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A STUDY ON PROPAGATION OF LOVE WAVES IN MICRO-ISOTROPIC, MICRO-ELASTIC LAYERED MEDIA

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ABSTRACT

In this article the Love wave propagation in a micro-isotropic, micro-elastic solid lying under another micro-isotropic, micro-elastic layer is studied. We found some additional waves which are not found in the classical theory of elasticity. The classical and micropolar results corresponding to this problem are obtained as particular cases.

Keywords: *Micro-isotropic, Micro-elastic, Layered Media.*

INTRODUCTION

The theory of micro-isotropic, micro-elastic materials is the simplified theory of micromorphic materials [1]. Koh[3] developed this theory by extending the concept of coincidence of principal directions of stress and strain in classical elasticity to the micro-elastic materials and assuming micro-isotropy. Assuming the micro-isotropy the frequency equations of the waves simplified considerably, but still it retains the characteristic features of the micromorphic model.

When surface waves are studied in layered medium these waves exhibit new features. They were shown to have the particle motion parallel to the surface and perpendicular to direction of propagation. A.E.H.Love [6] studied these waves comprehensively in the case of an elastic solid half space covered by a single solid layer. Since then these waves are known as Love waves. These waves found immediate applications in view of the large transverse displacements found in earth tremors. The existence of large transverse displacements was explained by Love and showed that these waves consisted of horizontally polarized shear waves trapped in superficial layer and propagated by multiple total reflections. A number of authors [5, 9, 10] studied the Love wave propagation assuming the velocity, rigidity and density to be functions of depth only.

Press et al[2] studied the propagation of Love waves in classical layered media. Mrithyunjaya Rao, Parameswara Rao and Kesava Rao [7] studied the propagation of Love waves in Micropolar layered media. Poonam Khurana and Anil K.Vashisth [8] studied the propagation of Love wave in a prestressed medium. S.Kundu, S.Gupta, A.Chattopadhyay and D.K.Majhi [4] studied the propagation of these waves in porous rigid layer lying over an initially stressed half-space. In this article the propagation of Love waves in a micro-isotropic, micro-elastic solid lying under another micro-isotropic, micro-elastic layer is studied. We found some additional waves which are not found in the classical theory of elasticity. The classical [2] and micropolar [7] results corresponding to this problem are obtained as particular cases of it.

NOTATION AND FORMULAE

Using the notation of Eringen the necessary equations for the formulation of this problem are listed below:

The strain measure e_{kl} is given by

$$e_{kl} = \frac{1}{2}\left(u_{l,k} + u_{k,l}\right) \tag{1}$$

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The equations of motion for the micro-isotropic, micro-elastic solid are given by

$$(A_1 + A_2 - A_3)u_{p,pm} + (A_2 + A_3)u_{m,pp} + 2A_3 \in_{pkm} \phi_{p,k} + \rho f_m = \rho \frac{\partial^2 u_m}{\partial t^2} \tag{2}$$

$$2B_3\phi_{p,mm} + 2(B_3 + B_5)\phi_{m,mp} - 4A_3(r_p + \phi_p) - \rho l_p = \rho j \frac{\partial^2 \phi_p}{\partial t^2} \tag{3}$$

$$B_1\phi_{pp,kk}\delta_{ij} + 2B_2\phi_{(ij),kk} - A_4\phi_{pp}\delta_{ij} - 2A_5\phi_{(ij)} + \rho f_{(ij)} = \frac{1}{2}\rho j \frac{\partial^2 \phi_{(ij)}}{\partial t^2} \tag{4}$$

The constitutive equations for this material are

$$t_{(km)} = A_1 e_{pp}\delta_{km} + 2A_2 e_{km} \tag{5}$$

$$t_{[km]} = \sigma_{[km]} = 2A_3 \in_{pkm} (r_p + \phi_p) \tag{6}$$

$$\sigma_{(km)} = -A_4\phi_{pp}\delta_{km} - 2A_5\phi_{(km)} \tag{7}$$

$$m_{kl} = -2\left(B_3\phi_{l,k} + B_4\phi_{k,l} + B_5\phi_{(mn),k}\delta_{kl}\right) \tag{8}$$

$$t_{k(mn)} = B_1\phi_{pp,k}\delta_{mn} + 2B_2\phi_{(mn),k} \tag{9}$$

where $$\phi_p = \frac{1}{2} \in_{pkm} \phi_{km} \tag{10}$$

$$r_p = \frac{1}{2} \in_{pkm} u_{m,k} \tag{11}$$

FORMULATION OF THE PROBLEM

We consider the waves which are propagating in the plane x=0 with an amplitude decay in the z-direction taking the origin at the free surface. The geometry of it is shown in fig.1.

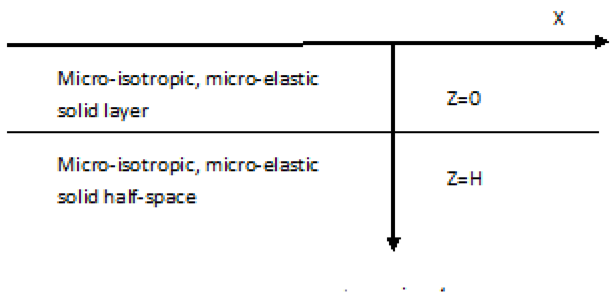

Fig.1

Now the displacement and micro-rotation components are given by

$u = w = 0, \quad v = v(x,z,t);$

$\phi_1 = \phi_1(x,z,t), \quad \phi_2 = 0, \quad \phi_3 = \phi_3(x,z,t)$

Now the equations of motion (2) and (3) become

$$(A_2 + A_3)\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial z^2}\right) + 2A_3\left(-\frac{\partial \phi_1}{\partial z} + \frac{\partial \phi_3}{\partial x}\right) - \rho\frac{\partial^2 v}{\partial t^2} = 0$$

$$2B_3\left(\frac{\partial^2 \phi_1}{\partial x^2} + \frac{\partial^2 \phi_1}{\partial z^2}\right) + 2(B_4 + B_5)\left(\frac{\partial^2 \phi_1}{\partial x^2} + \frac{\partial^2 \phi_3}{\partial z \partial x}\right) + 2A_3\frac{\partial v}{\partial z} - 4A_3\phi_1 - \rho j\frac{\partial^2 \phi_1}{\partial t^2} = 0 \tag{12}$$

$$2B_3\left(\frac{\partial^2 \phi_3}{\partial x^2} + \frac{\partial^2 \phi_3}{\partial z^2}\right) + 2(B_4 + B_5)\left(\frac{\partial^2 \phi_1}{\partial x \partial z} + \frac{\partial^2 \phi_3}{\partial z^2}\right) - 2A_3\frac{\partial v}{\partial x} - 4A_3\phi_3 - \rho j\frac{\partial^2 \phi_3}{\partial t^2} = 0$$

Since the propagation is in the xz-plane having amplitude decay in z-direction, we seek the solutions of (12) in the form

$$v = A\,exp(-mz + ik(x - ct))$$

$$\phi_1 = B\,exp(-mz + ik(x - ct)) \tag{13}$$

$$\phi_3 = iC\,exp\,(-mz + ik(x - ct))$$

where A,B and C are constants.

Substituting (13) in (12) we obtain

$$(A_2 + A_3)(-k^2A + m^2A) + 2A_3(mB - kC) + \rho k^2c^2A = 0$$

$$2B_3(-k^2B + m^2B) + 2(B_4 + B_5)(-k^2B + mkC) - 2A_3mA - 4A_3B + \rho jk^2c^2B = 0 \tag{14}$$

$$2B_3(-k^2C + m^2C) + 2(B_4 + B_5)(-mkB + m^2C) - 2A_3mA - 4A_3C + \rho jk^2c^2C = 0$$

A non-trivial solution for A,B,C exists if and only if the determinant of the coefficients is zero. Eliminating A, B and C we have

$$[2(B_3 + B_4 + B_5)(m^2 - k^2) - 4A_3 + \rho jk^2c^2]\begin{Bmatrix} [(A_2 + A_3)(m^2 - k^2) + \rho k^2c^2]\begin{bmatrix} 2B_3(m^2 - k^2) \\ -4A_3 + \rho jk^2c^2 \end{bmatrix} \\ +4A_3^2(m^2 + k^2) \end{Bmatrix} = 0$$

A set of approximate roots is obtained by neglecting A_3^2 term, since A_3^2 is negligibly small when compared with A_3, and they are

$$m_1^{(i)} = \sqrt{\frac{\rho^{(i)}c^2}{A_2^{(i)}}\left(1 - \frac{A_3^{(i)}}{A_2^{(i)}}\right) - 1}$$

$$m_2^{(i)} = \sqrt{\frac{\rho^{(i)}j^{(i)}c^2}{2(B_3^{(i)} + B_4^{(i)} + B_5^{(i)})} - \frac{2A_3^{(i)}}{k^2\left(B_3^{(i)} + B_4^{(i)} + B_5^{(i)}\right)} - 1}$$

$$m_3^{(i)} = \sqrt{\frac{\rho^{(i)}j^{(i)}c^2}{2B_3^{(i)}} - \frac{2A_3^{(i)}}{k^2B_3^{(i)}} - 1} \quad \text{for i=1,2} \tag{15}$$

where $A_p^{(1)}, B_q^{(1)}$ (p=2,3;q=3,4,5) denotes the elastic constants pertaining to solid layer and $A_p^{(2)}, B_q^{(2)}$ (p=2,3;q=3,4,5) denotes elastic constants pertaining to half space. We seek the solutions of v, ϕ_1, ϕ_3 in the form

$$v^{(1)} = \left\{Aexp\left(ikm_1^{(1)}z\right) + Bexp\left(-ikm_1^{(1)}z\right)\right\}\exp[-\mathrm{ik}(x - ct)]$$

$$\phi_1^{(1)} = \left\{Cexp\left(ikm_2^{(1)}z\right) + Dexp\left(-ikm_2^{(1)}z\right)\right\}\exp[-\mathrm{ik}(x - ct)]$$

$$\phi_3^{(1)} = \left\{Eexp\left(ikm_3^{(1)}z\right) + Fexp\left(-ikm_3^{(1)}z\right)\right\}\exp[-\mathrm{ik}(x - ct)]$$

$$v^{(2)} = Gexp\left(-ikm_1^{(2)}z\right)\exp[-\mathrm{ik}(x - ct)]$$

$$\phi_1^{(2)} = Jexp\left(-ikm_2^{(2)}z\right)\exp[-\mathrm{ik}(x - ct)]$$

$$\phi_3{}^{(2)} = Iexp\left(-ikm_3^{(2)}z\right)\exp[-\mathrm{ik}(x - ct)] \tag{16}$$

$$\phi_{22}^{(i)} = K_1^{(i)}exp\left(-iks^{(i)}z\right)\exp[-\mathrm{ik}(x - ct)]$$

$$\phi_{12}^{(i)} = K_2^{(i)}exp\left(-iks^{(i)}z\right)\exp[-\mathrm{ik}(x - ct)]$$

$$\phi_{32}^{(i)} = K_3^{(i)}exp\left(-iks^{(i)}z\right)\exp[-\mathrm{ik}(x - ct)] \tag{17}$$

where $s^{(i)} = \sqrt{\frac{A_5^{(i)}}{B_2^{(i)}} + (1 - \frac{\rho^{(i)}j^{(i)}c^2}{4B_2^{(i)}})k^2}$ for i=1,2

The boundary conditions for the problem under consideration will be

(i). at free surface i.e., at z=0

$$t_{zy}^{(1)} = m_{zz}^{(1)} = m_{zx}^{(1)} = 0$$

(ii). at the interface of two layers, i.e., at z=H

$$v^{(1)} = v^{(2)}, \quad \phi_1^{(1)} = \phi_1^{(2)}, \quad \phi_3^{(1)} = \phi_3^{(2)}$$

$$t_{zy}^{(1)} = t_{zy}^{(2)}, \quad m_{zz}^{(1)} = m_{zz}^{(2)}, \quad m_{zx}^{(1)} = m_{zx}^{(2)} \tag{18}$$

(iii). at the free surface z=0

$$t_{z(zy)}^{(1)} = t_{z(zz)}^{(1)} = t_{z(zx)}^{(1)} = 0$$

$$\phi_{(12)}^{(1)} = \phi_{(12)}^{(2)}, \qquad \phi_{(22)}^{(1)} = \phi_{(22)}^{(2)}, \quad \phi_{(23)}^{(1)} = \phi_{(23)}^{(2)} \tag{19}$$

By substituting (16) in (18) we obtain the nine equations as follows

$$\frac{1}{2}\left(A_2^{(1)} + A_3^{(1)}\right)ikm_1^{(1)}(A - B) - A_3^{(1)}(C + D) = 0$$

$$(E - F)m_3^{(1)}\left(B_3^{(1)} + B_4^{(1)} + B_5^{(1)}\right) + B_5^{(1)}(C + D) = 0$$

$$B_3^{(1)}m_2^{(1)}(C - D) - B_4^{(1)}(E + F) = 0$$

$$Aexp\left(ikm_1^{(1)}H\right) + Bexp\left(-ikm_1^{(1)}H\right) - Gexp\left(-ikm_1^{(2)}H\right) = 0$$

$$Cexp\left(ikm_2^{(1)}H\right) + Dexp\left(-ikm_2^{(1)}H\right) - Jexp\left(-ikm_2^{(2)}H\right) = 0$$

$$Eexp\left(ikm_3^{(1)}H\right) + Fexp\left(-ikm_3^{(1)}H\right) - Iexp\left(-ikm_3^{(2)}H\right) = 0$$

$$\frac{ikm_1^{(1)}}{2}\left(A_2^{(1)} + A_3^{(1)}\right)\left(Aexp\left(ikm_1^{(1)}H\right) - Bexp\left(-ikm_1^{(1)}H\right)\right) - A_3^{(1)}\left(Cexp\left(ikm_2^{(1)}H\right) + Dexp\left(-ikm_2^{(1)}H\right)\right) + \frac{ikm_1^{(2)}}{2}\left(A_2^{(2)} + A_3^{(2)}\right)Gexp\left(-ikm_1^{(2)}H\right) + A_3^{(2)}Jexp\left(-ikm_2^{(2)}H\right) = 0$$

$$\left(B_3^{(1)} + B_4^{(1)} + B_5^{(1)}\right)m_3^{(1)}\left(Eexp\left(ikm_3^{(1)}H\right) - Fexp\left(-ikm_3^{(1)}H\right)\right) + m_3^{(2)}\left(B_3^{(2)} + B_4^{(2)} + B_5^{(2)}\right)Iexp\left(-ikm_3^{(2)}H\right) - B_5^{(1)}\left(Cexp\left(ikm_2^{(1)}H\right) + Dexp\left(-ikm_2^{(1)}H\right)\right) + B_5^{(2)}Jexp\left(-ikm_2^{(2)}H\right) = 0$$

$$B_3^{(1)}m_2^{(1)}\left(Cexp\left(ikm_2^{(1)}H\right) - Dexp\left(-ikm_2^{(1)}H\right)\right) - B_4^{(1)}\left(Eexp\left(ikm_3^{(1)}H\right) + Fexp\left(-ikm_3^{(1)}H\right)\right) + B_3^{(2)}m_2^{(2)}Jexp\left(-ikm_2^{(2)}H\right) + B_4^{(2)}Iexp\left(-ikm_3^{(2)}H\right) = 0 \tag{20}$$

A non-zero solution for these equations exist if the determinant of these nine equations is zero.
i.e., $\left|P_{ij}\right| = 0$ for i, j=1, 2,…..,9

where

$P_{11} = \frac{ik}{2}m_1^{(1)}\left(A_2^{(1)} + A_3^{(1)}\right)$ $\quad$ $P_{12} = -\frac{ik}{2}m_1^{(1)}\left(A_2^{(1)} + A_3^{(1)}\right)$

$P_{13} = P_{14} = -A_3^{(1)}$ $\quad$ $P_{15} = P_{16} = P_{17} = P_{18} = P_{19} = 0$

$P_{21} = P_{22} = 0$ $\quad$ $P_{23} = P_{24} = B_5^{(1)}$

$P_{25} = m_3^{(1)}\left(B_3^{(1)} + B_4^{(1)} + B_5^{(1)}\right)$ $\quad$ $P_{26} = -m_3^{(1)}\left(B_3^{(1)} + B_4^{(1)} + B_5^{(1)}\right)$

$P_{27} = P_{28} = P_{29} = 0$ $\quad$ $P_{31} = P_{32} = 0$

$P_{33} = B_3^{(1)}m_2^{(1)}$ $\quad$ $P_{34} = -B_3^{(1)}m_2^{(1)}$

$P_{35} = P_{36} = -B_4^{(1)}$ $\quad$ $P_{37} = P_{38} = P_{39} = 0$

$P_{41} = iexp\left(ikm_1^{(1)}H\right)$ $\quad$ $P_{42} = iexp\left(-ikm_1^{(1)}H\right)$

$P_{43} = P_{44} = P_{45} = P_{46} = 0$ $\quad$ $P_{47} = -iexp\left(-ikm_1^{(2)}H\right)$

$P_{48} = P_{49} = 0$ $\quad$ $P_{51} = P_{52} = 0$

$P_{53} = exp\left(ikm_2^{(1)}H\right)$ $\quad$ $P_{54} = exp\left(-ikm_2^{(1)}H\right)$

$P_{55} = P_{56} = P_{57} = 0$ $\quad$ $P_{58} = -exp\left(-ikm_2^{(2)}H\right)$

$P_{59} = 0$ $\quad$ $P_{61} = P_{62} = P_{63} = P_{64} = 0$

$P_{65} = exp\left(ikm_3^{(1)}H\right)$ $\quad$ $P_{66} = exp\left(-ikm_3^{(1)}H\right)$

$P_{67} = P_{68} = 0$ $\quad$ $P_{69} = -exp\left(-ikm_3^{(2)}H\right)$

$P_{71} = \frac{ik}{2}m_1^{(1)}\left(A_2^{(1)} + A_3^{(1)}\right)exp\left(ikm_1^{(1)}H\right)$ $\quad$ $P_{72} = -\frac{ik}{2}m_1^{(1)}\left(A_2^{(1)} + A_3^{(1)}\right)exp\left(-ikm_1^{(1)}H\right)$

$P_{73} = -A_3^{(1)}exp\left(ikm_2^{(1)}H\right)$ $\quad$ $P_{74} = -A_3^{(1)}exp\left(-ikm_2^{(1)}H\right)$

$P_{75} = P_{76} = 0$ $\quad$ $P_{77} = \frac{ik}{2}m_1^{(2)}\left(A_2^{(2)} + A_3^{(2)}\right)exp\left(-ikm_1^{(2)}H\right)$

$P_{78} = A_3^{(2)}exp\left(-ikm_2^{(2)}H\right)$ $\quad$ $P_{79} = 0$

$P_{81} = P_{82} = 0$ $\quad$ $P_{83} = -B_5^{(1)}exp\left(ikm_2^{(1)}H\right)$

$P_{84} = -B_5^{(1)} exp\left(-ikm_2^{(1)} H\right)$ $\quad P_{85} = \left(B_3^{(1)} + B_4^{(1)} + B_5^{(1)}\right) m_3^{(1)} exp\left(ikm_3^{(1)} H\right)$

$P_{86} = -\left(B_3^{(1)} + B_4^{(1)} + B_5^{(1)}\right) m_3^{(1)} exp\left(-ikm_3^{(1)} H\right)$ $\quad P_{87} = 0$

$P_{88} = B_5^{(2)} exp\left(-ikm_2^{(2)} H\right)$ $\quad P_{89} = \left(B_3^{(2)} + B_4^{(2)} + B_5^{(2)}\right) m_3^{(2)} exp\left(-ikm_3^{(2)} H\right)$

$P_{91} = P_{92} = 0$ $\quad P_{93} = B_3^{(1)} m_2^{(1)} exp\left(ikm_2^{(1)} H\right)$

$P_{94} = -B_3^{(1)} m_2^{(1)} exp\left(-ikm_2^{(1)} H\right)$ $\quad P_{95} = -B_4^{(1)} exp\left(ikm_3^{(1)} H\right)$

$P_{96} = -B_4^{(1)} exp\left(-ikm_3^{(1)} H\right)$ $\quad P_{97} = 0$

$P_{98} = B_3^{(2)} m_2^{(2)} exp\left(-ikm_2^{(2)} H\right)$ $\quad P_{99} = B_4^{(2)} exp\left(-ikm_3^{(2)} H\right)$ (21)

By simplifying this periodic function we get two equations as follows:

$$\frac{km_1^{(1)}}{2}\left(A_2^{(1)} + A_3^{(1)}\right) \exp\left(-ikm_1^{(1)} H\right) \left[\frac{-k}{2} m_1^{(1)}\left(A_2^{(1)} + A_3^{(1)}\right) + \frac{k}{2} m_1^{(2)}\left(A_2^{(2)} + A_3^{(2)}\right)\right]$$
$$+ \frac{km_1^{(1)}}{2}\left(A_2^{(1)} + A_3^{(1)}\right) \exp\left(ikm_1^{(1)} H\right) \left[\frac{k}{2} m_1^{(1)}\left(A_2^{(1)} + A_3^{(1)}\right) + \frac{k}{2} m_1^{(2)}\left(A_2^{(2)} + A_3^{(2)}\right)\right] = 0 \quad (22)$$

and

$$-4s_1 s_3^3 m_2^{(1)} m_3^{(1)} - s_2 s_3^2 [R_1 + L_5(-R_2 + R_3)] + 2s_1 s_3 m_2^{(1)^2} m_3^{(1)^2} L_2 L_3^2 - s_2 s_3^3 (L_1 L_2 - L_6 R_4) - 2s_1^2 s_3 m_2^{(1)^2} m_3^{(1)^2} L_2 L_3^2$$
$$+\exp(\mathrm{ik}\left(-m_2^{(1)} + m_3^{(1)}\right) H)\{2s_1 s_2 s_3^2 m_2^{(1)} m_3^{(1)} L_2 L_3 + s_1 s_3 L_3 m_2^{(1)} m_3^{(1)} [R_1 - L_6(-R_2 + R_3)]$$
$$+s_1 s_3^2 L_3 m_2^{(1)} m_3^{(1)} (L_1 L_2 - L_6 R_4)\} + \exp(\mathrm{ik}\left(m_2^{(1)} - m_3^{(1)}\right) H)\{2s_1 s_3 m_2^{(1)} m_3^{(1)} L_3 [R_1 + L_5(R_2 - R_3)]$$
$$-s_1 s_3 m_2^{(1)} m_3^{(1)} L_3 [R_1 + L_5(-R_2 + R_3)] + 2s_1 s_2 s_3^2 L_2 L_3 m_2^{(1)} m_3^{(1)}\}$$
$$+\exp(\mathrm{ik}\left(m_2^{(1)} + m_3^{(1)}\right) H)\{-2s_1 s_3^2 m_2^{(1)} (L_5 R_4 - L_1 L_2) - s_1 s_3 m_2^{(1)} m_3^{(1)} L_3 [R_1 - L_5(R_2 + R_3)]\}$$
$$+(\exp(-ikm_2^{(1)}))^2 \{s_2 s_3^2 [R_1 - L_6(-R_2 + R_3)]\} + (\exp(ikm_2^{(1)}))^2 \{R_1 - L_5(R_2 + R_3)\} = 0 \quad (23)$$

where
$R_1 = L_1 L_2 s_3$ $\quad R_2 = L_3 s_3 m_3^{(1)}$
$R_3 = L_4 m_3^{(2)}$ $\quad R_4 = L_3 m_3^{(1)} + L_4 m_3^{(2)}$
$L_1 = -B_4^{(1)} + B_4^{(2)}$ $\quad L_2 = -B_5^{(1)} + B_5^{(2)}$
$L_3 = B_3^{(1)} + B_4^{(1)} + B_5^{(1)}$ $\quad L_4 = B_3^{(2)} + B_4^{(2)} + B_5^{(2)}$
$L_5 = B_3^{(1)} m_2^{(1)} + B_3^{(2)} m_2^{(2)}$ $\quad L_6 = -B_3^{(1)} m_2^{(1)} + B_3^{(2)} m_2^{(2)}$
$s_1 = B_3^{(1)}$ $\quad s_2 = B_4^{(1)}$ $\quad s_3 = B_5^{(1)}$

By simplifying equation (22) we get

$$tankm_1^{(1)} H = \frac{A_2^{(2)} + A_3^{(2)}}{A_2^{(1)} + A_3^{(1)}} \frac{\sqrt{1 - \frac{\rho^{(2)} c^2}{A_2^{(2)}}\left(1 - \frac{A_3^{(2)}}{A_2^{(2)}}\right)}}{\sqrt{\frac{\rho^{(1)} c^2}{A_2^{(1)}}\left(1 - \frac{A_3^{(1)}}{A_2^{(1)}}\right) - 1}}$$

By substituting (17) in (19) we get

$$\frac{\omega^2}{k^2} = \frac{4B_2\omega^2}{\rho j(\omega^2 - \omega^{*2})}$$

where $\omega^* = \sqrt{\frac{4A_5}{\rho j}}$ and $\omega^2 = c^2 k^2$

where ω^* is the cut off frequency.

By assuming $\varphi_{kl} = -\varphi_{lk}$, $A_2^{(i)} = \frac{\mu^{(i)}}{2}$, $A_3^{(i)} = \frac{\kappa^{(i)}}{2}$, $B_3 = \frac{\gamma}{2}$, $B_4 = \frac{\beta}{2}$, $B_5 = \frac{\alpha}{2}$ for i=1, 2 the results of [7] and by taking $\kappa \to 0$ the result of [2] can be obtained as particular cases.

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