag{10}$$

$$W_n(0) = \alpha\lambda\int_0^\infty I_n(x,t)dx + \alpha\int_0^\infty I_{n+1}(x,t)r(x)dx + \beta\int_0^\infty V_1(x,t)\eta(x)dx, n \geq 1 \tag{11}$$

$$R_1(0) = \bar{\alpha}\lambda I_0 + \bar{\alpha}\int_0^\infty I_1(x,t)r(x)dx \tag{12}$$

$$R_n(0) = \bar{\alpha}\lambda\int_0^\infty I_{n-1}(x,t)r(x)dx + \bar{\alpha}\int_0^\infty I_n(x,t)r(x)dx, n \geq 2 \tag{13}$$

$$V_n(0) = \eta\int_0^\infty W_n(x,t)\mu(x)dx, n \geq 0 \tag{14}$$

Normalizing condition is

$$I_0 + \sum_{n=1}^{\infty}\int_0^\infty I_n(x,t)dx + \sum_{n=1}^{\infty}\int_0^\infty W_n(x,t)dx + \sum_{n=1}^{\infty}\int_0^\infty R_n(x,t)dx + \sum_{n=1}^{\infty}\int_0^\infty V_n(t,x)dx = 1 \tag{15}$$

Generating function of the queue length: The time dependent solution.

In this section we obtain the transient solution for the above set of differential-difference equations.

Theorem: The system of differential-difference equations to describe an single-server retrial queue with the server being subject to failure, breakdown and multiple vacations are given by equations (1)-(14) with initial condition(15) and generating functions of transient solution are given

Proof:

We define the probability generating functions,
$$\left.\begin{aligned} I(x,z) &= \sum_{n=1}^{\infty} I_n(x) z^n \\ W(x,z) &= \sum_{n=1}^{\infty} W_n(x) z^n \\ R(x,z) &= \sum_{n=1}^{\infty} R_n(x) z^n \\ V(x,z) &= \sum_{n=1}^{\infty} V_n(x) z^n \end{aligned}\right\} \tag{16}$$

Which are convergent inside the circle given by $|z| \le 1$ and define the Laplace transform of the function f(t) as $\bar{f}(s) = \int_0^{\infty} e^{-st} f(t)dt, s > 0$

Taking the Laplace transforms of equations (1)-(14) and using (15), we obtain

$$(\lambda+\eta)\bar{I}_0(x) = \int_0^{\infty} \bar{W}_0(x,s)\mu(x)dx + \int_0^{\infty} \bar{R}_0(x,s)\beta(x)dx + \int_0^{\infty} \bar{V}_0(x,s)\eta(x)dx \tag{17}$$

$$\frac{\partial}{\partial x}\bar{I}_n(x,s) = -(\lambda + r(x))\bar{I}_n(x,t), n \ge 1 \tag{18}$$

$$\frac{\partial}{\partial x}\bar{W}_0(x,s) = -(\lambda + \mu(x))\bar{W}_0(x,s) \tag{19}$$

$$\frac{\partial}{\partial x}\bar{W}_n(x,s) = -(\lambda + \mu(x))\bar{W}_n(x,s) + \lambda\bar{W}_{n-1}(x,s), n \ge 1 \tag{20}$$

$$\frac{\partial}{\partial x}\bar{R}_1(x,s) = -(\lambda + \beta(x))\bar{R}_1(x,s) \tag{21}$$

$$\frac{\partial}{\partial x}\bar{R}_n(x,s) = -(\lambda + \beta(x))\bar{R}_n(x,s) + \lambda\bar{R}_{n-1}(x,s), n \ge 2 \tag{22}$$

$$\frac{\partial}{\partial x}\bar{V}_0(x,s) = -(\lambda + \eta(x))\bar{V}_0(x,s) \tag{23}$$

$$\frac{\partial}{\partial x}\bar{V}_n(x,s) = -(\lambda + \eta(x))\bar{V}_n(x,s) + \lambda\bar{V}_{n-1}(x,s), n \ge 1 \tag{24}$$

With boundary conditions

$$\bar{I}_0(0) = (1-\eta)\int_0^{\infty} \bar{W}_n(x,s)\mu(x)dx + (1-\beta)\int_0^{\infty} \bar{R}_0(x)\beta(x)dx + \int_0^{\infty} \bar{V}_0(x)\eta(x)dx, n \ge 1 \tag{25}$$

$$\bar{W}_0(0) = \alpha\lambda\bar{I}_0 + \lambda\int_0^{\infty} \bar{I}_n(x,s)r(x)dx + \int_0^{\infty} \bar{V}_1(x,s)\eta(x)dx \tag{26}$$

$$\bar{W}_n(0) = \alpha\lambda\int_0^{\infty} \bar{I}_n(x,s)dx + \alpha\int_0^{\infty} \bar{I}_{n+1}(x,s)r(x)dx + \beta\int_0^{\infty} \bar{V}_1(x,s)\eta(x)dx, n \ge 1 \tag{27}$$

$$\bar{R}_1(0) = \bar{\alpha}\lambda\bar{I}_0 + \bar{\alpha}\int_0^{\infty} \bar{I}_1(x,s)r(x)dx \tag{28}$$

$$\overline{R}_n(0) = \overline{\alpha}\lambda\int_0^\infty \overline{I}_{n-1}(x,t)r(x)dx + \overline{\alpha}\int_0^\infty \overline{I}\lim_{x\to\infty}(x,t)r(x)dx, n\ge 2 \quad (29)$$

$$\overline{V}_n(0) = \eta\int_0^\infty \overline{W}_n(x,s)\mu(x)dx, n\ge 0 \quad (30)$$

Normalizing condition is

$$\overline{I}_0 + \sum_{n=1}^\infty\int_0^\infty \overline{I}_n(x,s)dx + \sum_{n=1}^\infty\int_0^\infty \overline{W}_n(x,s)dx + \sum_{n=1}^\infty\int_0^\infty \overline{R}_n(x,s)dx + \sum_{n=1}^\infty\int_0^\infty \overline{V}_n(t,s)dx = 1 \quad (31)$$

Generating function of the queue length: The time dependent solution Appling probability generating function (18)-(24)

$$\frac{\partial}{\partial x}\overline{I}_q(x,s,z) = -(\lambda + r(x))\overline{I}_q(x,s,z), n\ge 1 \quad (32)$$

$$\frac{\partial}{\partial x}\overline{W}_0(x,s,z) = -(\lambda + \mu(x))\overline{W}_0(x,s,z) \quad (33)$$

$$\frac{\partial}{\partial x}\overline{W}_q(x,s,z) = -(\lambda + \mu(x))\overline{W}_q(x,s,z) + \lambda\overline{W}_{q-1}(x,s,z), n\ge 1 \quad (34)$$

$$\frac{\partial}{\partial x}\overline{R}_1(x,s,z) = -(\lambda + \beta(x))\overline{R}_1(x,s,z) \quad (35)$$

$$\frac{\partial}{\partial x}\overline{R}_q(x,s,z) = -(\lambda + \beta(x))\overline{R}_q(x,s,z) + \lambda\overline{R}_{q-1}(x,s,z), n\ge 2 \quad (36)$$

$$\frac{\partial}{\partial x}\overline{V}_0(x,s,z) = -(\lambda + \eta(x))\overline{V}_0(x,s,z) \quad (37)$$

$$\frac{\partial}{\partial x}\overline{V}_q(x,s,z) = -(\lambda + \eta(x))\overline{V}_q(x,s,z) + \lambda\overline{V}_{q-1}(x,s,z), n\ge 1 \quad (38)$$

$$\overline{I}_0(s) = (1-\eta)\int_0^\infty \overline{W}_q(x,s,z)\mu(x)dx + (1-\beta)\int_0^\infty \overline{R}_0(x)\beta(x)dx + \int_0^\infty \overline{V}_0(x)\eta(x)dx, n\ge 1 \quad (39)$$

For the boundary conditions, we multiply both sides of the equation (33) by z, multiply both sides of equation (34) by z^{n+1} Sum over n from 1 to ∞, add the two results and use equation (16) to get

$$z\overline{I}_q(0,s,z) = \lambda z\overline{I}(s) + \beta\overline{R}_q(s,z) - \beta\overline{R}_0(s) + \int_0^\infty \overline{V}_q(x,s,z)\eta(x)dx - \int_0^\infty \overline{V}_q(x,s)\eta(x)dx \quad (40)$$

Performing similar operations on equations (36) and (38), we obtain

$$\overline{I}_q(0,s,z) = \int_0^\infty \overline{I}_q(x,s,z)\gamma_1(x)dx \quad (41)$$

$$\overline{V}_q(0,s,z) = \int_0^\infty \overline{I}_q(x,s,z)\gamma_1(x)dx \quad (42)$$

Using equation (37) in (40)

$$z\overline{I}_q(0,s,z) = (1 - s\overline{W}(s)) + \lambda(z-1)\overline{W}(s) + \beta\overline{R}_q(s,z) + \int_0^\infty \overline{V}_q(x,s,z)\eta(x)dx \quad (43)$$

Integrating equation (32) from 0 to x yields

$$\overline{I}_q(x,s,z) = \overline{I}_q(0,s,z)e^{-(s+\lambda+\lambda z+\alpha)x - \int_0^\infty \mu_1(t)dt}, \quad (44)$$

Where $\overline{I}_q(0,s,z)$ is given by equation (43). Again integrating equation (44) by parts with respect to x yields,

$$\overline{I}_q(s,z)=\overline{I}_q(0,s,z)\left[\frac{1-\overline{B}_1(s+\lambda-\lambda z+\alpha)}{s+\lambda-\lambda z+\alpha}\right] \tag{45}$$

$$\overline{B}_1(s+\lambda-\lambda z+\alpha)=\int_0^\infty e^{-(s+\lambda-\lambda z+\alpha)}dB_1(x) \tag{46}$$

Apply the same procedure we get service, repair and vacation time are

$$\overline{R}_q(s,z)=\overline{R}_q(0,s,z)\left[\frac{1-\overline{R}_1(s+\lambda-\lambda z+\alpha)}{s+\lambda-\lambda z+\alpha}\right] \tag{47}$$

$$\overline{V}_q(s,z)=\overline{V}_q(0,s,z)\left[\frac{1-\overline{V}(s+\lambda-\lambda z+\alpha)}{s+\lambda-\lambda z}\right] \tag{48}$$

$$\overline{R}_1(s+\lambda-\lambda z+\alpha)=\int_0^\infty e^{-(s+\lambda-\lambda z+\alpha)}dR_1(x) \tag{49}$$

Is the Laplace –Stieltjes of the repeated attempt, when the server is idle,

Using the above result we get,

$$\overline{I}_q(0,s,z)=\frac{f_1(z)f_2(z)[(1-s\overline{W}(s))+\lambda(z-1)\overline{W}(s)]}{dr} \tag{50}$$

Where

$$d \; r= f_1(z)f_2(z)\{z-\overline{W}[f_1(z)]\overline{W}[f_2(z)]\overline{V}(s+\lambda-\lambda z)\}-\beta\alpha z\{1-\overline{W}[f_1(z)]\overline{W}[f_2(z)\} \tag{51}$$

$$f_1(z)=s+\lambda-\lambda z+\alpha \; and \; f_2(z)=s+\lambda-\lambda z+\beta$$

Substituting the value of $\overline{P}_q(0,s,z)$ from the equation (50) into equations (45), (47), (48). we get,

$$\overline{I}_q(s,z)=\frac{f_2(z)[(1-s\overline{W}(s))+\lambda(z-1)\overline{W}(s)][1-\overline{B}_1[f_1(z)]]}{dr} \tag{52}$$

$$\overline{W}_q(s,z)=\frac{f_2(z)[(1-s\overline{W}(s))+\lambda(z-1)\overline{W}(s)]\overline{B}_1[f_1(z)][1-\overline{B}_2[f_1(z)]]}{dr} \tag{53}$$

$$\overline{R}_q(s,z)=\frac{\alpha z[(1-s\overline{W}(s))+\lambda(z-1)\overline{W}(s)][1-\overline{B}_1[f_1(z)]]\overline{B}_2[f_1(z)]}{dr} \tag{54}$$

$$\overline{V}_q(s,z)=\frac{f_1(z)f_2(z)[(1-s\overline{W}(s))+\lambda(z-1)\overline{W}(s)]\overline{B}_1[f_1(z)]\overline{B}_2[f_1(z)]}{dr} \tag{55}$$

Where dr is given by equation (51).

Thus
$\overline{I}_q(s,z)$, $\overline{W}_q(s,z)$, $\overline{R}_q(s,z)$ and $\overline{V}_q(s,z)$ are completely determined from equations(52)-(55)which completes the proof of the theorem.

5. THE STEADY STATE RESULTS

In this section we shall derive the steady state probability distribution for our queueing model. To define the steady state probabilities, suppress the argument 't' where ever it appears in the time dependent analysis. By using well known Tauberian property as follows:

$$\lim_{t\to\infty} f(t)=\lim_{s\to 0}\overline{f}(s) \tag{56}$$

Thus, multiplying both sides of the equation (52) by s taking limit as s tends to 0 applying the property and simplifying we have $I_q(z) = V(z)\left[\dfrac{N(z)}{D(Z)}\right]$, z=1, $I_q(z)$ in equation(57) is indeterminate of the form % . (57)

Applying L'Hosptial rule and using $\overline{B}_i(0) = 1, \overline{B}_j^{'}(0) = E(v_j), \overline{B}_j^{''}(0) = E(v_j^2), j = 1,2,3,....$

Where $E(v_i^2)$ is the second moment of the service time for the i th service.

$$= \lim_{z\to 1} I_q(z)$$

$$= \frac{f_2(z)[(1-s\overline{W}(s)) + \lambda(z-1)\overline{W}(s)][1-\overline{B}_1[f_1(z)]]}{dr} v(1)$$

Next, we have $I_q(z) + V(1) = 1$

Adding V(1) to $P_q(z)$ and equating (1), on simplification we get

V(1) = 1 - $\dfrac{f_2(z)[(1-s\overline{W}(s)) + \lambda(z-1)\overline{W}(s)][1-\overline{B}_1[f_1(z)]]}{dr}$ V(1) is the stead state probability that the server is under Vacation.

Utilization factor is given by $\rho = 1 - V(1)$

MEAN NUMBER IN THE SYSTEM

Let L_q denote the mean number of customers in the queue, from equation (57)

$L_q = \dfrac{d}{dz}(I_q(z))$ at z=1

Since $I_q(z)$ in (57) takes the form $0/0$, so we get

$$L_q = \lim_{z\to 1} \frac{d}{dz}(I_q(z)) = I_q^{'}(z) \tag{58}$$

And again use little formula we get average system size L is calculated.

We have $L = L_q + \rho$ (59)

The mean waiting time in the queue and in the system obtained by little formula.

$$W_q = \frac{L_q}{\lambda}$$

$$W = \frac{L}{\lambda}$$

Where L, L_q have been found in equations (57),(58).

CONCULSION

In this paper we analyzed a single server retrial queue with starting failure, breakdown and multiple vacations. It is remarkable that our solution does involve the summation of the infinite number of terms, if we have the closed-form expression for the probability generating function. Extensions of this work in several directions (e.g., to model public transportation situation production industries system, bank service, computer communication networks,) are possible. This service is advantageous of customers. The customer arrive from fresh or if he in orbit he get service. This type of service will give a complete satisfaction to the customer.

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