International Journal of Mathematical Archive-6(1), 2014, 62-66
Available online through www.ijma.info ISSN 2229 – 5046

Fuzzy hyper bi -Γ- ideals in Γ-hypernear-rings

[1]A. Kalaiyarasi, [2]N. Meenakumari* and [3]M. Navaneethakrishnan

[1, 3]*Department of Mathematics, Kamaraj College, Thoothukudi, India.*

[2]*Department of Mathematics, A. P. C. Mahalaxmi College for Women, Thoothukudi, India.*

(Received On: 16-12-14; Revised & Accepted On: 19-01-15)

ABSTRACT

In this paper, we introduce Fuzzy hyper bi -Γ- ideal inΓ- hyper near – rings and obtain their properties.

1. INTRODUCTION

The Theory of fuzzy sets proposed by Zadeh[6] has achieved a great success in various fields. W. Liu [2] has studied fuzzy ideals of a ring, and many researchers are engaged in extending the concepts. Gamma near- rings were defined by Bh. Sathyanarayana [5] and G. L. Booth. Fuzzy bi-Γ- ideals in semi groups were studied by Prince Williams, K. B. Latha and E. Chandrasekeran [4]. N. Meeakumari and T. Tamizh Chelvam [3] studied about fuzzy bi- ideals in Gamma near–rings. In this paper, we introduce Fuzzy hyper bi-Γ-ideals in Γ- hyper near – rings and obtain their properties.

2. PRELIMINARIES

Definition 2.1: By a near– ring we mean a non empty set N with two binary operations '+' and'.' satisfying the following axioms:
(i) (N, +) is a group
(ii) (N, .) is a semi group
(iii) x.(y+z) = x.y +x.z for all x, y, z Є N
Precisely speaking, it is a left near – ring and we will use the word near – ring to mean left near – ring.

Definition 2.2 [5]: A Γ-near- ring is a triple (M, +, Γ) where
(i) (M, +) is a group.
(ii) Γis a non empty set of binary operators on M such that, for each α ЄΓ, (M, +, α) is a near – ring.
(iii) x α (y β z) = (x α y) β z for all x, y, z Є M and α, β ЄΓ

Definition 2.3 [5]: A Γ-near – ring M is said to be zero symmetric, if 0 γ m = 0 for all m Є M and for all γ ЄΓ.

Throughout this paper, we assume that M is a Zero symmetric Γ- near- ring.
Definition 2.4: A canonical hyper group is an algebraic structure (H, +) satisfying the following conditions:
(i) for every x, y, z ЄH, x + (y + z) =(x+y)+ z
(ii) there exists a 0 Є H such that 0+x = x+0 = x for all x Є H
(iii) for every x ЄH there exists a unique element x' Є H. Such that 0 Є (x + x') ∩ (x' + x), (we call the element x' the opposite of x).
(iv) zЄ x + y implies y Є -x + z and x Є z – y

Definition 2.5: A hyper near-ring is an algebraic structure (R, +,.) satisfying the following axioms:
(i) (R,+) is a canonical hyper group
(ii) With respect to the multiplication, (R, .) is a semi group
(iii) x .(y+z) = x.y +x.z for all x, y, z ЄR

Definition 2.6 [1]: A Γ-hypernear – ring is a triple (M, +,Γ) where
(i) Γis a non-empty set of binary operators such that (M, +, α) is a hyper near-ring for each α Є Γ
(ii) x α (yβz) = (xαy)βz for all x, y, z Є M and α, β ЄΓ

*Corresponding Author: [2]N. Meenakumari**

Definition 2.7[1]: If M and M′ are Γ– hypernear – rings then a mapping f: M → M′ such that f(x+ y) = f(x) + f(y) and f (x α y) = f (x) α f(y) for all x, y ЄM & α Є Γ is called a Γ- hypernear–ring homomonphism

Definition 2.8 [6]: A Fuzzy set in a set M is a function μ: M →[0, 1].

Definition 2.9 [6]: A level subset of μ denoted by μ_t is defined as {x Є M / μ(x) ≥t} where t Є [0, 1].

Definition 2.10 [6]: The complement of a fuzzy set μ denoted by μ′ is the fuzzy set in M defined by μ' (x) = 1 – μ(x) for all x Є M

Definition 2.11 [6]: If μ is a fuzzy set in M and f is a function defined on M then the fuzzy set Ѵ inf (M), defined by Ѵ(y) = $\sup_{x \in f^{-1}(y)} \mu(x)$ for all y Є f(M) is called the image of μ under f.

Definition 2.12 [6]: If Ѵ is a fuzzy set in f(M), then the fuzzy set μ defined by μ (x) = Ѵ (f(x)) for all x Є M. That is μ = Ѵ of in M and is called the pre image of Ѵ under f [3]

Definition 2.13 [7]: Let A and B be fuzzy subsets of a hyper Γ-near ring M. Then the direct product of A and B, denoted by A x B, is the function defined by (Ax B) (x, y) = min {A(x), B (x)}.

3. Fuzzy hyper bi – Γ– ideals

Definition 3.1: A hyper subgroup H of (M, +) is a hyper bi-Γ- ideal if and only if H ΓM ΓH ⊆ H.

Definition 3.2: A fuzzy set μ of M is called a fuzzy hyper bi- Γ- ideal of M if

(i) $\inf_{z \in x-y} \mu(z) \geq \min\{\mu(x), \mu(y)\}$

(ii) $\mu(x\alpha y\beta z) \geq \min\{\mu(x), \mu(z)\}$ for all x, y, z ЄM, α, βЄΓ.

Example 3.3: Consider the Γ – hyper near- ring (M, +, Γ), Let M = {0, a, b} and Γ be the non- empty set of binary operators such that α, β϶ Γ are defined as follows:

+	0	a	b
0	{0}	{a}	{b}
a	{a}	{0,a,b}	{a, b}
b	{b}	{a, b}	{0,a,b}

α	0	a	b
0	0	0	0
a	0	a	b
b	0	a	b

β	0	a	b
0	0	0	0
a	0	0	0
b	0	0	0

Then (M, +, Γ) is a Γ- hyper near – ring.

We define a fuzzy set μ by μ (a) = μ (b) = .3, μ (0) = 7

By routine calculations, we can verify that μ is a fuzzy hyper bi –Γ- ideal of M.

Lemma 3.4: Let H be a non- empty subset of a Γ– hyper near- ring M. Then H is a hyper bi-Γ-ideal of M if and only if χ_H is a fuzzy hyper bi-Γ- ideal.

Proof: Straight forward.

Proposition 3.5: Let μ be a fuzzy set in a Γ-hypernear – ring M. Then μ is a fuzzy hyper bi- Γ- ideal of M if and only if each level subset μ_t, t ϵ Im(μ) is a hyper bi-Γ- ideal of M.

Proof: Let μ be a fuzzy hyper bi -Γ- ideal of M. Let tϵIm(μ).

We claim that μ_t is a hyper bi – Γ– ideal of M.

Let x, yϵμ_t . Then μ(x) ≥ t and μ(y) ≥ t

Hence

$$\inf_{z\in x-y} \mu(z) \geq \min\{\mu(x), \mu(y)\} \geq t$$

for all $z \in x - y$ we have $z \in \mu_t$ and so $x - y \subseteq \mu_t$. Hence μ_t is a hyper subgroup.

Let $x, z \in \mu_t$.$y \in M$ and $\alpha, \beta \in \Gamma$. Then $\mu(x \alpha y \beta z) \geq \min\{\mu(x), \mu(z)\} \geq t \Rightarrow x\alpha y\beta z \in \mu_t$ hence μ_t is a hyper bi –Γ-ideal of M.

Conversely, Assume that μ_t is a hyper bi –Γ– ideal of M. We claim that μ is a fuzzy hyper bi – Γ– ideal of M.

Putting $t_0=\min\{\mu(x), \mu(y)\}$ then $x, y \in \mu_{t0}$ and so $x - y \subseteq \mu_{t0}$, hence for any $z \in x-y$ we have $\mu(z) \geq t_0$ which implies,

$$\inf_{z\in y+x-y} \mu(z) \geq \min\{\mu(x), \mu(y)\}$$

Suppose it is not true, then for a fixed $\alpha, \beta \in \Gamma$, there exist $x_0\ y_o z_o \in M$. such that $\mu(x_0 \alpha y_o \beta z_o) < \min\{\mu(x_0), \mu(z_o)\}$.

Let $w_0 = \dfrac{\{\mu(x_0 \alpha y_0 \beta z_0) + \min\{\mu(x_0), \mu(z_0)\}}{2}$

Then $\mu(x_0 \alpha y_0 \beta z_0) < w_0$ & $\mu(x_0) > w_0$

$\mu(z_0) > w_0$ hence $x_0 \alpha y_o \beta z_o \notin \mu_{w0}$

$x_0 \in \mu_{w0}$, $z_0 \in \mu_{w0}$ a contradition

$\mu(x \alpha y \beta z) \geq \min\{\mu(x), \mu(z)\}$

Thus μ is a fuzzy hyper bi - Γ- ideal of M.

Proposition 3.6: Let μ be a fuzzy hyper bi–Γ- ideal in hyper gamma near ring M. Then the omplement $\mu'(x) = 1-\mu(x)$ is also a fuzzy hyper bi-Γ- ideal.

Proof: For $x, y \in M$

We have
$$\begin{aligned}\mu'(x - y) &= 1- \mu(x - y)\\ &\geq 1-\min\{\mu(x), \mu(y)\}\\ &= \min\{1-\mu(x), 1- \mu(y)\}\\ &= \min\{\mu'(x), \mu'(y)\}\end{aligned}$$

Further for $x, y, z \in M$ and $\alpha, \beta \in \Gamma$,

We have
$$\begin{aligned}\mu'(x \alpha y \beta z) &= 1- \mu(x \alpha y \beta z)\\ &\geq 1-\min\{\mu(x), \mu(z)\}\\ &= \min\{1-\mu(x), 1 - \mu(z)\}\\ &= \min\{\mu'(x), \mu'(z)\}\end{aligned}$$

Hence μ′ is also a fuzzy hyper bi-Γ- ideal.

Proposition 3.7: Γ-hypernear- ring homomorphic pre image of a fuzzy hyper bi –Γ- ideal is a fuzzy hyper bi-Γ- ideal.

Proof: Let $\psi: M \longrightarrow N$ be a Γ-hyper near- ring homomorphism.

Let ν be a fuzzy hyper bi ideal of M and μ be the pre image of V under ψ.

Then

$$\begin{aligned}\mu(x - y) &= V(\psi(x)) - V(\psi(y))\\ &\geq \min\{(V(\psi(x)), V(\psi(y))\}\\ &= \min\{\mu(x), \mu(y)\}\end{aligned}$$

Further
$\mu (x \alpha y \beta z) = V (\psi (x \alpha y \beta z))$
$= V (\psi (x) \alpha \psi (y) \beta \psi (z))$
$\geq \min \{V(\psi (x)), V (\psi (z))\}$
$= \min \{\mu (x), \mu (z)\}, \alpha, \beta \in \Gamma$ for all $x, y, z \in M$

Hence μ is a fuzzy hyper bi –Γ-ideal of M.

Proposition 3.8: Let V and σ be two fuzzy hyper bi-Γ-ideals of M. The $(\nu \cap \sigma)(x) = \min\{V(x),\sigma(x)\}$ is also a fuzzy hyper bi-Γ-ideal.

Proof:
$(V \cap \sigma)(x-y) = \min\{V(x-y),\sigma(x-y)\}$
$\geq \min\{\min\{V(x),V (y)\},\min \{\sigma(x),\sigma(y)\}\}$
$= \min\{\min\{V(x),\sigma(x)\}, \min\{V(y),\sigma(y)\}\}$
$= \min\{(V\cap\sigma)(x), (V\cap\sigma)(y)\}$

Further $(V\cap\sigma)(x\alpha y\beta z) = \min\{V(x\alpha y\beta z),\sigma(x\alpha y\beta z)\}$
$\geq \min \{\min\{ \nu (x), \nu (z)\}, \min\{\sigma(x), \sigma(z)\}\}$
$= \min\{\min\{V(x),\sigma(x)\},\min\{V(z),\sigma(z)\}\}$
$= \min\{(\nu\cap \sigma)(x), (\nu\cap \sigma)(z)\}$

Hence V ∩σ is fuzzy hyper bi – Γ-ideal

Proposition 3.9: Let V *and* σ be two fuzzy hyper bi-Γ- ideals in Γ-hyper near-rings. Then the direct product VXσ is also a fuzzy hyper bi-Γ- ideal.

Proof:
Now $(VX\sigma)[(x_1, y_1)-(x_2,y_2)] = (VX\sigma)(x_1-x_2,y_1-y_2)$
$= \min\{V(x_1-x_2), \sigma(y_1-y_2)\}$
$\geq \min\{\min\{V(x_1),V(x_2)\}, \min\{\sigma(y_1),\sigma(y_2)\}$
$= \min\{\min\{V(x_1),\sigma(y_1)\}, \min\{V(x_2),\sigma(y_2)\}\}$
$= \min \{(VX\sigma)(x_1,y_1), (VX\sigma)((x_2,y_2)\}$

Further,
$(VX\sigma)[(x_1,y_1)\ \alpha (x_2,y_2)\beta(x_3,y_3)] = (VX\sigma) \{(x_1\alpha x_2\beta x_3, y_1\alpha y_2\beta y_3)\}$
$= \min\{V(x_1\alpha x_2 \beta x_3),\sigma(y_1\alpha y_2\beta y_3) \}$
$\geq \min\{\min\{V (x_1),V(x_3)\}, \min\{\sigma(y_1), \sigma(y_3) \}\}$
$= \min\{\min\{V(x_1),\sigma(y_1)\}, \min\{V(x_3),\sigma(y_3)\}\}$
$= \min \{(Vx\sigma) (x_1, y_1), (Vx\sigma)(x_3, y_3)\}$

Vx σ is also a fuzzy hyper bi – Γ – ideal.

Proposition 3.10: An onto Γ hyper near–ring homomorphic image of a fuzzy bi–Γ–ideal of M is a Fuzzy hyper bi – Γ – ideal of M′

Proof: Let x_o and y_o be such that
$\mu (x_{o)} = \sup_{z\in f^{-1}(x)} \mu (z)$ & $\mu (y_o) = \sup_{z \in f^{-1}(y)} \mu (z)$
$\mu(x -y) = \sup_{z \in f^{-1}(x-y)} \mu (z)$
$= \sup_{z\in f^{-1}(x) - f^{-1} (y)} \mu (z)$
$\geq\mu (x_0 -y_o)$
$\geq \min \{\mu (x_0), \mu (y_o)\}$
$= \min \{\sup_{z\in f^{-1}(x)} \mu (z) \sup_{z\in f^{-1}(y)} \mu(z)\}$
$= \min \{V (x) ,V (y)\}$

Let
$x_0 = \sup_{z\in f^{-1}(x)} \mu(z)$ & $y_0 = \sup_{z\in f^{-1}(y)} \mu (z)$

$z_0 = \sup_{s \in f^{-1}(z)} \mu(s)$

Further,

$$V(x\alpha y\beta z) = \sup_{t \in f^{-1}(x\alpha y\beta z)} \mu(t)$$
$$= \sup_{t \in f^{-1}(x)\,\alpha\, f^{-1}(y)\,\beta\, f^{-1}(z)} \mu(t)$$
$$\geq \mu(x_o y_o z_o)$$
$$\geq \min\{\mu(x_o), \mu(z_o)\}$$
$$= \min\{\sup_{z \in f^{-1}(x)} \mu(z), \sup_{s \in f^{-1}(z)} \mu(s)\}$$
$$= \min\{V(x), V(z)\}$$

V is a fuzzy hyper bi -Γ- ideal of M.

REFERENCES

1. BijanDavvaz. Jianming Zhan, Kyung Ho KIM, Fuzzy Γ–hypernear–rings, Computers and Mathematics with Applications, 59 (2010) 2846-2853.
2. W. Liu, Fuzzy invariant subgroups and fuzzy ideal, Fuzzy sets and systems, 8 (1982)133 -139.
3. N. Meena Kumari and T. Tamizhchelvam, Fuzzy bi – ideals in Gamma near–rings, Journal of Algebra and Discrete Structures, vol. 9(2011) No 1 & 2. Pp 43 -52.
4. D.R.Prince Williams, K. B Latha and E. Chandra sekaran, Fuzzy bi –Γ–ideals in gamma–semi groups, Hacettepe Journal of mathematics and statistics, 38 (2009), 1-15.
5. Bh.Satyanarayana, A note on Gamma near–rings, Indian J. Mathematics, 41 (1999), 427 – 433.
6. L. A Zadeh, Fuzzy sets, Information and control, 8 (1965) 338 – 353.
7. Zheng Young Kang and Liu wangjin, Some Characterizations of the Direct product of fuzzy subgroups, Fuzzy system and mathematics, 2 (1989) 32 – 37.

Source of support: Nil, Conflict of interest: None Declared