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GENERALIZED λ^*-CLOSED SETS IN INSTUITIONISTIC FUZZY TOPOLOGICAL SPACES

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ABSTRACT

In this paper, we introduce the new notion Instuitionistic fuzzy generalized λ^*-closed sets and Instuitionistic fuzzy generalized λ^*-open sets in Topological space. We study some of their basic properties.

Key words: *Instuitionistic fuzzy generalized λ^*-closed sets, Instuitionistic fuzzy generalized λ^*-open sets.*

INTRODUCTION

In 1965, Zadeh [8] introduced fuzzy sets and in 1968, Chang [3] introduced fuzzy topology. After the introduction of fuzzy set and fuzzy topology, several authors were conducted on the generalization of this notion. The notion of intuitionistic fuzzy sets was introduced by Atanassov [2] as generalization of fuzzy sets. In 1997, Coker [4] introduced the concept of intuitionistic fuzzy topological spaces. Now, we introduce the concept of Instuitionistic fuzzy generalized λ^*-closed sets and Instuitionistic fuzzy generalized λ^*-open sets in Topological space and study some of their properties.

I. PRELIMINARIES

Definition 1.1 [1]: Let X be a nonempty fixed set. An intutionistic fuzzy set (IFS) A in X is an object having the form $A = \{< x, \mu A(x), \upsilon A(x) > : x \in X\}$, where the function $\mu_A : X \rightarrow [0,1]$ and $\upsilon_A : X \rightarrow [0,1]$ denotes the degree of membership $\mu_A(x)$ and the degree of non membership $\gamma_A(x)$ of each element x∈X to the set A respectively and 0 ≤ $\mu_A(x) + \upsilon_A(x) \leq 1$ for each x∈ X.

Definition 1.2 [1]: Let X be a non empty fixed set. Let A and B be the intuitionistic fuzzy sets in the form $A = \{<x, \mu_A(x), \upsilon_A(x) >: x \in X\}$ and $B = \{<x, \mu_B(x), \upsilon_B(x) > : x \in X\}$ Then

(a) A= B if and only if $\mu_A(x) \leq \mu_B(x)$ and $\upsilon_A(x) \geq \upsilon_B(x)$ for all x∈X

(b) A = B if and only if A ⊆ B and B⊆A

(c) $A^C = \{<x, \upsilon A(x), \mu A(x) > / x \in X\}$

(d) $A \cap B = \{<x, \mu_A(x) \wedge \mu_B(x), \upsilon_A(x) \vee \upsilon_B(x)> / x \in X\}$

(e) $A \cup B = \{x, \mu_A(x) \vee \mu_B(x), \upsilon_A(x) \wedge \upsilon_B(x)> / x \in X\}$

(f) $0 = \{<x, 0, 1> / x \in X\}$ and $1 = \{<x, 1, 0> / x \in X\}$ are respectively the empty set and the whole set of X.

Definition 1.3 [4]: An intuitionistic fuzzy topology (IFT) on X is a family τ of intuitionistic fuzzy topology sets in X satisfying the following axioms.

(i) $0\sim, 1\sim \in \tau$

(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$

(iii) $\cup\, Gi \in \tau$ for any family $\{Gi / i \in I\} \subseteq \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space and each intuitionistic fuzzy set in τ is known as an intuitionistic fuzzy open set (IFOS) in X.

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Definition 1.4 [4]: The complement A^C of an intuitionistic fuzzy open set A in an intuitionistic fuzzy topological space (X, τ) is called intuitionistic fuzzy closed set (IFOS) in X.

Definition 1.5 [4]: Let (X, τ) be in intuitionistic fuzzy topological space and $A = \{< x, \mu A(x), \upsilon A(x) > : x \in X\}$ be an intuitionistic fuzzy set in X. Then the intuitionistic fuzzy interior and the intuitionistic fuzzy closure are defined by cl(A) = ∩{K / K is an intuitionistic fuzzy closed sets in X and A ⊆ K}

int(A) = ∪ {G / G is an intuitionistic fuzzy open sets in X and G⊆ A}.

Remark 1.6 [4]: For any intuitionistic fuzzy set A in (X, τ) we have
(i) $cl(A^C) = [int(A)]^C$,
(ii) $int(A^C) = [cl(A)]^C$,
(iii) A is an intuitionistic fuzzy closed in X ⇔ cl(A) = A
(iv) A is an intuitionistic fuzzy open set in X ⇔ int(A) = A

Definition 1.7: An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X, τ) is called
(i) intuitionistic fuzzy generalized closed set (IFGCS) [7] (intuitionistic fuzzy g – closed) if cl(A)⊆U whenever A ⊆ U and U is intuitionistic fuzzy open
(ii) intuitionistic fuzzy g–open set (IFGOS) [7] if the complement of an intuitionistic fuzzy g–closed set is called intuitionistic fuzzy g - open set.
(iii) intuitionistic fuzzy semi open set (IFSOS) [5](resp. intuitionistic fuzzy semi closed SET (IFSCS)) if there exists an intuitionistic fuzzy open set U (resp. intuitionistic fuzzy closed) such that U⊆A⊆cl (U) (resp. int (U) ⊆A⊆U).
(iv) intuitionistic fuzzy λ-closed set (IFλCS)[6] if A⊇ cl(U) whenever A⊇U and U is intuitionistic fuzzy open in X.

Remark 1.8 [7]: Every intuitionistic fuzzy closed set [7] (intuitionistic fuzzy open set) is intuitionistic fuzzy g-closed (intuitionistic fuzzy g- open set) but the converse may not be true.

II. GENERALIZED λ^*-CLOSED SETS IN INSTUITIONISTIC FUZZY TOPOLOGICAL SPACES

In this section, we introduce the concept of Instuitionistic fuzzy generalized λ^*-closed sets in topological spaces and study some of its properties.

Definition 2.1: An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X,τ) is called an Instuitionistic fuzzy generalized λ^*-closed set (IFGλ^*CS) in X if A ⊇ cl(G) whenever A ⊇ G and G is IFGOS in X.

Theorem 2.2: Every intuitionistic fuzzy closed set is Instuitionistic fuzzy generalized λ^*-closed set but not conversely.

Proof: Let A be an intuitionistic fuzzy set in IFTS (X, τ). Let G be an IFGOS in X such that A⊇G. Then cl(A) ⊇ cl(G). But cl(A) = A. Therefore A ⊇ cl(G) and hence A is Instuitionistic fuzzy generalized λ *-closed set in X.

The following example shows that the converse of the above theorem need not be true.

Example 2.3: Let X = {a, b} and $\tau = \{0_\sim, U, 1_\sim\}$ be an IFT on X where U = < x, (0.5, 0.5), (0.3, 0.4)>. Then the IFS A = < x, (0.5, 0.5), (0.4, 0.2) > is IFGλ^*CS but not IFCS.

Theorem 2.4: If A and B are IFGλ^*CS in a IFTS (X,τ), then A ∩ B is an IFGλ^*CS.

Proof: Let A ∩ B ⊇ G, where G is IFGOS.
⇒ A ⊇ G, B ⊇ G
⇒ A ⊇ cl(G), B ⊇ cl(G), since A & B are IFGλ^*CS.
⇒ A ∩ B ⊇ cl(G)
⇒ A ∩ B is IFGλ^*CS.

Remark 2.5: If A and B are < x, (0.5, 0.5), (0.3, 0.4)> in X, then their union is also IFGλ^*CS in X as seen from the following example.

Example 2.6: Let X = {a, b} and U, A and B be the IFT of X defined as follows:
U = < x, (0.5, 0.5), (0.5, 0.4)>; A= < x, (0.4,0.5), (0.6,0.5)>; B= < x, (0.4,0.3), (0.6,0.4)>. Let τ = {0~, U, 1~} be IFT on X. Now A and B are IFGλ^*CS. Then A∪B is also an IFGλ^*CS

Theorem 2.7: Every intuitionistic fuzzy generalized closed set is Instuitionistic fuzzy generalized λ^*-closed set but not conversely.

Proof: Let A be an intuitionistic fuzzy closed set in IFTS (X,τ). By theorem 2.2 and Remark 1.8. It follows the theorem.

Example 2.8: Let X = {a, b} and τ = {0~, U, 1~} be an IFT on X where U = < x, (0.5, 0.6), (0.5, 0.4)>. Then the IFS A = < x, (0.4, 0.5), (0.6, 0.5)> is IFGλ^*CS but not IFGCS.

Theorem 2.9: Every intuitionistic fuzzy λ closed set is Instuitionistic fuzzy generalized λ^*-closed set but not conversely.

Proof: Let A be an intuitionistic fuzzy set in IFTS (X,τ). Let G be an IFGOS in X such that A$\supseteq$G. Since A is IFλCS, A $\supseteq$ cl(G) and hence A is is Instuitionistic fuzzy generalized λ^*-closed set in X.

Example 2.10: Let X = {a, b} and τ = {0~, U, 1~} be an IFT on X where U = < x, (0.5, 0.5), (0.5, 0.3)>. Then the IFS A = < x, (0.5, 0.5), (0.5, 0.2)> is IFGλ^*CS but not IFλCS.

Remark 2.11: The concepts of IFGλ^*CS and IFSCS are independent as seen from the following examples.

Example 2.12: Let X = {a, b} and τ = {0~, U, 1~} be an IFT on X where U = < x, (0.5, 0.5), (0.5, 0.2)>. Then the IFS A = < x, (0.5, 0.5), (0.5, 0.4)> is IFGλ^*CS but not IFSCS.

Example 2.13: Let X = {a, b} and τ = {0~, U, V, 1~} be an IFT on X where U = < x, (0.5, 0.5), (0.3, 0.5)> and V = < x, (0.5, 0.5), (0.4, 0.5)>. Then the IFS A = < x, (0.5, 0.5), (0.3, 0.5) > is not IFGλ^*CS but IFSCS.

From the above results, we get the following implications.

$$\begin{tikzcd}
 & \text{IFGCS} \arrow[dr] & & \text{IF}\lambda\text{CS} \arrow[dl] & \\
\text{IFCS} \arrow[rr] & & \text{IFG}\lambda^*\text{CS} \arrow[rr, leftrightarrow, "|" marking] & & \text{IFSCS}
\end{tikzcd}$$

III GENERALIZED λ^*-OPEN SETS IN INSTUITIONISTIC FUZZY TOPOLOGICAL SPACES

In this section, we introduce the concept of intuitionistic fuzzy generalized λ^*-open sets and study some of its properties.

Definition 3.1: An instuitionistic set A of an intuitionistic fuzzy topological space X is called intuitionistic fuzzy generalized λ^*-open set (IFGλ^*OS) if A^c is intuitionistic fuzzy generalized λ^*-closed set (IFGλ^*CS) in X.

Theorem 3.2: An instuitionistic set A of an intuitionistic fuzzy topological space X is intuitionistic fuzzy generalized λ^*-open set if and only if A $\subseteq$ int(F) whenever A $\subseteq$ F and F is IFGCS in X.

Proof: Assume that A is IFGλ^*OS and F is a IFGCS in X such that A $\subseteq$ F.

$\Rightarrow A^c \supseteq F^c$
$\Rightarrow A^c \supseteq cl(F^c)$, since A^c is IFGλ^*CS.
$\Rightarrow A^c \supseteq (int(F))^c$
$\Rightarrow A \subseteq int(F)$

Conversely assume that A $\subseteq$ int(F) whenever A $\subseteq$ F and F is IFGCS in X. We have to prove that A^c is IFGλ^*CS in X. Let $A^c \supseteq G$, where G is IFGOS in X. Then $A \subseteq G^c$, which implies $A \subseteq int(G^c)$ by our hypothesis.

$\Rightarrow A \subseteq (cl(G))^c$, since $(cl(G))^c = int(G^c)$
$\Rightarrow A^c \supseteq cl(G)$
$\Rightarrow A^c$ is IFGλ^*CS. Thus A is IFGλ^*OS.

Theorem 3.5: If A and B are IFGλ^*OS in X, then $A \cup B$ is also an IFGλ^*OS in X.

Proof: Assume that A and B are IFGλ^*OS in X. Then A^c and B^c are IFGλ^*CS in X and so $A^c \cap B^c = (A \cup B)^c$ is IFGλ^*CS. Thus $A \cup B$ is IFGλ^*OS in X.

Remark 3.6: If A and B are IFGλ^*OS then their intersection, $A \cap B$ is also an IFGλ^*OS as seen from the following example.

Example 3.7: Let X = {a, b} and U, A and B be the intuitionistic fuzzy sets of X defined by. U = {<a, 0.5, 0.5>, <b, 0.5, 0.3>}, A = {<a, 0.5, 0.5>, <b, 0.3, 0.5>} and B = {<a, 0.5, 0.5>, <b, 0.4, 0.5>}. Let τ = {~0, ~1, U} be intuitionistic fuzzy topology on X. Then A and B are IFGλ^*OS in (X, τ). But $A \cap B$ is IFGλ^*CS

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