# ON $\tilde{g}$ (1, 2)\*-HOMEOMORPHISMS

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# ABSTRACT

In this present paper, we introduce two new classes of bitopological functions called  $\tilde{g}(1, 2)^*$ -homeomorphisms and strongly  $\tilde{g}(1, 2)^*$ -homeomorphisms by using  $\tilde{g}(1, 2)^*$ -closed sets.

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# **1. INTRODUCTION**

Njastad introduced  $\alpha$ -open sets. Maki *et al.* [7] generalized the concepts of closed sets to  $\alpha$ -generalized closed (briefly  $\alpha$ g-closed) sets which are strictly weaker than  $\alpha$ -closed sets. Veera Kumar [19] defined  $\hat{g}$ -closed sets in topological spaces. Thivagar *et al.* [20] introduced  $\alpha \hat{g}$ -closed sets which lie between  $\alpha$ -closed sets and  $\alpha$ g-closed sets in topological spaces.

Maki *et al.* introduced the notion of generalized homeomorphisms (briefly g-homeomorphism) which are generalizations of homeomorphisms in topological spaces. Subsequently, Devi *et al.* [2] introduced two classes of functions called generalized semi-homeomorphisms (briefly gs-homeomorphisms) and semi-generalized homeomorphisms (briefly sg-homeomorphisms). Quite recently, Zbigniew Duszynski [20] has introduced  $\alpha$ ĝ-homeomorphisms in topological spaces.

It is well-known that the above mentioned topological sets and functions have been generalized to bitopological settings due to the efforts of many modern topologists [see 1, 3, 4, 5, 6, 8, 9, 13, 14]. In this present paper, we introduce two new classes of bitopological functions called  $\tilde{g}$  (1, 2)\*-homeomorphisms and strongly  $\tilde{g}$  (1,2)\*-homeomorphisms by using  $\tilde{g}$  (1,2)\*-closed sets. Basic properties of these two functions are studied and the relation between these types and other existing ones are established.

# 2. PRELIMINARIES

Throughout this paper,  $(X, \tau_1, \tau_2)$  (briefly, X) will denote bitopological space.

**Definition 2.1:** Let S be a subset of X. Then S is said to be  $\tau_{1,2}$ -open [11] if  $S = A \cup B$  where  $A \in \tau_1$  and  $B \in \tau_2$ .

The complement of  $\tau_{1,2}$ -open set is called  $\tau_{1,2}$ -closed.

Notice that  $\tau_{1,2}$ -open subsets of X need not necessarily form a topology.

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Definition 2.2 [11]: Let S be a subset of a bitopological space X. Then

- (i) the  $\tau_{1,2}$ -closure of S, denoted by  $\tau_{1,2}$ -cl(S), is defined as  $\cap \{F: S \subseteq F \text{ and } F \text{ is } \tau_{1,2}\text{-closed}\}$ .
- (ii) the  $\tau_{1,2}$ -interior of S, denoted by  $\tau_{1,2}$ -int(S), is defined as  $\cup \{F: F \subseteq S \text{ and } F \text{ is } \tau_{1,2}\text{-open}\}.$

### Definition 2.3: A subset A of a bitopological space X is called

- (i)  $(1,2)^*$ -semi-open set [10] if  $A \subseteq \tau_{1,2}$ -cl( $\tau_{1,2}$ -int(A));
- (ii)  $(1,2)^*-\alpha$  -open set [6] if  $A \subseteq \tau_{1,2}-int(\tau_{1,2}-cl(\tau_{1,2}-int(A)));$
- (iii)  $(1,2)^*$ - $\beta$ -open set [12] if  $A \subseteq \tau_{1,2}$ -cl $(\tau_{1,2}$ -int $(\tau_{1,2}$ -cl(A))).

The complements of the above mentioned open sets are called their respective closed sets.

The  $(1,2)^*$ -semi-closure [8] (resp.  $(1,2)^*$ - $\alpha$  -closure [8],  $(1,2)^*$ - $\beta$ -closure [12]) of a subset A of X, denoted by  $(1,2)^*$ -scl(A) (resp.  $(1,2)^*$ - $\alpha$  cl(A),  $(1,2)^*$ - $\beta$ cl(A)), is defined to be the intersection of all  $(1,2)^*$ -semi-closed (resp.  $(1,2)^*$ - $\alpha$  -closed,  $(1,2)^*$ - $\beta$ -closed) sets of (X,  $\tau_1$ ,  $\tau_2$ ) containing A. It is known that  $(1, 2)^*$ -scl(A) (resp.  $(1, 2)^*$ - $\alpha$  cl(A),  $(1,2)^*$ - $\beta$ cl(A)) is a  $(1,2)^*$ -semi-closed (resp.  $(1,2)^*$ - $\alpha$  -closed,  $(1,2)^*$ - $\beta$ cl(A)) is a  $(1,2)^*$ -semi-closed (resp.  $(1,2)^*$ - $\alpha$  -closed,  $(1,2)^*$ - $\beta$ closed) set.

**Definition 2.4:** A subset A of a bitopological space  $(X, \tau_1, \tau_2)$  is called

- (i)  $(1,2)^*$ -g-closed set [18] if  $\tau_{1,2}$ -cl(A)  $\subseteq U$  whenever A  $\subseteq U$  and U is  $\tau_{1,2}$ -open in X. The complement of  $(1,2)^*$ -g-closed set is called  $(1,2)^*$ -g-open set;
- (ii) (1,2)\*-sg-closed set 10] if (1,2)\*-scl(A) ⊆ U whenever A⊆U and U is (1,2)\*-semi-open in X. The complement of (1,2)\*-sg-closed set is called (1,2)\*-sg-open set;
- (iii) (1,2)\*-gs-closed set [10] if (1,2)\*-scl(A)  $\subseteq$  U whenever A  $\subseteq$  U and U is  $\tau_{1,2}$ -open in X. The complement of (1,2)\*-gs-closed set is called (1,2)\*-gs-open set;
- (iv)  $(1,2)^*-\alpha$  g-closed set [16] if  $(1,2)^*-\alpha$  cl(A)  $\subseteq$  U whenever A  $\subseteq$  U and U is  $\tau_{1,2}$ -open in X. The complement of  $(1,2)^*-\alpha$  g-closed set is called  $(1,2)^*-\alpha$  g-open set;
- (v) (1,2)\*-ĝ-closed set [3] or (1,2)\*-ω-closed set [4] if τ<sub>1,2</sub>-cl(A) ⊆ U whenever A ⊆ U and U is (1,2)\*-semi-open in X. The complement of (1,2)\*-ĝ-closed (or (1,2)\*-ω-closed) set is called (1,2)\*-ĝ-open (or (1,2)\*-ω-open) set;
- (vi)  $(1,2)^* \psi$ -closed set [9] if  $(1,2)^*$ -scl(A)  $\subseteq U$  whenever A  $\subseteq U$  and U is  $(1,2)^*$ -sg-open in X. The complement of  $(1,2)^* \psi$ -closed set is called  $(1,2)^* \psi$ -open set;
- (vii)(1,2)\*- $\ddot{g}_{\alpha}$ -closed set [9] if (1,2)\*- $\alpha$  cl(A)  $\subseteq$  U whenever A  $\subseteq$  U and U is (1,2)\*-sg-open in X. The complement

of  $(1,2)^*$ - $\ddot{g}_{\alpha}$ -closed set is called  $(1,2)^*$ - $\ddot{g}_{\alpha}$ -open set;

(viii) (1,2)\*-gsp-closed set [12] if (1,2)\*- $\beta$ cl(A)  $\subseteq$  U whenever A  $\subseteq$  U and U is  $\tau_{1,2}$ -open in X. The complement of (1,2)\*-gsp-closed set is called (1,2)\*-gsp-open set.

**Remark 2.5:** The collection of all  $(1,2)^*$ - $\ddot{g}_{\alpha}$ -closed (resp.  $(1,2)^*$ - $\hat{g}$ -closed,  $(1,2)^*$ -g-closed,  $(1,2)^*$ -gs-closed,  $(1,2)^*$ - $\alpha$  g-closed,  $(1,2)^*$ -g-closed,  $(1,2)^*$ - $\ddot{G}_{\alpha}$  (1,2)\*-seg-closed,  $(1,2)^*$ - $\psi$ -closed,  $(1,2)^*$ - $\alpha$ -closed,  $(1,2)^*$ -semi-closed) sets is denoted by  $(1,2)^*$ - $\ddot{G}_{\alpha}$  C(X) (resp.  $(1,2)^*$ - $\hat{G}$  C(X),  $(1,2)^*$ -G C(X),  $(1,2)^*$ - $\psi$  C(X),  $(1,2)^*$ - $\chi$  C(X)).

We denote the power set of X by P(X).

**Definition 2.6 [17]:** A subset S of a bitopological space X is said to be locally  $(1,2)^*$ -closed if S = U  $\cap$  F, where U is  $\tau_{1,2}$ -open and F is  $\tau_{1,2}$ -closed in X.

### Remark 2.7

- (i) Every  $\tau_{1,2}$ -closed set is  $(1,2)^*$ -semi-closed but not conversely [10].
- (ii) Every  $\tau_{1,2}$ -closed set is  $(1,2)^*$ - $\alpha$ -closed but not conversely [10].
- (i) Every  $(1,2)^*$ -semi-closed set is  $(1,2)^* \psi$ -closed but not conversely [9].
- (ii) Every  $(1,2)^*$ -semi-closed set is  $(1,2)^*$ -sg-closed but not conversely [9].
- (iv) Every  $(1,2)^*$ - $\hat{g}$ -closed set is  $(1,2)^*$ -g-closed but not conversely [3].
- (v) Every  $(1,2)^*$ -sg-closed set is  $(1,2)^*$ -gs-closed but not conversely [15].
- (iii) Every  $(1,2)^*$ -g-closed set is  $(1,2)^*$   $\alpha$  g-closed but not conversely [16].
- (ix) Every  $(1,2)^*$ -g-closed set is  $(1,2)^*$ -gs-closed but not conversely [15].
- (x) Every  $\tau_{1,2}$ -closed set is  $(1,2)^*$   $\hat{g}$ -closed but not conversely [3].
- (xi) Every  $(1,2)^*$   $\hat{g}$  -closed set is  $(1,2)^*$ -sg-closed but not conversely [3].

**Definition 2.8:** A subset A of a bitopological space X is called regular  $(1,2)^*$ -open set [13] if A =  $\tau_{1,2}$ -int( $\tau_{1,2}$ -cl(A)).

The complement of regular  $(1,2)^*$ -open set is called regular  $(1,2)^*$ -closed.

Definition 2.9: A subset A of a bitopological space X is called

- (i)  $(1,2)^*-\alpha \hat{g}$ -closed [4] or  $\tilde{g}$   $(1,2)^*$ -closed if  $(1,2)^*-\alpha cl(A) \subseteq U$  whenever  $A \subseteq U$  and U is  $(1,2)^*-\hat{g}$ -open in X. The complement of  $\tilde{g}$   $(1,2)^*$ -closed set is called  $\tilde{g}$   $(1,2)^*$ -open.
- (ii)  $\widetilde{g}(1,2)^*$ -open if the image of every  $\tau_{1,2}$ -open set of X is  $\widetilde{g}(1,2)^*$ -open in Y.

**Definition 2.10:** A function  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  is called  $(1,2)^*$ -g-open [14] (resp.  $(1,2)^*$ - $\hat{g}$ -open [4],  $(1,2)^*$ -open [4],  $(1,2)^*$ -sg-open [10],  $(1,2)^*$ -gs-open [5],  $(1,2)^*$ - $\alpha$ -open [4],  $(1,2)^*$ - $\alpha$ -open,  $(1,2)^*$ - $\alpha$ -open, (1

**Definition 2.11:** A function f:  $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  is called

- (i)  $(1,2)^*$ -g-continuous [14] if  $f^1(V)$  is  $(1,2)^*$ -g-closed in X, for every  $\sigma_{1,2}$ -closed set V of Y.
- (ii)  $(1,2)^*$ -sg-continuous [10] if  $f^1(V)$  is  $(1,2)^*$ -sg-closed in X, for every  $\sigma_{1,2}$ -closed set V of Y.
- (iii)  $(1,2)^*$ -gs-continuous [15] if  $f^1(V)$  is  $(1,2)^*$ -gs-closed in X, for every  $\sigma_{1,2}$ -closed set V of Y.
- (iv)  $(1,2)^*$ -ĝ-continuous [14] if  $f^1(V)$  is  $(1,2)^*$ -ĝ-closed in X, for every  $\sigma_{1,2}$ -closed set V of Y.
- (v) (1,2)\*-continuous [14] if  $f^{1}(V)$  is  $\tau_{1,2}$ -closed in X, for every  $\sigma_{1,2}$ -closed set V of Y.
- (iii)  $\tilde{g}$  (1,2)\*-continuous or (1,2)\*- $\alpha \hat{g}$ -continuous [4] if  $f^{-1}(V)$  is  $\tilde{g}$  (1,2)\*-closed in X, for every  $\sigma_{1,2}$ -closed set V of Y.

**Definition 2.12 [15]:** A function f:  $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  is called

- (i)  $(1,2)^*$ -g-homeomorphism if f is bijection,  $(1,2)^*$ -g-open and  $(1,2)^*$ -g-continuous.
- (ii)  $(1,2)^*$ -sg-homeomorphism if f is bijection,  $(1,2)^*$ -sg-open and  $(1,2)^*$ -sg-continuous.
- (iii)  $(1,2)^*$ -gs-homeomorphism if f is bijection,  $(1,2)^*$ -gs-open and  $(1,2)^*$ -gs-continuous.
- (iv)  $(1,2)^*$ -homeomorphism if f is bijection,  $(1,2)^*$ -open and  $(1,2)^*$ -continuous.

**Definition 2.13 [15]:** A function f:  $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  is called

- (i)  $(1,2)^*$ - $\alpha$ -continuous if  $f^{-1}(V)$  is  $(1,2)^*$ - $\alpha$ -open in X, for every  $\sigma_{1,2}$ -open set V of Y.
- (ii)  $\tilde{g}(1,2)^*$ -continuous if  $f^{(1)}(V)$  is  $\tilde{g}(1,2)^*$ -closed in X, for every  $\sigma_{1,2}$ -closed set V of Y.
- (iii)  $\tilde{g}(1,2)^*$ -irresolute if  $f^1(V)$  is  $\tilde{g}(1,2)^*$ -closed in X, for every  $\tilde{g}(1,2)^*$ -closed set V of Y.

**Definition 2.14 [15]:** A function  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  is called

- (i) pre-(1,2)\*-α-closed (resp. pre (1,2)\*-α-open) if the image of every (1,2)\*-α-closed (resp. (1,2)\*-α-open) in X is (1,2)\*-α-closed ( resp. (1,2)\*-α-open) in Y.
- (ii)  $(1,2)^*-\alpha$ -irresolute if  $f^1(V)$  is  $(1,2)^*-\alpha$ -open in X, for every  $(1,2)^*-\alpha$ -open set V of Y.
- (iii)  $(1,2)^*$ -gc-irresolute if  $f^{-1}(V)$  is  $(1,2)^*$ -g-closed in X, for every  $(1,2)^*$ -g-closed set V of Y.
- (iv)  $(1,2)^*-\alpha$ -homeomorphism if f is bijection,  $(1,2)^*-\alpha$ -irresolute and pre- $(1,2)^*-\alpha$ -closed.

#### Remark 2.15

- (i) Every  $(1,2)^*$ - $\alpha$ -closed set is  $\widetilde{g}(1,2)^*$ -closed but not conversely.
- (ii) Every  $\tilde{g}$  (1,2)\*-open set is (1,2)\*-gs-open but not conversely.

# $3.\tilde{g}$ (1, 2)\*-HOMEOMORPHISMS

#### **Definition 3.1**

- (i) A bijective function f: (X, τ<sub>1</sub>, τ<sub>2</sub>) → (Y, σ<sub>1</sub>, σ<sub>2</sub>) is called a strongly g̃ (1,2)\*-closed (resp. strongly g̃ (1,2)\*-open) if the image of every g̃ (1,2)\*-closed (resp. g̃ (1,2)\*-open) set in X is g̃ (1,2)\*-closed (resp. g̃ (1,2)\*-open) of Y.
- (ii) A bijective function  $f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$  is called an  $\widetilde{g}(1,2)^*$ -homeomorphism if f is both  $\widetilde{g}(1,2)^*$ -open and  $\widetilde{g}(1,2)^*$ -continuous.

**Theorem 3.2:** Every  $(1,2)^*$ -homeomorphism is  $\widetilde{g}(1,2)^*$ -homeomorphism.

**Proof:** Let f:  $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  be  $(1,2)^*$ -homeomorphism. Then f is bijective,  $(1,2)^*$ -open and  $(1,2)^*$ -continuous function. Let U be an  $\tau_{1,2}$ -open set in X. Since f is  $(1,2)^*$ -open function, f(U) is an  $\sigma_{1,2}$ -open set in Y. Every  $\tau_{1,2}$ -open set is  $\tilde{g}$   $(1,2)^*$ -open and hence f(U) is  $\tilde{g}$   $(1,2)^*$ -open in Y. This implies f is  $\tilde{g}$   $(1,2)^*$ -open. Let V be a  $\sigma_{1,2}$ -closed set in Y. Since f is  $(1,2)^*$ -continuous,  $f^1(V)$  is  $\tau_{1,2}$ -closed in X. Thus  $f^1(V)$  is  $\tilde{g}$   $(1,2)^*$ -closed in X and therefore, f is  $\tilde{g}$   $(1,2)^*$ -continuous. Hence, f is an  $\tilde{g}$   $(1,2)^*$ -homeomorphism.

Remark 3.3: The converse of Theorem 3.2 need not be true as shown in the following example.

**Example 3.4:** Let X = {a, b, c},  $\tau_1 = \{\phi, X\}$  and  $\tau_2 = \{\phi, X, \{a, b\}\}$ . Then the sets in  $\{\phi, X, \{a, b\}\}$  are called  $\tau_{1,2}$ -open and the sets in  $\{\phi, X, \{c\}\}$  are called  $\tau_{1,2}$ -closed. Also the sets in  $\{\phi, X, \{c\}, \{a, c\}, \{b, c\}\}$  are called  $\tilde{g}$  (1,2)\*-closed in X and the sets in  $\{\phi, X, \{a\}, \{b\}, \{a, b\}\}$  are called  $\tilde{g}$  (1,2)\*-open in X. Let Y = {a, b, c},  $\sigma_1 = \{\phi, Y, \{a\}\}$  and  $\sigma_2 = \{\phi, Y, \{b\}\}$ . Then the sets in  $\{\phi, Y, \{a\}, \{b\}, \{a, b\}\}$  are called  $\tilde{g}$  (1,2)\*-open and the sets in  $\{\phi, Y, \{c\}, \{a, c\}, \{b, c\}\}$  are called  $\sigma_{1,2}$ -open and the sets in  $\{\phi, Y, \{c\}, \{a, c\}, \{b, c\}\}$  are called  $\sigma_{1,2}$ -closed. Also the sets in  $\{\phi, Y, \{c\}, \{a, c\}, \{b, c\}\}$  are called  $\sigma_{1,2}$ -closed in Y and the sets in  $\{\phi, Y, \{c\}, \{a, c\}, \{b, c\}\}$  are called  $\tilde{g}$  (1,2)\*-closed in Y and the sets in  $\{\phi, Y, \{a\}, \{b\}, \{a, b\}\}$  are called  $\tilde{g}$  (1,2)\*-closed in Y and the sets in  $\{\phi, Y, \{a\}, \{b\}, \{a, b\}\}$  are called  $\tilde{g}$  (1,2)\*-closed in Y and the sets in  $\{\phi, Y, \{a\}, \{b\}, \{a, b\}\}$  are called  $\tilde{g}$  (1,2)\*-closed in Y and the sets in  $\{\phi, Y, \{a\}, \{b\}, \{a, b\}\}$  are called  $\tilde{g}$  (1,2)\*-closed in Y and the sets in  $\{\phi, Y, \{a\}, \{b\}, \{a, b\}\}$  are called  $\tilde{g}$  (1,2)\*-closed in Y and the sets in  $\{\phi, Y, \{a\}, \{b\}, \{a, b\}\}$  are called  $\tilde{g}$  (1,2)\*-closed in Y and the sets in  $\{\phi, Y, \{a\}, \{b\}, \{a, b\}\}$  are called  $\tilde{g}$  (1,2)\*-closed in Y and the sets in  $\{\phi, Y, \{a\}, \{b\}, \{a, b\}\}$  are called  $\tilde{g}$  (1,2)\*-closed in Y and the sets in  $\{\phi, Y, \{a\}, \{b\}, \{a, b\}\}$  are called  $\tilde{g}$  (1,2)\*-closed in Y and the sets in  $\{\phi, Y, \{a\}, \{b\}, \{a, b\}\}$  are called  $\tilde{g}$  (1,2)\*-closed in Y and the sets in  $\{\phi, Y, \{a\}, \{b\}, \{a, b\}\}$  are called  $\tilde{g}$  (1,2)\*-closed in Y and the sets in  $\{\phi, Y, \{a\}, \{b\}, \{a, b\}\}$  are called  $\tilde{g}$  (1,2)\*-closed in Y and the sets in  $\{\phi, Y, \{a\}, \{b\}, \{a, b\}\}$  are called  $\tilde{g}$  (1,2)\*-closed in Y and the sets in  $\{b, Y, \{a\}, \{b\}, \{a, b\}\}$  are called  $\tilde{g}$  (1,2)\*-closed in Y and the sets in  $\{b, Y, \{a\}, \{b\}, \{a, b\}\}$  are called  $\tilde{g}$  (1,2)\*-closed in Y and the sets in  $\{b, Y, \{a\}, \{b\}, \{a,$ 

**Proposition 3.5:** For any bijective function f:  $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  the following statements are equivalent.

- (i)  $f^{1}: (Y, \sigma_{1}, \sigma_{2}) \rightarrow (X, \tau_{1}, \tau_{2})$  is  $\tilde{g}(1,2)^{*}$ -continuous function.
- (ii) f is a  $\tilde{g}$  (1,2)\*-open function.
- (iii) f is a  $\widetilde{g}$  (1,2)\*-closed function.

### Proof

(i)  $\Rightarrow$  (ii): Let U be an  $\tau_{1,2}$ -open set in X. Then X – U is  $\tau_{1,2}$ -closed in X. Since f<sup>1</sup> is  $\tilde{g}(1,2)^*$ -continuous, (f<sup>1</sup>)<sup>-1</sup>(X – U) is  $\tilde{g}(1,2)^*$ -closed in Y. That is f(X - U) = Y - f(U) is  $\tilde{g}(1,2)^*$ -closed in Y. This implies f(U) is  $\tilde{g}(1,2)^*$ -open in Y. Hence f is  $\tilde{g}(1,2)^*$ -open function.

(ii)  $\Rightarrow$  (iii): Let F be a  $\tau_{1,2}$ -closed set in X. Then X – F is  $\tau_{1,2}$ -open in X. Since f is  $\tilde{g}$  (1,2)\*-open, f(X – F) is  $\tilde{g}$  (1,2)\*-open set in Y. That is Y–f(F) is  $\tilde{g}$  (1,2)\*-open in Y. This implies that f(F) is  $\tilde{g}$  (1,2)\*-closed in Y. Hence f is  $\tilde{g}$  (1,2)\*-closed.

(iii)  $\Rightarrow$  (i): Let V be a  $\tau_{1,2}$ -closed set in X. Since f is  $\widetilde{g}$  (1,2)\*-closed function, f(V) is  $\widetilde{g}$  (1,2)\*-closed in Y. That is  $(f^{-1})^{-1}(V)$  is  $\widetilde{g}$  (1,2)\*-closed in Y. Hence  $f^{-1}$  is  $\widetilde{g}$  (1,2)\*-continuous.

**Proposition 3.6:** Let f:  $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  be a bijective and  $\tilde{g}(1,2)^*$ -continuous function. Then the following statements are equivalent:

- (i) f is a  $\tilde{g}$  (1,2)\*-open function.
- (ii) f is a  $\tilde{g}$  (1,2)\*-homeomorphism.
- (iii) f is a  $\tilde{g}$  (1,2)\*-closed function.

#### **Proof:**

(i)  $\Rightarrow$  (ii): Let f be a  $\tilde{g}(1,2)^*$ -open function. By hypothesis, f is bijective and  $\tilde{g}(1,2)^*$ -continuous. Hence f is a  $\tilde{g}(1,2)^*$ -homeomorphism.

(ii)  $\Rightarrow$  (iii): Let f be a  $\tilde{g}(1,2)^*$ -homeomorphism. Then f is  $\tilde{g}(1,2)^*$ -open. By Proposition 3.5, f is  $\tilde{g}(1,2)^*$ -closed function.

(iii)  $\Rightarrow$  (i): It is obtained from Proposition 3.5.

**Theorem 3.7:** Every  $(1,2)^*$ - $\alpha$ -homeomorphism is  $\widetilde{g}(1,2)^*$ -homeomorphism.

**Proof:** Let f:  $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  be a  $(1,2)^*$ - $\alpha$ -homeomorphism. Then f is bijective,  $(1,2)^*$ - $\alpha$ -irresolute and pre-(1,2)\*- $\alpha$ -closed. Let F be  $\tau_{1,2}$ -closed in X. Then F is  $(1,2)^*$ - $\alpha$ -closed in X. Since f is pre- $(1,2)^*$ - $\alpha$ -closed, f(F) is  $(1,2)^*$ - $\alpha$ -closed in Y. Every  $(1,2)^*$ - $\alpha$ -closed set is  $\tilde{g}$   $(1,2)^*$ -closed and hence f(F) is  $\tilde{g}$   $(1,2)^*$ -closed in Y. This implies

f is  $\tilde{g}(1,2)^*$ -closed function. Let V be a  $\sigma_{1,2}$ -closed set of Y. Thus V is  $(1,2)^*$ - $\alpha$ -closed in Y. Since f is  $(1,2)^*$ - $\alpha$ -irresolute f<sup>1</sup>(V) is  $(1,2)^*$ - $\alpha$ -closed in X. Thus f<sup>1</sup>(V) is  $\tilde{g}(1,2)^*$ -closed in X. Therefore f is  $\tilde{g}(1,2)^*$ -continuous. Hence f is a  $\tilde{g}(1,2)^*$ -homeomorphism.

Remark 3.8: The following Example shows that the converse of Theorem 3.7 need not be true.

**Example 3.9:** Let X = {a, b, c},  $\tau_1 = \{\phi, X\}$  and  $\tau_2 = \{\phi, X, \{a\}\}$ . Then the sets in  $\{\phi, X, \{a\}\}$  are called  $\tau_{1,2}$ -open and the sets in  $\{\phi, X, \{b\}, \{c\}, \{a, b\}, \{a, c\}\}$  are called  $\tau_{1,2}$ -closed. Also the sets in  $\{\phi, X, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$  are called  $\tilde{g}(1,2)^*$ -closed in X and the sets in  $\{\phi, X, \{a\}, \{c\}, \{a, b\}, \{a, c\}\}$  are called  $(1,2)^*$ - $\alpha$ -open in X. Moreover, the sets in  $\{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$  are called  $(1,2)^*$ - $\alpha$ -open in X and the sets in  $\{\phi, X, \{b\}, \{c\}, \{b, c\}\}$  are called  $(1,2)^*$ - $\alpha$ -open in X. Let Y = {a, b, c},  $\sigma_1 = \{\phi, Y\}$  and  $\sigma_2 = \{\phi, Y, \{a, b\}\}$ . Then the sets in  $\{\phi, Y, \{a, b\}\}$  are called  $(1,2)^*$ - $\alpha$ -open and the sets in  $\{\phi, Y, \{c\}\}$  are called  $\sigma_{1,2}$ -closed. Also the sets in  $\{\phi, Y, \{c\}, \{a, c\}, \{b, c\}\}$  are called  $\tilde{g}(1,2)^*$ -closed in Y and the sets in  $\{\phi, Y, \{a\}, \{b\}, \{a, b\}\}$  are called  $\tilde{g}(1,2)^*$ -open in Y. Moreover, the sets in  $\{\phi, Y, \{a, b\}\}$  are called  $(1,2)^*$ - $\alpha$ -closed in Y and the sets in  $\{\phi, Y, \{c\}\}$  are called  $(1,2)^*$ - $\alpha$ -open in Y. Let f:  $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  be the identity function. Then f is a  $\tilde{g}(1,2)^*$ -homeomorphism but f is not a  $(1,2)^*$ - $\alpha$ -homeomorphism.

**Remark 3.10:** Next Example shows that the composition of two  $\tilde{g}(1,2)^*$ -homeomorphisms is not always a  $\tilde{g}(1,2)^*$ -homeomorphism.

**Example 3.11:** Let  $X = \{a, b, c\}, \tau_1 = \{\phi, X, \{a\}\}$  and  $\tau_2 = \{\phi, X, \{a, c\}\}$ . Then the sets in  $\{\phi, X, \{a\}, \{a, c\}\}$  are called  $\tau_{1,2}$ -open and the sets in  $\{\phi, X, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}$  are called  $\tau_{1,2}$ -closed. Also the sets in  $\{\phi, X, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}$  are called  $\tilde{g}$  (1,2)\*-open in X. Let  $Y = \{a, b, c\}, \sigma_1 = \{\phi, Y\}$  and  $\sigma_2 = \{\phi, Y, \{a\}\}$ . Then the sets in  $\{\phi, Y, \{a\}\}$  are called  $\sigma_{1,2}$ -open and the sets in  $\{\phi, Y, \{b\}, \{c\}, \{a, b\}, \{c\}, \{a, b\}, \{c\}, \{a, b\}, \{c\}, \{a, b, c\}\}$  are called  $\sigma_{1,2}$ -open in X. Let  $Y = \{a, b, c\}, \sigma_1 = \{\phi, Y\}$  and  $\sigma_2 = \{\phi, Y, \{a\}\}$ . Then the sets in  $\{\phi, Y, \{a\}\}$  are called  $\sigma_{1,2}$ -open and the sets in  $\{\phi, Y, \{b\}, \{c\}, \{a, b\}, \{a, c\}\}$  are called  $\tilde{g}$  (1,2)\*-closed in Y and the sets in  $\{\phi, Z, \{a, b\}\}$  are called  $\tilde{g}$  (1,2)\*-closed in Z and the sets in  $\{\phi, Z, \{a\}, \{b\}, \{a, b\}\}$  are called  $\tilde{g}$  (1,2)\*-closed in Z and the sets in  $\{\phi, Z, \{a\}, \{b\}, \{a, b\}\}$  are called  $\tilde{g}$  (1,2)\*-closed in Z and the sets in  $\{\phi, Z, \{a\}, \{b\}, \{a, b\}\}$  are called  $\tilde{g}$  (1,2)\*-open in Z. Let f:  $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  and g:  $(Y, \sigma_1, \sigma_2) \rightarrow (Z, \eta_1, \eta_2)$  be two identity functions. Then both f and g are  $\tilde{g}$  (1,2)\*-homeomorphisms. The set  $\{a, c\}$  is  $\tau_{1,2}$ -open in X, but (g o f)( $\{a, c\}$ ) = $\{a, c\}$  is not  $\tilde{g}$  (1,2)\*-open in Z. This implies that g o f is not  $\tilde{g}$  (1,2)\*-open and hence g o f is not  $\tilde{g}$  (1,2)\*-homeomorphism.

**Theorem 3.12:** Every  $\widetilde{g}$  (1,2)\*-homeomorphism is (1,2)\*-gs-homeomorphism but not conversely.

**Proof:** Let f:  $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  be a  $\tilde{g}(1,2)^*$ -homeomorphism. Then f is a bijective,  $\tilde{g}(1,2)^*$ -open and  $\tilde{g}(1,2)^*$ -continuous function. Let U be an  $\tau_{1,2}$ -open set in X. Then f(U) is  $\tilde{g}(1,2)^*$ -open in Y. Every  $\tilde{g}(1,2)^*$ -open set is  $(1,2)^*$ -gs-open and hence, f(U) is  $(1,2)^*$ -gs-open in Y. This implies f is  $(1,2)^*$ -gs-open function. Let V be  $\sigma_{1,2}$ -closed set in Y. Then f<sup>1</sup>(V) is  $\tilde{g}(1,2)^*$ -closed in X. Hence f<sup>1</sup>(V) is  $(1,2)^*$ -gs-closed in X. This implies f is  $(1,2)^*$ -gs-closed in X. This i

Remark 3.13: The following Example shows that the converse of Theorem 3.12 need not be true.

**Example 3.14:** Let X = {a, b, c},  $\tau_1 = \{\phi, X, \{a\}\}$  and  $\tau_2 = \{\phi, X, \{b\}\}$ . Then the sets in  $\{\phi, X, \{a\}, \{b\}, \{a, b\}\}$  are called  $\tau_{1,2}$ -open and the sets in  $\{\phi, X, \{c\}, \{a, c\}, \{b, c\}\}$  are called  $\tau_{1,2}$ -closed. Also the sets in  $\{\phi, X, \{c\}, \{a, c\}, \{b, c\}\}$  are called  $\tilde{g}$  (1,2)\*-open in X. Moreover, the sets in  $\{\phi, X, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$  are called (1,2)\*-gs-closed in X and the sets in  $\{\phi, X, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$  are called (1,2)\*-gs-closed in X and the sets in  $\{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$  are called (1,2)\*-gs-closed in X and the sets in  $\{\phi, Y, \{a\}, \{b, c\}\}$ . Moreover, the sets in  $\{\phi, Y, \{a\}, \{b, c\}\}$  are called  $\sigma_{1,2}$ -open and  $\sigma_{1,2}$ -closed. Also the sets in  $\{\phi, Y, \{a\}, \{b, c\}\}$  are called  $\tilde{g}$  (1,2)\*-closed and  $\tilde{g}$  (1,2)\*-open in Y. Moreover, the sets in  $\{\phi, Y, \{a\}, \{b\}, \{a, c\}, \{b, c\}\}$  are called  $(1,2)^*$ -g-closed and  $(1,2)^*$ -g-open in Y. Let f:  $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  be the identity function. Then f is a (1,2)\*-gs-open memorphism but f is not a  $\tilde{g}$  (1,2)\*-homeomorphism.

**Remark 3.15:** The following Examples show that the concepts of  $\tilde{g}(1,2)^*$ -homeomorphisms and  $(1,2)^*$ -g-homeomorphisms are independent of each other.

**Example 3.16:** Let X = {a, b, c},  $\tau_1 = \{\phi, X, \{a\}, \{a, b\}\}$  and  $\tau_2 = \{\phi, X, \{a, c\}\}$ . Then the sets in { $\phi, X, \{a\}, \{a, b\}$ , {a, c}} are called  $\tau_{1,2}$ -open and the sets in { $\phi, X, \{b\}, \{c\}, \{b, c\}\}$  are called  $\tau_{1,2}$ -closed. Also the sets in { $\phi, X, \{b\}, \{c\}, \{b, c\}\}$  are called  $\tilde{g}(1,2)^*$ -closed and  $(1,2)^*$ -g-closed in X. Moreover, the sets in { $\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$  are called  $\tilde{g}(1,2)^*$ -open and  $(1,2)^*$ -g-open in X. Let Y = {a, b, c},  $\sigma_1 = \{\phi, Y, \{b\}\}$  and  $\sigma_2 = \{\phi, Y, \{a, b\}\}$ . Then the sets in { $\phi, Y, \{b\}, \{a, b\}\}$  are called  $\sigma_{1,2}$ -open and the sets in { $\phi, Y, \{c\}, \{a, c\}\}$  are called  $\sigma_{1,2}$ -closed. Also the sets in { $\{\phi, Y, \{a\}, \{c\}, \{a, c\}, \{b, c\}\}$  are called  $\sigma_{1,2}$ -open and the sets in { $\{\phi, Y, \{c\}, \{a, c\}\}$  are called  $\sigma_{1,2}$ -closed. Also the sets in { $\{\phi, Y, \{a\}, \{c\}, \{a, c\}, \{b, c\}\}$  are called  $\tilde{g}(1,2)^*$ -closed in Y and the sets in { $\{\phi, Y, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$  are called  $\tilde{g}(1,2)^*$ -closed in Y and the sets in { $\{\phi, Y, \{a\}, \{b\}, \{a, b\}\}$  are called  $\tilde{g}(1,2)^*$ -closed in Y and the sets in { $\{\phi, Y, \{a\}, \{b\}, \{a, b\}, \{a, b\}\}$  are called  $\tilde{g}(1,2)^*$ -closed in Y and the sets in { $\{\phi, Y, \{a\}, \{b\}, \{a, b\}\}$  are called  $(1,2)^*$ -g-open in Y. Moreover, the sets in { $\{\phi, Y, \{c\}, \{a, c\}, \{b, c\}\}$  are called  $(1,2)^*$ -g-closed in Y and the sets in { $\{\phi, Y, \{a\}, \{b\}, \{a, b\}\}$  are called  $(1,2)^*$ -g-open in Y. Define a function f:  $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  by f(a) = b, f(b) = a and f(c) = c. Then f is a  $\tilde{g}(1,2)^*$ -homeomorphism but f is not a  $(1,2)^*$ -g-homeomorphism.

**Example 3.17:** Let X = {a, b, c},  $\tau_1 = \{\phi, X, \{a\}\}$  and  $\tau_2 = \{\phi, X\}$ . Then the sets in { $\phi$ , X, { $a\}}$ } are called  $\tau_{1,2}$ -open and the sets in { $\phi$ , X, { $b\}$ , {c}, {a, b}, {a, c}, {b, c}} are called  $\tilde{\tau}_{1,2}$ -closed. Also the sets in { $\phi$ , X, {b}, {c}, {a, b}, {a, c}, {b, c}} are called  $\tilde{g}$  (1,2)\*-closed and (1,2)\*-g-closed in X. Moreover, the sets in { $\phi$ , X, {a}, {b}, {c}, {a, b}, {a, c}} are called  $\tilde{g}$  (1,2)\*-open and (1,2)\*-g-open in X. Let Y = {a, b, c},  $\sigma_1 = \{\phi, Y, \{a\}\}$  and  $\sigma_2 = \{\phi, Y, \{b, c\}\}$ . Then the sets in { $\phi$ , Y, {a}, {b}, c}} are called  $\tilde{g}$  (1,2)\*-closed and  $\sigma_{1,2}$ -open and  $\sigma_{1,2}$ -closed. Also the sets in{{ $\phi$ , Y, {a}, {a}, {b, c}} are called  $\tilde{g}$  (1,2)\*-closed and  $\tilde{g}$  (1,2)\*-closed in Y. Moreover, the sets in { $\phi$ , Y, {a}, {a, b}, {a, c}} are called  $\tilde{g}$  (1,2)\*-closed and  $\tilde{g}$  (1,2)\*-open in Y. Moreover, the sets in { $\phi$ , Y, {a}, {b}, {c}, {a, b}, {a, c}} are called (1,2)\*-gs-closed and (1,2)\*-gs-open in Y. Define a function f : (X,  $\tau_1, \tau_2$ )  $\rightarrow$  (Y,  $\sigma_1, \sigma_2$ ) by f(a) = b, f(b) = c, f(c) = a. Then f is a (1,2)\*-g-homeomorphism but f is not a  $\tilde{g}$  (1,2)\*-homeomorphism.

**Remark 3.18:**  $\tilde{g}$  (1,2)\*-homeomorphisms and (1,2)\*-sg-homeomorphisms are independent of each other as shown below.

**Example 3.19:** The function f defined in Example 3.16 is  $\tilde{g}(1,2)^*$ -homeomorphism but not  $(1,2)^*$ -sg-homeomorphism.

**Example 3.20:** Let X = {a, b, c},  $\tau_1 = \{\phi, X, \{a\}\}$  and  $\tau_2 = \{\phi, X, \{b\}\}$ . Then the sets in  $\{\phi, X, \{a\}, \{b\}, \{a, b\}\}$  are called  $\tau_{1,2}$ -open and  $\tilde{g}$  (1,2)\*-open in X; the sets in  $\{\phi, X, \{c\}, \{a, c\}, \{b, c\}\}$  are called  $\tau_{1,2}$ -closed and  $\tilde{g}$  (1,2)\*-closed in X. Also, the sets in  $\{\phi, X, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$  are called (1,2)\*-sg-closed in X and the sets in  $\{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$  are called (1,2)\*-sg-closed in X and the sets in  $\{\phi, X, \{a\}, \{b\}, \{a, c\}, \{b, c\}\}$  are called (1,2)\*-sg-closed. Also the sets in  $\{\phi, Y, \{a\}, \{b, c\}\}$  are called  $\tilde{g}$  (1,2)\*-closed and  $\tilde{g}$  (1,2)\*-open in Y. Moreover, the sets in  $\{\phi, Y, \{a\}, \{b\}, \{a, c\}, \{a, c\}, \{a, c\}, \{b, c\}\}$  are called (1,2)\*-sg-closed and (1,2)\*-sg-open in Y. Define a function f:  $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  by f(a) = b, f(b) = a and f(c) = c. Then f is (1,2)\*-sg-homeomorphism but not  $\tilde{g}$  (1,2)\*-homeomorphism.

### 4. STRONGLY $\tilde{g}$ (1, 2)\*-HOMEOMORPHISMS

**Definition 4.1:** A bijection f:  $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  is said to be strongly  $\tilde{g}(1,2)^*$ -homeomorphism if f is  $\tilde{g}(1,2)^*$ -irresolute and its inverse f<sup>-1</sup> is also  $\tilde{g}(1,2)^*$ -irresolute.

**Theorem 4.2:** Every strongly  $\tilde{g}(1,2)^*$ -homeomorphism is  $\tilde{g}(1,2)^*$ -homeo-morphism.

**Proof:** Let f:  $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  be strongly  $\tilde{g}(1,2)^*$ -homeomorphism. Let U be  $\tau_{1,2}$ -open in X. Then U is  $\tilde{g}(1,2)^*$ -open in X. Since f<sup>1</sup> is  $\tilde{g}(1,2)^*$ -irresolute,  $(f^1)^{-1}(U)$  is  $\tilde{g}(1,2)^*$ -open in Y. That is f(U) is  $\tilde{g}(1,2)^*$ -open in Y. This implies f is  $\tilde{g}(1,2)^*$ -open function. Let F be a  $\sigma_{1,2}$ -closed in Y. Then F is  $\tilde{g}(1,2)^*$ -closed in Y. Since f is  $\tilde{g}(1,2)^*$ -irresolute,  $f^1(F)$  is  $\tilde{g}(1,2)^*$ -closed in X. This implies f is  $\tilde{g}(1,2)^*$ -closed in Y. Then F is  $\tilde{g}(1,2)^*$ -closed in Y. Since f is  $\tilde{g}(1,2)^*$ -irresolute,  $f^1(F)$  is  $\tilde{g}(1,2)^*$ -closed in X. This implies f is  $\tilde{g}(1,2)^*$ -continuous function. Hence f is  $\tilde{g}(1,2)^*$ -homeomorphism.

Remark 4.3: The following Example shows that the converse of Theorem 4.2 need not be true.

**Example 4.4:** Let  $X = \{a, b, c\}, \tau_1 = \{\phi, X, \{a\}\}$  and  $\tau_2 = \{\phi, X, \{a, c\}\}$ . Then the sets in  $\{\phi, X, \{a\}, \{a, c\}\}$  are called  $\tau_{1,2}$ -open and the sets in  $\{\phi, X, \{b\}, \{b, c\}\}$  are called  $\tau_{1,2}$ -closed. Also the sets in  $\{\phi, X, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}$  are called  $\tilde{g}$  (1,2)\*-closed in X and the sets in  $\{\phi, X, \{a\}, \{c\}, \{a, b\}, \{a, c\}\}$  are called  $\tilde{g}$  (1,2)\*-closed in X and the sets in  $\{\phi, X, \{a\}, \{c\}, \{a, b\}, \{a, c\}\}$  are called  $\tilde{g}$  (1,2)\*-open in X. Let  $Y = \{a, b, c\}, \sigma_1 = \{\phi, Y, \{a\}\}$  and  $\sigma_2 = \{\phi, Y\}$ . Then the sets in  $\{\phi, Y, \{a\}\}$  are called  $\sigma_{1,2}$ -open and the sets in

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 $\{\phi, Y, \{b, c\}\}$  are called  $\sigma_{1,2}$ -closed. Also the sets in  $\{\phi, Y, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$  are called  $\tilde{g}(1,2)^*$ -closed in Y and the sets in  $\{\phi, Y, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}\}$  are called  $\tilde{g}(1,2)^*$ -open in Y. Let f:  $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  be the identity function. Then f is a  $\tilde{g}(1,2)^*$ -homeomorphism but f is not a strongly  $\tilde{g}(1,2)^*$ -homeomorphism.

# **Theorem 4.5:** The composition of two strongly $\tilde{g}$ (1,2)\*-homeomorphisms is a strongly $\tilde{g}$ (1,2)\*-homeomorphism.

**Proof:** Let  $f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$  and  $g: (Y, \sigma_1, \sigma_2) \to (Z, \eta_1, \eta_2)$  be two strongly  $\tilde{g}(1,2)^*$ -homeomorphisms. Let F be a  $\tilde{g}(1,2)^*$ -closed set in Z. Since g is  $\tilde{g}(1,2)^*$ -irresolute,  $g^{-1}(F)$  is  $\tilde{g}(1,2)^*$ -closed in Y. Since f is a  $\tilde{g}(1,2)^*$ -irresolute,  $f^1(g^{-1}(F))$  is  $\tilde{g}(1,2)^*$ -closed in X. That is  $(g \circ f)^{-1}(F)$  is  $\tilde{g}(1,2)^*$ -closed in X. This implies that g o f:  $(X, \tau_1, \tau_2) \to (Z, \eta_1, \eta_2)$  is  $\tilde{g}(1,2)^*$ -irresolute. Let V be a  $\tilde{g}(1,2)^*$ -closed in X. Since  $f^{-1}$  is a  $\tilde{g}(1,2)^*$ -irresolute,  $(f^{-1})^{-1}(V)$  is  $\tilde{g}(1,2)^*$ -closed in Y. That is f(V) is  $\tilde{g}(1,2)^*$ -closed in Y. Since  $g^{-1}$  is a  $\tilde{g}(1,2)^*$ -irresolute,  $(g^{-1})^{-1}(f(V))$  is  $\tilde{g}(1,2)^*$ -closed in Z. That is g(f(V)) is  $\tilde{g}(1,2)^*$ -closed in Z. So,  $(g \circ f)(V)$  is  $\tilde{g}(1,2)^*$ -closed in Z. This implies that  $((g \circ f)^{-1})^{-1}(V)$  is  $\tilde{g}(1,2)^*$ -closed in Z. This shows that  $(g \circ f)^{-1} : (Z, \eta_1, \eta_2) \to (X, \tau_1, \tau_2)$  is  $\tilde{g}(1,2)^*$ -irresolute. Hence g o f is a strongly  $\tilde{g}(1,2)^*$ -homeomorphism.

**Remark 4.6:** The concepts of strongly  $\tilde{g}(1,2)^*$ -homeomorphisms and  $(1,2)^*-\alpha$ -homeomorphisms are independent notions as shown in the following examples.

**Example 4.7:** Let  $X = \{a, b, c\}, \tau_1 = \{\phi, X\}$  and  $\tau_2 = \{\phi, X, \{a, b\}\}$ . Then the sets in  $\{\phi, X, \{a, b\}\}$  are called  $\tau_{1,2}$ -open and  $(1,2)^*-\alpha$ -open; and the sets in  $\{\phi, X, \{c\}\}$  are called  $\tau_{1,2}$ -closed and  $(1,2)^*-\alpha$ -closed. Also the sets in  $\{\phi, X, \{c\}, \{a, c\}, \{b, c\}\}$  are called  $\tilde{g}$   $(1,2)^*$ -closed in X and the sets in  $\{\phi, X, \{a\}, \{b\}, \{a, b\}\}$  are called  $\tilde{g}$   $(1,2)^*$ -open in X. Let  $Y = \{a, b, c\}, \sigma_1 = \{\phi, Y, \{a\}\}$  and  $\sigma_2 = \{\phi, Y, \{b\}\}$ . Then the sets in  $\{\phi, Y, \{a\}, \{b\}, \{a, b\}\}$  are called  $\sigma_{1,2}$ -open and  $(1,2)^*-\alpha$ -open; and the sets in  $\{\phi, Y, \{c\}, \{a, c\}, \{b, c\}\}$  are called  $\sigma_{1,2}$ -closed and  $(1,2)^*-\alpha$ -closed in Y. Also the sets in  $\{\phi, Y, \{c\}, \{a, c\}, \{b, c\}\}$  are called  $\tilde{g}$   $(1,2)^*$ -a-closed in Y. Also the sets in  $\{\phi, Y, \{c\}, \{a, c\}, \{b, c\}\}$  are called  $\tilde{g}$   $(1,2)^*$ -a-closed in Y. Let f:  $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  be the identity function. Then f is a strongly  $\tilde{g}$   $(1,2)^*$ -homeomorphism but f is not  $(1,2)^*-\alpha$ -homeomorphism.

**Example 4.8:** Let X = {a, b, c},  $\tau_1 = \{\phi, X, \{a\}\}$  and  $\tau_2 = \{\phi, X, \{a, b\}\}$ . Then the sets in { $\phi, X, \{a\}, \{a, b\}\}$  are called  $\tau_{1,2}$ -open and the sets in { $\phi, X, \{c\}, \{b, c\}\}$  are called  $\tau_{1,2}$ -closed. Also the sets in { $\phi, X, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$  are called  $\tilde{g}$  (1,2)\*-closed in X and the sets in { $\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$  are called  $\tilde{g}$  (1,2)\*-open in X. Moreover, the sets in { $\phi, X, \{b\}, \{c\}, \{b, c\}\}$  are called (1,2)\*- $\alpha$ -closed in X and the sets in { $\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$  are called (1,2)\*- $\alpha$ -open in X. Let Y = {a, b, c},  $\sigma_1 = \{\phi, Y\}$  and  $\sigma_2 = \{\phi, Y, \{a\}\}$ . Then the sets in { $\phi, Y, \{a\}\}$  are called  $\sigma_{1,2}$ -open and the sets in { $\phi, Y, \{b, c\}\}$  are called  $\sigma_{1,2}$ -closed. Also the sets in { $\phi, Y, \{a\}, \{a, c\}, \{a, c\}, \{b, c\}\}$  are called  $\sigma_{1,2}$ -closed in X and the sets in { $\phi, Y, \{a\}\}$  are called  $\sigma_{1,2}$ -open and the sets in { $\phi, Y, \{b, c\}\}$  are called  $\sigma_{1,2}$ -closed. Also the sets in { $\phi, Y, \{b\}, \{c, b, c\}\}$  are called  $\sigma_{1,2}$ -open and the sets in { $\phi, Y, \{b, c\}$  are called  $\sigma_{1,2}$ -closed. Also the sets in { $\phi, Y, \{b\}, \{c, b, c\}\}$  are called  $\sigma_{1,2}$ -open in X. Let Y = {a, b, c},  $\sigma_1 = \{\phi, Y, \{a\}, \{c\}, \{c\}, \{a, b\}, \{a, c\}, \{a, c\}, \{b, c\}\}$  are called  $\sigma_{1,2}$ -closed in Y and the sets in { $\{\phi, Y, \{b, c\}\}}$  are called  $\sigma_{1,2}$ -closed in Y and the sets in { $\{\phi, Y, \{b\}, \{c\}, \{b, c\}\}}$  are called (1,2)\*- $\alpha$ -closed in Y and the sets in { $\{\phi, Y, \{a\}, \{a, b\}, \{a, c\}\}}$  are called (1,2)\*- $\alpha$ -open in Y. Let f: (X,  $\tau_1, \tau_2$ ) $\rightarrow$ (Y,  $\sigma_1, \sigma_2$ ) be the identity function. Then f is a (1,2)\*- $\alpha$ -homeomorphism but not strongly  $\tilde{g}$  (1,2)\*-homeomorphism.

**Definition 4.9:** A bijective function  $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  is called  $(1,2)^*$ -gc-homeomorphism if f is  $(1,2)^*$ -gc-irresolute and  $f^1$  is  $(1,2)^*$ -gc-irresolute.

**Remark 4.10:** The concepts of strongly  $\tilde{g}$  (1,2)\*-homeomorphisms and (1,2)\*-gc-homeomorphisms are independent of each other as the following examples show.

**Example 4.11:** Let X = {a, b, c},  $\tau_1 = \{\phi, X, \{a\}\}$  and  $\tau_2 = \{\phi, X, \{a, b\}\}$ . Then the sets in  $\{\phi, X, \{a\}, \{a, b\}\}$  are called  $\tau_{1,2}$ -open and the sets in  $\{\phi, X, \{c\}, \{b, c\}\}$  are called  $\tau_{1,2}$ -closed. Also the sets in  $\{\phi, X, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$  are called  $\tilde{g}$  (1,2)\*-closed in X and the sets in  $\{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$  are called  $\tilde{g}$  (1,2)\*-open in X. Moreover, the sets in  $\{\phi, X, \{c\}, \{a, c\}, \{b, c\}\}$  are called (1,2)\*-g-closed in X and the sets in  $\{\phi, X, \{a\}, \{b\}, \{a, b\}\}$  are called (1,2)\*-g-open in X. Let Y = {a, b, c},  $\sigma_1 = \{\phi, Y, \{b\}, \{a, b\}\}$  and  $\sigma_2 = \{\phi, Y, \{a\}, \{a, c\}\}$ . Then the sets in  $\{\phi, Y, \{a\}, \{b\}, \{a, c\}\}$  are called  $\sigma_{1,2}$ -open and the sets in  $\{\phi, Y, \{a\}, \{b, c\}\}$  are called  $\sigma_{1,2}$ -closed. Also the sets in  $\{\phi, Y, \{b\}, \{c, \{a, c\}, \{b, c\}\}$  are called  $\sigma_{1,2}$ -open and the sets in  $\{\phi, Y, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$  are called  $\sigma_{1,2}$ -closed. Also the sets in  $\{\phi, Y, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$  are called  $\tilde{g}$  (1,2)\*-closed, the sets in  $\{\phi, Y, \{b\}, \{c\}, \{b, c\}\}$  are called (1,2)\*-g-closed in Y and the sets in  $\{\phi, Y, \{a\}, \{b\}, \{a, b\}, \{a, b\}, \{a, c\}\}$  are called (1,2)\*-g-open in Y. Let f:  $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  be the identity function. Then f is a strongly  $\tilde{g}$  (1,2)\*-homeomorphism but not (1,2)\*-gc-homeomorphism.

**Example 4.12:** Let X = {a, b, c},  $\tau_1 = \{\phi, X, \{a\}\}$  and  $\tau_2 = \{\phi, X, \{b\}\}$ . Then the sets in  $\{\phi, X, \{a\}, \{b\}, \{a, b\}\}$  are called  $\tau_{1,2}$ -open and the sets in  $\{\phi, X, \{c\}, \{a, c\}, \{b, c\}\}$  are called  $\tau_{1,2}$ -closed. Also the sets in  $\{\phi, X, \{c\}, \{a, c\}, \{b, c\}\}$  are called  $\tilde{g}$  (1,2)\*-closed and (1,2)\*-g-closed in X, and the sets in  $\{\phi, X, \{a\}, \{b\}, \{a, b\}\}$  are called  $\tilde{g}$  (1,2)\*-open and (1,2)\*-g-open in X. Let Y = {a, b, c},  $\sigma_1 = \{\phi, Y, \{a\}\}$  and  $\sigma_2 = \{\phi, Y, \{a, b\}\}$ . Then the sets in  $\{\phi, Y, \{a\}, \{a, b\}\}$  are called  $\sigma_{1,2}$ -open and the sets in  $\{\phi, Y, \{a\}, \{a, c\}, \{b, c\}\}$  are called  $\sigma_{1,2}$ -open and the sets in  $\{\phi, Y, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$  are called  $\sigma_{1,2}$ -closed. Also the sets in  $\{\phi, Y, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$  are called  $\sigma_{1,2}$ -closed. Also the sets in  $\{\phi, Y, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$  are called  $\tilde{g}$  (1,2)\*-open in Y. Moreover, the sets in  $\{\phi, Y, \{c\}, \{a, c\}, \{b, c\}\}$  are called (1,2)\*-g-closed in Y and the sets in  $\{\phi, Y, \{a\}, \{b\}, \{a, b\}\}$  are called (1,2)\*-g-open in Y. Let f:  $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  be the identity function. Then f is a (1,2)\*-gc-homeomorphism but not strongly  $\tilde{g}$  (1,2)\*-homeomorphism.

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