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ON PROPERTIES OF b$\alpha\hat{g}$- CLOSED SETS IN TOPOLOGICAL SPACES

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ABSRTACT

In this paper a new class of closed sets namely b$\alpha\hat{g}$-closed sets in topological spaces is introduced. Some properties and applications of b$\alpha\hat{g}$-closed sets are characterized. Also new classes of spaces, based on the class of b$\alpha\hat{g}$-closed sets are introduced and their properties are analyzed.

Key words: *α-closed sets, $\hat{g}$-open sets, $\alpha\hat{g}$-open sets, b$\alpha\hat{g}$-closed sets, b$\alpha\hat{g}$- continuous, and b$\alpha\hat{g}$ open map.*

AMS subject classification: *54A05, 54F65, 54C55.*

1. INTRODUCTION

Norman Levine introduced and studied generalized closed (briefly g-closed) sets[6] and semi-open sets[7] in 1963 and 1970 respectively. M.K.R.S.Veera Kumar [15] introduced $\hat{g}$ -closed sets in topological spaces in 2003. M.E.Abd El-Monsef, S.Rose Mary and M.L. Thivagar [1] introduced $\alpha\hat{g}$ -closed sets in topological spaces in 2007. Followed by these, we have introduced a new class of closed sets namely, b$\alpha\hat{g}$ -closed sets and characterized their properties.

2. PRELIMINARIES

Throughout this paper, (X, τ) denote a topological space with topology τ. For a subset A of X the interior of A and closure of A are denoted by int(A) and cl(A) respectively.

Definition 2.1.1: A subset A of a topological space (X, τ) is called

1. a semi open set [7] if A $\subseteq$ cl(int(A)) and a semi closed set if int(cl(A)) $\subseteq$ A.
2. a pre-open set [11] if A $\subseteq$ int(cl(A)) and a pre-closed set if cl(int(A)) $\subseteq$ A.
3. an α-open set [12] if A $\subseteq$ int(cl(int(A)) and an α- closed set if cl(int(cl(A)) $\subseteq$ A.
4. a b-open set [3] if A $\subseteq$ cl (int(A)) $\cup$ int(cl(A)) and a b -closed set if int(cl(A)) $\cap$ cl(int(A)) $\subseteq$ A.

The intersection of all semi-closed (resp α-closed, b-closed) sets of X containing A is called the semi-closure (resp. α-closure, b-closure) of A and is denoted by scl(A) (resp. αcl(A), bcl(A)).

Definition 2.1.2: A subset A of a topological space (X, τ) is called,

1. a generalized closed set (briefly g-closed) [6] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ). The complement of a g-closed set is called a g-open set.
2. a generalized semi-closed set (briefly gs-closed) [4] if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ).
3. an α- generalized closed set (briefly αg-closed) [10] if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ).
4. a generalized pre-closed set (briefly gp-closed) [8] if pcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ).
5. a strongly g-closed set [13] if cl(intA) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ).
6. a $\hat{g}$ -closed set [15] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is a semi-open set in (X, τ).
7. an $\alpha\hat{g}$ -closed set [1] if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is $\hat{g}$-open set in (X, τ).
8. a gb-closed set [2] if bcl (A) $\subseteq$ U whenever A $\subseteq$ U and U is open set in (X, τ).
9. a b$\hat{g}$ - closed set [14] if bcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is $\hat{g}$-open set in (X, τ).

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Definition 2.1.3: A function f: (X, τ) → (Y, σ) is called

1. α-continuous [9] if f^{-1} (V) is α-closed in (X, τ) for every closed set V of (Y, σ).
2. b-continuous [2] if f^{-1} (V) is b-closed in (X, τ) for every closed set V of (Y, σ).
3. pre- continuous [11] if f^{-1} (V) is pre-closed in (X, τ) for every closed set V of (Y, σ).
4. Semi-continuous [7] if f^{-1} (V) is semi-closed in (X, τ) for every closed set V of (Y, σ).
5. gb- continuous [2] if f^{-1}(V) is gb-closed in (X, τ) for every closed set V of (Y, σ).
6. gb- open map [2] if f (U) is gb-open in (Y,σ), for every open set U of (X, τ).

Definition 2.1.4: A bijection f: (X, τ) → (Y, σ) is called

1. a homeomorphism [5] if f is both continuous and open.
2. a gb-homeomorphism [2] if f is gb-continuous and gb-open.

Definition 2.1: A space (X, τ) is called

1. a T_{gs} – space [2] if every gb-closed set in it is b-closed.
2. a $T_b\hat{g}$ - space [14] if every b$\hat{g}$ - closed set in it is b-closed.
3. a $T_\alpha\hat{g}$ - space [1] if every α$\hat{g}$ - closed set in it is α-closed.

3. bα$\hat{g}$- CLOSED SETS

In this section we introduce a new class of closed sets called bα$\hat{g}$- closed sets which lies between the class of α-closed sets and the class of gb-closed sets.

Definition 3.1.1: A subset A of a topological space (X, τ) is said to be bα$\hat{g}$- closed set if bcl(A)⊆U, whenever A⊆U and U is a α$\hat{g}$- open set in (X, τ).

3.1. Relationship of bα$\hat{g}$ - closed sets with other known sets.

Theorem 3.1.1: Every pre-closed set is bα$\hat{g}$- closed.

Proof: Let A be pre-closed. Then A^c is pre open and $A^c \subseteq int(cl(A^c)) \subseteq int(cl(A^c)) \cup cl(int(A^c)$. Hence A^c is b-open and therefore whenever U is a α$\hat{g}$- open set and A ⊆ U then bcl(A) = A ⊆ U and hence A is bα$\hat{g}$ - closed.

Theorem 3.1.2: Every semi-closed set is bα$\hat{g}$ -closed.

Proof: Since every semi- closed set A is b- closed, bcl(A) = A. Therefore, bcl(A) = A⊆U, whenever A ⊆ U and U is α$\hat{g}$- open. Thus, every semi-closed set is bα$\hat{g}$- closed.

Theorem 3.1.3: Every closed set is bα$\hat{g}$- closed.

Proof: A be closed then A is b-closed and hence whenever A ⊆ U, and U is α$\hat{g}$-open, bcl(A) = A ⊆ U. Thus, A is bα$\hat{g}$ - closed.

Theorem 3.1.4: Every α-closed set is bα$\hat{g}$- closed.

Proof: Let A be α-closed and U be α$\hat{g}$- open, such that A ⊆ U. Since every α-closed set is b- closed, bcl(A) = A ⊆ U. Hence A is bα$\hat{g}$- closed.

Note: The converse part of the above Theorems 3.1.1, 3.1.2, 3.1.3, and 3.1.4 need not be true. The following examples show that the class of bα$\hat{g}$- closed sets properly contains the class of pre-closed, semi-closed, closed, α-closed sets.

Theorem 3.1.5: Let A bα$\hat{g}$ be closed set. Then A need not be (i) pre-closed (ii) semi-closed (iii)closed (iv) α- closed.

Proof:
(i) **Example 3.1.1:** X = {a, b, c}, τ = {X, ϕ, {a},{b},{a, b}}. A = {a} is bα$\hat{g}$- closed, but not pre-closed.

(ii) **Example 3.1.2:** X= {a, b, c}, τ = {X, ϕ, {a, c}}. In X, the set A = {a, b} is bα$\hat{g}$- closed, but not semi-closed.

(iii) **Example 3.1.3:** X = {a, b, c}, τ = {X, ϕ, {a},{ a, b}}. In X, the set A = {b} is bα$\hat{g}$- closed, but not closed.

(iv) **Example 3.1.4:** X = {a, b, c}, τ = {X, ϕ, { a, c}}. In X, the set A = {b, c} is bα$\hat{g}$- closed, but not α-closed.

Theorem 3.1.6: Every b-closed set is b$\alpha\hat{g}$-closed set.

Proof: Let A be b-closed, so that whenever A $\subseteq$ U, U is $\alpha\hat{g}$-open, bcl(A) = A $\subset$ U, thus A is b$\alpha\hat{g}$ closed.

Theorem 3.1.7: Every b$\alpha\hat{g}$- closed set is gb- closed. The converse is not true.

Proof: Let U be open such that A $\subseteq$ U. Since every open set is $\alpha\hat{g}$- open, and A is b$\alpha\hat{g}$- closed set, A $\subset$ U implies bcl(A) $\subset$ U and hence A is gb-closed.

Example 3.1.5: X= {a, b, c}, τ = {X, ϕ, {a}, {a, b}}. A = {a, c} is gb-closed, but not b$\alpha\hat{g}$- closed.

Remark: The following examples reveal that b$\alpha\hat{g}$- closed sets are independent from g- closed, gs- closed, gp -closed, αg-closed, $\alpha\hat{g}$ -closed and strongly g -closed sets.

Example 3.1.6:
(i) Let X = {a, b, c}, τ = {X, ϕ, {a, c}}. A_1 = {a} is b$\alpha\hat{g}$ -closed but not g- closed.
X = {a, b, c}, τ = {X, ϕ, {a, c}}. A_2 = {a, c} is g-closed but not b$\alpha\hat{g}$ -closed.
(ii) Let X = {a, b, c}, τ = {X, ϕ, {a}, {a, b}}. A_1={ a, c} is gs-closed but not b$\alpha\hat{g}$- closed. A_2= {b} is b$\alpha\hat{g}$ -closed but not gs- closed.
(iii) Let X = {a, b, c}, τ = {X, ϕ, {a}}. A_1 = {a, b} is gp_closed but not b$\alpha\hat{g}$- closed.
X = {a, b, c}, τ ={X, ϕ, {a}, {b}, {a, b}}. A_2 ={a} is b$\alpha\hat{g}$ - closed but not gp- closed.
(iv) Let X = {a, b, c}, τ = {X, ϕ, {a}}. A_1 = {a, b} is αg-closed but not b$\alpha\hat{g}$- closed.
X = {a, b, c}, τ = {X, ϕ, {a, c}}. A_2 = {a} is b$\alpha\hat{g}$ -closed but not αg -closed.
(v) Let X = {a, b, c}, τ = {X, ϕ, {a}, {a, b}}. A_1 = {a, c} is αg -closed but not b$\alpha\hat{g}$ -closed.
X = {a, b, c}, τ = {X, ϕ, {a, c}}. A_2 = {c} is b$\alpha\hat{g}$ -closed but not αg- closed.
(vi) Let X = {a, b, c}, τ = {X, ϕ, {a}}. A_1={a, b} is strongly g-closed but not b$\alpha\hat{g}$ - closed.
X = {a, b, c}, τ = {X, ϕ, {a},{b},{a, b}}. A_2 = {b} is b$\alpha\hat{g}$ -closed but not strongly g- closed.

3.2 Basic properties of b$\alpha\hat{g}$-closed sets.

Theorem 3.2.1:
(i) The finite union of b$\alpha\hat{g}$ - closed set need not be b$\alpha\hat{g}$- closed set.
(ii) Intersection of any two b$\alpha\hat{g}$- closed sets need not be b$\alpha\hat{g}$- closed.

Proof: Example 3.2.1: (i) Let X = {a, b, c}, τ = {X, ϕ, {a, c}}. In (X , τ) the sets X, ϕ, {a},{b},{c},{a, b},{b, c} are b$\alpha\hat{g}$ - closed sets, but {a} $\cup$ {c} = {a, c} is not b$\alpha\hat{g}$ closed.

Example 3.2.2: (ii) Let X = {a, b, c} and τ = {X, ϕ, {a},{b, c}}, In (X, τ), the sets X, ϕ, {a},{a, b},{b, c},{c, a}} are b$\alpha\hat{g}$- closed sets, but {a, b} $\cap$ {b, c}= {b} is not b$\alpha\hat{g}$ -closed.

Remark: The collection of b$\alpha\hat{g}$ - open sets is not a topology.

Theorem 3.2.2: Let A be a b$\hat{g}$ - closed set in (X, τ). Then A need not be a b$\alpha\hat{g}$ closed set of (X, τ).

Proof: Example 3.2.3: Let X = {a, b, c} and τ = {X, ϕ, {a}}. In X, the set {a,b} is b$\hat{g}$ – closed but not b$\alpha\hat{g}$ -closed.

Theorem 3.2.3: Let A be a b$\alpha\hat{g}$ closed set of (X, τ). Then bcl (A) - A does not contain any non- empty $\alpha\hat{g}$ -closed set.

The converse part does not hold.

Proof: Suppose G is a $\alpha\hat{g}$- closed subset of (X, τ) such that G$\subseteq$ bcl (A) –A. Then G$\subseteq$X - A, and this implies A$\subseteq$ X -G. Now X - G is $\alpha\hat{g}$ open set of (X, τ) such that A $\subseteq$ X - G. Since A is b$\alpha\hat{g}$ -closed set of (X, τ), bcl (A) $\subseteq$ X-G. Thus G $\subseteq$ X-bcl (A). Now G $\subseteq$ bcl(A) $\cap$ (X -bcl(A)) = ϕ.

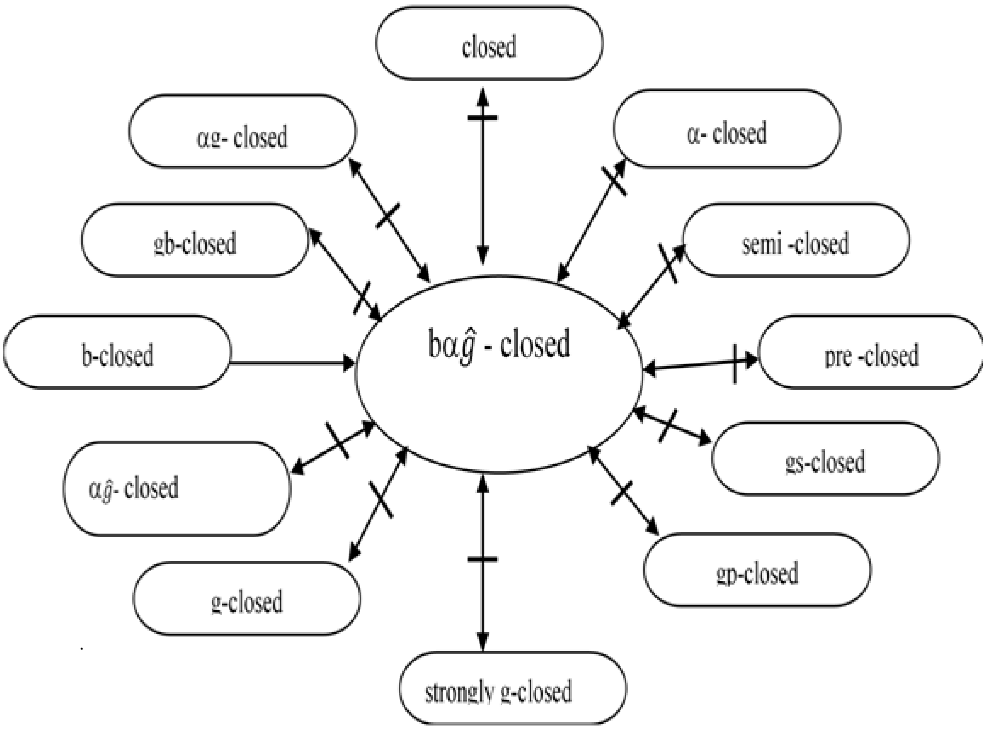
Example 3.2.4: Let X = {a, b, c}, τ = {X, ϕ, {a}, {a, b}}, and A = {b}, then bcl (A) - A= ϕ, which does not contain any non-empty $\alpha\hat{g}$- closed set. But A is not b$\alpha\hat{g}$- closed set of (X, τ).

Corollary 3.2.4: Let A be a b$\alpha\hat{g}$-closed set of (X, τ), then A is b- closed if and only if bcl (A) - A is $\alpha\hat{g}$ closed.

Theorem 3.2.5: If A is a bα$\hat{g}$- closed set in a space (X, τ) and A ⊆ B ⊆ bcl(A), then B is also a bα$\hat{g}$- closed set.

Proof: Let U be a α$\hat{g}$- open set such that B ⊆ U, then A ⊆ U. Since A is bα$\hat{g}$- closed, bcl(A) ⊆ U. and B ⊆ bcl(A) implies bcl(B) ⊆ bcl (bcl(A)) = bcl(A) ⊆ U. Hence B is also a bα$\hat{g}$- closed set.

Relationships of bα$\hat{g}$-closed sets with other closed sets are represented by the following diagram



In the above diagram, A→B denotes, A implies B. A ←+→ B represents, A and B are independent. A ←+→B denotes, B implies A, but A does not imply B. A ←+→ B means, B does not imply A, but A implies B.

3.3 bα$\hat{g}$ −Continuous Functions:

We introduce the following definition.

Definition 3.3.1: A function f : (X, τ) → (Y,σ) is called bα$\hat{g}$- continuous if $f^{-1}(V)$ is bα$\hat{g}$- closed set of (X, τ) for every closed set V of (Y, σ).

Theorem 3.3.1: Every continuous map f: (X, τ) → (Y,σ) is bα$\hat{g}$- continuous but not conversely.

Proof: Let V be a closed set in (Y, σ), then f^{-1} (V) is closed set in (X, τ) . By Theorem 3.1.3, $f^{-1}(V)$ is bα$\hat{g}$ - closed.

Example 3.3.1: X = {a, b, c} = Y, τ = {X, ϕ, {a}}, σ = {Y, ϕ, {a, b},{b, c}, {b}}. In X, the bα$\hat{g}$- Closed sets are {X, φ, {b}, {c}, {b, c}}. Define f: (X, τ) → (Y, σ) by f(a) = b, f(b) = a ,f(c) = c**.** Then f is *b*α$\hat{g}$- continuous but not continuous, for the closed set **{**a} in (Y, σ), $f^{-1}(a)$ = b is bα$\hat{g}$- closed in (X, τ), but not closed in (X, τ).

Theorem 3.3.2: Every α- continuous map f is bα$\hat{g}$- continuous but not conversely.

Proof: Let f: (X, τ) → (Y, σ) be α - continuous. Let A be closed in (Y, σ), then f^{-1} (A) is α-closed in (X, τ).

By Theorem 3.1.4, f^{-1} (A) is bα$\hat{g}$ closed. Hence f is bα$\hat{g}$ - continuous.

Example 3.3.2: X = {a, b, c} = Y,τ = {X, ϕ, {a}, {b}, {a, b}}, σ = {Y, ϕ, {a, c}}. In X, the bα$\hat{g}$- Closed sets are {X, φ, {a}, {b}, {c}, {a, c},{b, c}} and α-closed sets are {X, φ, {c}, {a, c},{b, c}}. Let f: (X, τ) → (Y, σ) be the identity map, then f is bα$\hat{g}$- continuous, but not α- continuous. For the closed set {b} in (Y,σ), f^{-1} (b) is bα$\hat{g}$- closed in (X, τ), but not α- closed in (X, τ).

Theorem 3.3.3: Every b-continuous map f is bα$\hat{g}$- continuous.

Proof: Let V be a closed set in (Y, σ), then f^{-1} (V) is b- closed in (X, τ). By Theorem 3.1.6, f^{-1} (V) is bα$\hat{g}$ - closed.

Theorem 3.3.4: Every pre-continuous map f is bα$\hat{g}$- continuous but not conversely.

Proof: The first assertion follows by Theorem 3.1.1.

Example 3.3.3: X= {a, b, c} = Y, τ ={X, ϕ, {a}, {b}, {a, b}}, σ = {Y, ϕ, {a}, {b, c}}. In X, the bα$\hat{g}$- closed sets are X, φ, {a}, {b}, {c}, {a, c}, {b, c} and pre-closed sets are X, φ, {c}, {b, c}, {a, c}. Let f: (X, τ) → (Y, σ) be the identity map, then f is bα$\hat{g}$- continuous but not pre- continuous. For the closed set {a} in (Y,σ), f^{-1} {a} = a is bα$\hat{g}$ closed in (X, τ), but not pre-closed in (X, τ).

Theorem 3.3.5: Every semi- continuous map f is bα$\hat{g}$- continuous but not conversely.

***Proof*:** The first part of the Theorem follows from Theorem 3.1.2.

Example 3.3.4: X = {a, b, c} = Y, τ = {X, ϕ, {a}, {b, c}}, σ = {Y, ϕ, {a, c}}.

In X, all the subsets are bα$\hat{g}$- closed sets and semi-closed sets are {X, φ, {a}, {b, c}}. Let f: (X, τ) → (Y, σ) be the identity map. Then f is bα$\hat{g}$ -continuous but not semi continuous. For the closed set {b} in (Y, σ), f^{-1} {b} = b is bα$\hat{g}$-closed in (X, τ) but not semi-closed in (X, τ).

Theorem 3.3.6: Every bα$\hat{g}$- continuous map is gb- continuous but not conversely.

Proof: Let V be closed in (Y, σ), then f^{-1} (V) is bα$\hat{g}$-closed in (X, τ). According to Theorem 3.1.7, f^{-1}(V) is gb – closed. Hence every bα$\hat{g}$ -continuous map is gb- continuous. The converse of the above Theorem need not be true.

Example 3.3.5: Let X = {a, b, c} = Y, τ = {X, ϕ, {a}}, σ = {Y, ϕ, {a}, {a, b}}.In X , the bα$\hat{g}$- closed sets are {X, φ, {b}, {c}, {b, c}} and gb-closed sets are {X, φ, {b}, {c}, {a, b}, {b, c}, {a, c}}. Define f : (X, τ) → (Y,σ) by f (a) = b, f (b) = c, f(c) = a . For the closed set {b, c} in {Y,σ}, f^{-1} {b, c} = {a, b} is gb- closed, but not bα$\hat{g}$- closed in (X, τ).

Theorem 3.3.7: Composition of two bα$\hat{g}$- continuous mapping need not be bα$\hat{g}$- continuous.

***Proof:* Example 3.3.6:** Let X= {a, b, c}, τ ={X, ϕ { a, c}} , σ = {X, ϕ,{a},{b, c}}, η = { X, ϕ,{a}}. The bα$\hat{g}$- closed sets in (X, τ) are {X, φ, {a}, {b}, {c}, {a, b}, {b, c}} and in (X, σ), bα$\hat{g}$- closed sets are all the subsets of X.

Define f: (X, τ) → (X, σ) by f(a) = a, f(b) =c, f(c) = b, and g : (X, σ) → (X, η) by g(a) = c, g(b) = b, g(c) = a,. Then both f and g are bα$\hat{g}$ - continuous. For the closed set {b, c} in (X, η), $(gof)^{-1}$ {b, c} = {a, c} is not bα$\hat{g}$- closed in (X, τ). Hence composition of two bα$\hat{g}$-continuous mapping need not be bα$\hat{g}$ -continuous.

Definition 3.3.2: A function f: (X, τ) → (Y, σ) is called a bα$\hat{g}$ - irresolute if f^{-1} (V) is bα$\hat{g}$ -closed in (X, τ) for every bα$\hat{g}$- closed set V of (Y, σ).

Theorem 3.3.8: Let f: (X, τ) → (Y, σ) and h : (Y, σ) → (Z, η) be any two functions. Then

(i) hof is bα$\hat{g}$- continuous if h is continuous and f is bα$\hat{g}$ -continuous.
(ii) hof is bα$\hat{g}$ - irresolute if h and f are bα$\hat{g}$- irresolute.
(iii) hof is bα$\hat{g}$- continuous if h is bα$\hat{g}$- continuous and f is bα$\hat{g}$- irresolute.

Proof: The statements in (i), (ii) and (iii) easily follow from the respective definitions.

3.4. bα$\hat{g}$- open maps and homeomorphism

We introduce the following definitions.

Definition 3.4.1: A map f : (X, τ) → (Y, σ) is called a bα$\hat{g}$ - open map if f (U) is bα$\hat{g}$ open in (Y, σ) for every open set U of (X, τ).

Definition 3.4.2: A bijection f:(X, τ) $\rightarrow$ (Y, σ) is called a b$\alpha\hat{g}$- homeomorphism if f is b$\alpha\hat{g}$-continuous and b$\alpha\hat{g}$- open.

Theorem 3.4.1: Every open map is b$\alpha\hat{g}$- open map but not conversely.

Proof: Let f: (X, τ) $\rightarrow$ (Y, σ) be an open map. Let U be an open set in (X, τ), then f (U) be an open set in (Y, σ). By Theorem 3.1.3, f is b$\alpha\hat{g}$- open map.

Example 3.4.1: Let X = {a, b, c} = Y, τ = {X, ϕ, {a}, {b}, {a, b}}, σ = {Y, φ, {a}, {b, c}}. Let f: (X, τ) $\rightarrow$ (Y, σ) be the identity map. For the open set {b} in (X, τ), f(b) = {b} is b$\alpha\hat{g}$- open in (Y, σ) **.** But f(b) = b is not open in (Y, σ) and f is not an open map.

Theorem 3.4.2: Every b$\alpha\hat{g}$ open map is a gb- open map.

Proof: Let f: (X, τ) $\rightarrow$ (Y, σ) be a map. Let U be an open set in (X, τ), then f(U) is b$\alpha\hat{g}$ - open in (Y, σ). By Theorem 3.1.7, f (U) is gb-open. Hence every b$\alpha\hat{g}$_open map is gb-open map. The converse of the above Theorem is not true.

Example 3.4.2: Let X = {a, b, c} = Y, τ = {X, ϕ, {a},{b, c}}, σ = {Y, ϕ, {a}, {a, b}}. Define f: (X, τ) $\rightarrow$ (Y,σ) by f (a) = b, f (b) = c, f (c) = a. For the open set {a} in (X, τ), f (a) = b is gb- open in (Y, σ). But f(a) = b is not b$\alpha\hat{g}$ open in (Y, σ) and hence f is not a b$\alpha\hat{g}$- open map.

Theorem 3.4.3: Let A ⇒ B represents, A implies B but not conversely. Let f :(X, τ) $\rightarrow$ (Y,σ) be a mapping. Then f is homeomorphism ⇒ f is b$\alpha\hat{g}$ - homeomorphism ⇒ f is **gb**- homeomorphism.

Proof: The first relation holds by the definition of homeomorphism and Theorem 3.4.1.

The second relation holds by the following Theorems, namely, every b$\alpha\hat{g}$- continuous is gb-continuous and every b$\alpha\hat{g}$ open map is gb-open map.

The fact that the converse of the above implications do not hold is evident from the following examples.

Example 3.4.3: Let X = {a, b, c}, τ = {X, ϕ, {a}}, σ = {X, ϕ, {a, b}, {b, c}, {b}}and f: (X, τ) $\rightarrow$ (X, σ).

By Example 3.3.1, f is b$\alpha\hat{g}$- continuous, but not continuous. Let τ = {X, ϕ, {a},{b},{a, b}}, σ = {X, ϕ, {a},{b, c}}

By Example 3.4.1, f is b$\alpha\hat{g}$ -open map but not an open map. Thus, f is b$\alpha\hat{g}$ - homeomorphism ≠>f is homeomorphism.

Let τ = {X, ϕ, {a}}, σ = (X, ϕ, {a}, {a, b}}. By Example 3.3.5, f is gb- continuous but not b$\alpha\hat{g}$- continuous.

Let τ = {X, ϕ, {a},{b, c}}, σ = (X, ϕ,{a},{a, b}}. By example 3.4.2, f is gb- open map but not b$\alpha\hat{g}$- open map. f is gb- homeomorphism ≠>f is b$\alpha\hat{g}$- homeomorphism.

3.5 Applications of b$\alpha\hat{g}$**- closed sets**

As an application of b$\alpha\hat{g}$ - closed sets we introduce three new spaces namely, $T^{c}_{b\alpha\hat{g}}$ -space, $T^{gb}_{b\alpha\hat{g}}$-space and $T^{b\hat{g}}_{b\alpha\hat{g}}$ - space.

Definition 3.5.1: A topological space (X, τ) is called

1. a $T^{c}_{b\alpha\hat{g}}$ - space if every b$\alpha\hat{g}$ - closed set is closed.
2. a $T^{gb}_{b\alpha\hat{g}}$ - space if every gb- closed set is b$\alpha\hat{g}$ - closed.
3. a $T^{b\hat{g}}_{b\alpha\hat{g}}$ - space if every b$\hat{g}$ - closed set is b$\alpha\hat{g}$- closed.

Theorem 3.5.1: Every T_{gs} - space is a $T^{gb}_{b\alpha\hat{g}}$ - space.

Proof: Let A be gb-closed. In T_{gs} - space, A is b – closed. By Theorem 3.1.6, A is b$\alpha\hat{g}$- closed. Hence A is in $T^{gb}_{b\alpha\hat{g}}$ - space.

Theorem 3.5.2: Every $T_{b\hat{g}}$ - space is $T^{b\hat{g}}_{b\alpha\hat{g}}$- space.

Proof: Let A be b$\hat{g}$ - closed. In $T_{b\hat{g}}$ - space, A is b -closed and hence b$\alpha\hat{g}$ −closed.

Theorem 3.5.3: If (X, τ) is a $T^{gb}_{b\alpha\hat{g}}$ - space and a $T^{c}_{b\alpha\hat{g}}$ - space, then it is a T_{gs} - space, the converse need not hold.

Proof: Let A be gb-closed, then in a $T^{gb}_{b\alpha\hat{g}}$ - space, A is b$\alpha\hat{g}$ - closed. In $T^{c}_{b\alpha\hat{g}}$ - space, A is closed and hence b - closed implies A is in T_{gs} - space.

Example 3.5.1: Let X = {a, b, c} τ = {X, ϕ, {a},{b},{a, b}}. In (X, τ), the gb-closed sets are {X, φ, {a}, {b}, {c}, {b, c}, {a, c}} and b- closed sets are {X, ϕ, {a}, {b}, {c},{b, c},{a, c}}. So, every gb- closed set is b-closed set. Hence X is a T_{gs}- space. The closed sets are{X, ϕ, {c},{b, c},{a, c}}. The b$\alpha\hat{g}$ −closed sets are {X, φ, {a}, {b}, {c}, {b, c}, {a, c}}. Since {a} is b$\alpha\hat{g}$ -closed but not closed, (X, τ) is not a $T^{c}_{b\alpha\hat{g}}$ - space.

Theorem 3.5.4: If (X, τ) is a $T^{b\hat{g}}_{b\alpha\hat{g}}$ - space and a $T^{c}_{b\alpha\hat{g}}$ - space, then it is a $T_{\alpha\hat{g}}$ - space, but not conversely.

Proof: Let A be $\alpha\hat{g}$closed, then it is b$\hat{g}$- closed and in a $T^{b\hat{g}}_{b\alpha\hat{g}}$ - space, A is b$\alpha\hat{g}$- closed set. In $T^{c}_{b\alpha\hat{g}}$ - space A is α- closed.

Example 3.5.2: Let X = {a, b, c} τ = {X, ϕ, {a},{b},{a, b}}. The $\alpha\hat{g}$-closed sets are X, ϕ, {c},{b, c},{a, c}. The α - closed sets are X, ϕ, {c},{b, c},{a, c}. In (X, τ), the $\alpha\hat{g}$-closed sets are the same as α-closed sets. So, it is a $T_{\alpha\hat{g}}$ - space. But not $T^{c}_{b\alpha\hat{g}}$ - space, since {b} is b$\alpha\hat{g}$ closed but not closed.

Corollary 3.5.5: If (X, τ) is a $T_{b\hat{g}}$ - space and a $T^{c}_{b\alpha\hat{g}}$ - space, then it is a $T_{\alpha\hat{g}}$ - space.

Proof: The assertion follows by Theorem 3.5.2 and Theorem 3.5.4.

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