International Journal of Mathematical Archive-5(11), 2014, 22-24
IJMA Available online through www.ijma.info ISSN 2229 – 5046

ON QUASI-BI IDEALS OF BI-NEAR SUBTRACTION SEMIGROUP

Firthous Fatima.S*[1] and Jayalakshmi.S[2]

[1]Department of Mathematics, Sadakathullah Appa College, Tirunelveli- 11, Tamil Nadu, India.

[2]Department of Mathematics,
Sri Parasakthi College for Women, Courtallam-627802, Tamil Nadu, India.

(Received On: 17-11-14; Revised & Accepted On: 30-11-14)

ABSTRACT

In this paper we introduce the notion of Quasi-bi ideals in bi-near subtraction semigroup. Also we give characterizations of Quasi -bi ideals in bi-near subtraction semigroup.

Mathematical subject classification: *06F35.*

Key words*: Quasi-ideal, Bi-regular, Bi-near subtraction semigroup, Quasi-bi ideal.*

1. INTRODUCTION

In 2007, Dheena[1] introduced Near Subtraction Algebra, Throughout his paper by a Near Subtraction Algebra, we mean a Right Near Subtraction Algebra. For basic definition one may refer to Pillz[5].Tamil chelvam and Ganesan[6] introduced the notation of Bi-ideals in Near ring Maharasi[4] introduced the notation of Quasi-bi ideals in Near ring.. Recently Firthous *et.al* [2] introduced the notation of Bi bi-ideals in Bi near subtraction semigroup. In this paper we shall obtained equivalent conditions for regularity in terms of Quasi-bi ideals.

Given two subsets A and B of X the product AB = {ab /a∈A and b∈B}. Also we define another operator "$_*$" on the class of subsets of X given by A_*B= {ab-a(a′-b)/a,a′∈A, b∈B}.

2. PRELIMINARIES

Definition 2.1: A non-empty subset X together with two binary operations "–"and"." is said to be subtraction semigroup If
(i) (X,–) is a subtraction algebra
(ii) (X, .) is a semi group (iii) x(y–z)=xy–xz and (x–y)z= xz–yz for every x, y, z∈X.

Definition 2.2: A non-empty subset X together with two binary operations "–"and "." is said to be near subtraction semigroup if
(i) (X,–) is a subtraction algebra
(ii) (X,.) is a semi group and
(iii) (x–y)z= xz–yz for every x, y, z∈X.

Definition 2.3: A non-empty subset $X=X_1\cup X_2$ together with two binary operations "–" and "." is said to be bi-near subtraction semigroup (right) if
(i) $(X_1,–,.)$ is a near-subtraction semigroup
(ii) $(X_2,–,.)$ is a subtraction semigroup

. *Corresponding Author: Firthous Fatima.S*[1]*
[1]Department of Mathematics, Sadakathullah Appa College, Tirunelveli- 11, Tamil Nadu, India.

Definition 2.4:A non-empty subset S of X Is said to be Subalgebra if $x-y\in S$ whenever $x, y\in S$.

Definition 2.5: A non-empty subset I of X is said to be Right ideal if (i) $x-y\in I$ for every $x\in I$ and $y\in X$ and (ii) $IX \subseteq I$.

Definition 2.6: A non-empty subset I of X is said to be Left ideal if (i) $x-y\in I$ for every $x\in I$ and $y\in X$ and (ii) $XI \subseteq I$.

Definition 2.7: A non-empty subset I of X is said to be ideal if (i) $IX \subseteq I$ and (ii) $XI \subseteq I$.

Definition 2.8: Let (X_1, $-$, .) be a bi-near subtraction semigroup. We say X is a Bi-regular bi-near subtraction semigroup if for any $x\in X$ their exist $y\in X$ such that $xyx=x$.

If both X_1 and X_2 are regular subtraction semigroup then the bi-near subtraction semigroup X is regular

Definition 2.9: An subalgebra Q of X is said to be Quasi-ideal if $Q\, X\cap X\, Q\cap X * Q \subseteq Q$.

3. QUASI-BI IDEALS OF BI-NEAR SUBTRACTION SEMIGROUP

Definition 3.1: A non-empty subset $Q= Q_1\cup Q_2$ of X is said to be Quasi-bi ideal, if Q_1 is Quasi-ideal in X_1 and Q_2 is ideal in X_2.

Example 3.2: Let X_1={0,a,b,c} in which "–" and "." be defined by

–	0	a	b	c
0	0	0	0	0
a	a	0	a	a
b	b	b	0	b
c	c	c	c	0

.	0	a	b	c
0	0	0	0	0
a	0	a	0	0
b	0	0	b	0
c	0	0	0	c

Then Q_1={0,b} is Quasi-ideal in X_1

Let X_2={0,a,b,1} in which "–" and "." be defined by

–	0	a	b	1
0	0	0	0	0
a	a	0	a	0
b	b	b	0	0
1	1	b	a	0

.	0	a	b	1
0	0	0	0	0
a	0	a	0	a
b	0	0	b	b
1	0	a	b	1

Then Q_2 = {0, a, b} is ideal in X_2.

Note 3.3: Obviously, every quasi-bi ideal is Bi-bi ideal in a bi-near subtraction semigroup. But the converse is not true

Example 3.4: Let X_1={0,a,b,c } in which "–" and "." be defined by

–	0	a	b	c
0	0	0	0	0
a	a	0	c	b
b	b	b	0	b
c	c	o	c	0

.	0	a	b	c
0	0	0	0	0
a	0	a	0	0
b	0	0	b	0
c	0	0	0	c

Here S_1={0,a} is bi-ideal but not Quasi-ideal in X_1

Let X_2={0,a,b,1} in which "–" and "." be defined by

–	0	a	b	1
0	0	0	0	0
a	a	0	a	0
b	b	b	0	0
1	1	b	a	0

.	0	a	b	1
0	0	0	0	0
a	0	a	0	a
b	0	0	b	b
1	0	a	b	1

Then S_2= {0, a, b} is an ideal in X_2. Hence, every Bi_ bi ideal need not be a Quasi_bi ideal.

Proposition 3.5: The set of all Quasi-bi ideals of a Bi-near subtraction semi group form a Moore System on X.

Proof: Let Q_i be a set of all Quasi-bi ideals in X. Let Q= $\cap Q_i$, $i \in I$ .Then Q= $\cap(Q_i' \cup Q_i'')$.

Since Intersection of all Q_i' are Quasi-ideal and Intersection of all Q_i'' are ideal, B is a Quasi -bi ideal of X.

Proposition 3.6: If Q is a Quasi -bi ideal of a Bi-near subtraction semi group X and S is a subalgebra of X, then $Q \cap S$ is a Quasi -bi ideal of S.

Proof: Let Q be a Quasi -bi ideal of X. Then $Q \cap S= (Q_1 \cup Q_2) \cap S = (Q_1 \cap S) \cup (Q_2 \cap S)$ .Since $Q_1 \cap S$ is a Quai-ideal of S and $Q_2 \cap S$ is an ideal of S, $B \cap S$ is a Quasi -bi ideal of S.

Theorem 3.7: Let $X=X_1 \cup X_2$ be a Bi-near subtraction semi group and let Q be a Quasi -ideal of X. Then Q is a Quasi-bi ideal of X if and only if there exist two proper subsets X_1 and X_2 of X such that
(i) $X=X_1 \cup X_2$ where X_1 and X_2 are proper subsets of X
(ii) $(Q \cap X_1)$ is a Quasi-ideal of $(X_1,-,.)$
(iii) $(Q \cap X_2)$ is a ideal of $(X_2,-,.)$

Proof: Assume that Q is a Quasi –bi ideal of X. Thus there exist two subsets Q_1 and Q_2 of Q such that $Q = Q_1 \cup Q_2$ Where Q_1 is a Quasi -ideal of X_1 and Q_2 is a ideal of X_2. Taking $Q_1= Q \cap X_1$ and $Q_2= Q \cap X_2$.

Conversely, let Q be a nonempty subset of X a satisfying conditions (i), (ii) and (iii).Hence

$$\begin{aligned}(Q \cap X_1) \cup (Q \cap X_2) &= ((Q \cap X_1) \cup Q) \cap ((Q \cap X_2) \cup X_2)\\ &= ((Q \cup Q) \cap (X_1 \cup Q)) \cap ((Q \cup X_2) \cap (X_1 \cup X_2))\\ &= (Q \cap (Q \cup X_1)) \cap ((Q \cup X_2) \cap X)\\ &= Q \cap (Q \cup X_2) \text{ (since } Q \subseteq Q \cup X_1 \text{and } Q \cup X_2 \subseteq X)\\ &= Q. \text{ (since } Q \subseteq Q \cup X_2)\end{aligned}$$

Thus, $(Q \cap X_1) \cup (Q \cap X_2) = B$. Hence, Q is a Quasi -bi-ideal of X.

Theorem 3.8: Let X be a Zero- Symmetric Bi-Near subtraction semigroup. If B is a Bi-bi ideal of X. If the element of B are bi- regular then B is a Quasi-ideal.

Proof: Let $x \in BX \cap X B$ Then $x=bx=x'b'$ for some b,b' in B and x,x' in X. Since B is bi-regular, (i.e.,) $bb_1b=b_1$ for some $b_1$in B. Now, $x = bx = (bb_1b) x = (bb_1) (b x) = (bb_1) (x'b) \in BXB \subseteq B$. Hence B is a Quasi-ideal.

Theorem 3.9: Let X be a Zero- Symmetric Bi- Near subtraction semigroup. If B is a bi-ideal of X. If the element of B are bi- regular then B is a Quasi-bi ideal.

Proof: Let $x \in B$. Since B is Bi-bi ideal, therefore $B=B_1 \cup B_2$ Where B_1 is a bi-ideal of X_1 and B_2 is a ideal of X_2 Every element of B is a Bi-regular, then every element of B_1 and B_2 is a Bi-regular. By theorem3.8, B_1 is a Quasi-ideal of X_1 and B_2 is a ideal of X_2. Hence B is a Quasi-bi ideal.

REFERENCES

1. Dheena.P and Sathesh kumar.G On Strongly Regular Near- Subtracttion semi groups, Commun. Korean Math. Soc.22 (2007), No.3, pp.323-330.
2. Firthous Fatima.S,Mahalakshmi.V, Jayalakshmi.S On Bi-bi ideal of bi-near subtraction semigroup(To appear in Advances in Algebra)
3. Mahalakshmi.V, Maharasi.S, Jayalakshmi.S Bi-ideals of Near-Subtraction semigroup, Indian Advances in Algebra Volume6, number 1(2013), pp.35-43.
4. Maharasi.S, Mahalakshmi.V, Jayalakshmi.S On Quasi--bi ideals of Near-Ring, GJMMS Volume 6, number1 (2013), pp. 35-38
5. Pilz Gunter,Near-Rings, North Holland,Amsterdam,1983
6. Tamil Chelvam.T and .Ganesan.N On Bi-Ideals Of Near-Rings, Indian- J.pure appl. Math..18(11), 1002-1005, November 1987.

Source of support: Nil, Conflict of interest: None Declared