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LAPLACE TRANSFORM TECHNIQUE ON RADIATIVE AND CHEMICAL REACTION EFFECTS OVER EXPONENTIALLY ACCELERATED VERTICAL PLATE AND TEMPERATURE WITH CONSTANT MASS FLUX

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ABSTRACT

An analytical solution of radiative and chemical effects over exponentially accelerated infinite vertical plate and temperature has been presented in the presence of constant mass flux. The dimensionless governing equations are solved using Laplace technique. The velocity, temperature, and concentration profiles are studied for different physical parameters like radiation parameter R, accelerating parameter a, thermal Grashof number Gr, mass Grashof number Gc, chemical reaction parameter K, Prandtl number Pr and Schmidt number Sc. It is observed that the velocity increases with increase in Gc, R, a and Sc. It is also observed that temperature rise with increasing a and R while concentration increases with decreasing Sc and K.

Key words: *Accelerated, Vertical plate, radiation, Chemical reaction, Heat transfer, Mass flux,*

1. INTRODUCTION

The voracious study of heat transfer and mass flux with radiative and chemical reaction effects is of great practical important to engineers and scientists because of its almost universal occurrence in many branches of science and engineering. Mazumdar and Deka [7] considered MHD flow past an impulsively started infinite vertical plate in presence of thermal radiation. Muthucumaraswamy *et al.* [8] worked on the diffusion and Heat transfer effects on exponentially accelerated vertical plate with variable temperature. Sharma *et al.* [12] studied the influence of chemical reaction and radiation on unsteady MHD free convection flow and mass transfer through viscous incompressible fluid past a heated vertical plate immersed in porous medium in presence of heat source. Chamkha [5] studied the effects of heat absorption and thermal radiation on heat transfer in a fluid particle flow past a surface in the presence of a gravity field. Muthucumaraswamy and Valliamal [9] considered first order chemical reaction on exponentially accelerated isothermal vertical plate with mass diffusion, the dimensionless governing equations are solved using laplace-transform technique. Chandrakala [3] studied thermal radiation effects on moving infinite vertical plate with uniform heat flux. Further studies were made by Chandrakala and Bhaskar [4] they considered analytically thermal radiation effects on moving infinite vertical plate with uniform heat flux. Das *et al.* [6] studied radiation effects on flow past an impulsively started vertical infinite plate. Mass transfer effects on the flow past an exponentially accelerated vertical plate with constant heat flux were considered by Basanth et al. [2]. Rajesh and Vijaya [10] presented radiation and mass transfer effects on MHD free convection flow past an exponentially accelerated vertical plate with variable temperature. Sahin and Dimbeswar [1] recently examine Laplace technique on Magnetohydrodynamica radiating and chemically reacting fluid over an infinite vertical surface. Asogwa *et al.* [11] worked on the flow past on an exponentially accelerated infinite vertical plate and temperature with variable mass diffusion. This study examines radiation and chemical reaction effects on exponentially accelerated isothermal vertical plate cum mass flux. The dimensionless governing equations are solved using Laplace transform technique. The solutions are obtained in terms of exponential and complementary error functions

This study examines radiative and chemical reaction effects over exponentially accelerated vertical plate and temperature with constant mass flux. The dimensionless governing equations are solved using Laplace transform technique. The solutions are obtained in terms of exponential and complementary error functions

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2. FORMULATION OF THE PROBLEM

Radiative and chemical effects over exponentially accelerated infinite vertical plate and temperature with constant mass flux has been considered. The x' - axis is taken along the plate in the vertically upward direction and also the y' -axis is taken normal to the plate. At $t' > 0$, the plate is accelerated with a velocity $u = u_0 \exp(at)$ in its own plane and the temperature of the plane is raise at a uniform rate to velocity, and the level of concentration near the plate is raised to C'_ω . Then under the usual Boussinesq's approximation the time dependent flow equations are momentum equation, energy equation, and mass equation respectively.

$$\frac{\partial u}{\partial t'} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta\left(T - T_\infty\right) + g\beta^*\left(C' - C'_\infty\right) \tag{1}$$

$$\frac{\partial T}{\partial t'} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y^2} - K^* C' \tag{3}$$

where u is the velocity of the fluid, T is the fluid temperature, C' is the concentration, g is gravitational constant, β and β^* are the thermal expansions of fluid and concentration , t' is the time, ρ is the fluid density, C_p is the specific heat capacity, v is the viscosity of the fluid, k is the thermal conductivity, D is the diffusion term, K^* is chemical reaction parameter and q_r is the radiative heat flux.

This research work is an extension of the work of Asogwa *et al*. [11] *International Journal of Computer Applications.* By Rosseland approximation, we assume that the temperature differences within the flow are such that T^{*4} may be expressed as a linear function of the temperature T^*. This is accomplished by expanding Taylor series about T_d^* neglecting higher order terms.

The initial and boundary conditions are;

$$\left.\begin{array}{ll} & u = 0, \quad T = T_\infty, \quad C' = C'_\infty, \quad for\,all\;\; y, t \le 0 \\ t' > 0: & u = u_0 e^{at}, T = T_\infty + \left(T_\omega - T_\infty\right)e^{at}, C' = C'_\omega \quad y = 0 \\ & u^* \to 0, T^* \to 0, C^* \to 0 \qquad as\; y \to \infty \end{array}\right\} \tag{4}$$

where the values for accelerating parameter a in the boundary conditions is not large for temperature.

The thermal radiation heat flux gradient used before by Sigel and Howell (1993) and also Beg and Ghosh (2009) which may be expressed as follows

$$-\frac{\partial q_r}{\partial y'} = 4a\sigma^*\left(T_\infty^{*4} - T^{*4}\right) \tag{5}$$

where q_r is the radiative heat flux, a is the absorption coefficient of the fluid and σ^*Is the Stefan-Boltzmann constant. We assumed that the temperature differences within the flow are sufficiently small such that T^{*4} may be expressed as a linear function of the temperature. Expanding T^{*4} about T_∞^* in a Taylor's series and neglecting higher order terms, we have

$$T^{*4} \cong 4T_\infty^{*3}T^* - 3T_\infty^{*4} \tag{6}$$

By using equation (5) and (6).equation (2) reduces to

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} + \frac{16\sigma^* T_d^3}{3a_R}\frac{\partial^2 T}{\partial y^2} \tag{7}$$

On introducing the following non-dimensional quantities

$$\left.\begin{aligned}
&U=\frac{u}{\left(\nu u_0\right)^{\frac{1}{3}}}\quad t=t'\left(\frac{u_0^2}{\nu}\right)^{\frac{1}{3}}\quad Y=y\left(\frac{u_0}{\nu^2}\right)^{\frac{1}{3}}\\
&\theta=\frac{T-T_\infty}{T_\omega-T_\infty},C=\frac{C'-C'_\infty}{C'_\omega-C'_\infty},\\
&Gr=\frac{g\beta\left(T_\omega-T_\infty\right)}{u_0},Gc=\frac{g\beta^*\left(C'_\omega-C'_\infty\right)}{u_0},\\
&\Pr=\frac{\mu C_p}{k},\ Sc=\frac{\nu}{D},\ R=\frac{4\sigma^*T_d^3}{ka_R}
\end{aligned}\right\}\tag{8}$$

Substituting the non-dimensional quantities of equation (8) into (1) to (4), leads to

$$\frac{\partial U}{\partial t}=Gr\theta+GcC+\frac{\partial^2 U}{\partial y^2}\tag{9}$$

$$\frac{\partial\theta}{\partial t}=\frac{1}{\lambda}\frac{\partial^2\theta}{\partial y^2}\tag{10}$$

$$\frac{\partial C}{\partial t}=\frac{1}{Sc}\frac{\partial^2 C}{\partial y^2}-KC\tag{11}$$

where $\lambda=\dfrac{3\Pr}{3+4R}$, Gr is the thermal Grashof number, Gc is the mass Grashof number, Sc is the Schmidt number, Pr is the Prandtl number and R is the radiation parameter

The initial and boundary conditions reduce:

$$\left.\begin{array}{lllll}
 & U=0,\ \theta=0,\ C=0, & for\ all & y,t\le 0\\
t>0: & U=e^{at},\theta=e^{at},C=1 & at & y=0\\
 & U\to 0,\theta\to 0,\ C\to 0 & as & y\to\infty
\end{array}\right\}\tag{12}$$

3. SOLUTION TO THE PROBLEM

To solve equations (6) to (8), subjected to the boundary conditions of (9), the solutions are obtained for concentration, temperature and velocity flow in terms of exponential and complementary error function using the Laplace- transform technique as follows;'

$$C\left(y\right)=\frac{1}{2}\left[e^{2\eta\sqrt{ScKt}}erfc\left(\eta\sqrt{Sc}+\sqrt{Kt}\right)+e^{-2\eta\sqrt{ScKt}}erfc\left(\eta\sqrt{Sc}-\sqrt{Kt}\right)\right]\tag{13}$$

$$\theta\left(y,t\right)=\frac{e^{at}}{2}\left[e^{2\eta\sqrt{\lambda at}}erfc\left(\eta\sqrt{\lambda}+\sqrt{at}\right)+e^{-2\eta\sqrt{\lambda at}}erfc\left(\eta\sqrt{\lambda}-\sqrt{at}\right)\right]\tag{14}$$

$$\begin{aligned}
U\left(y,t\right)=&\frac{e^{at}}{2}\left[e^{2\eta\sqrt{at}}erfc\left(\eta+\sqrt{at}\right)+e^{-2\eta\sqrt{at}}erfc\left(\eta-\sqrt{at}\right)\right]\\
&+\frac{Gre^{at}}{2a\left(\lambda-1\right)}\left\{\left[e^{2\eta\sqrt{at}}erfc\left(\eta+\sqrt{at}\right)+e^{-2\eta\sqrt{at}}erfc\left(\eta-\sqrt{at}\right)-\frac{2erfc\left(\eta\right)}{e^{at}}\right]\right.\\
&\left.-\left[e^{2\eta\sqrt{\lambda at}}erfc\left(\eta\sqrt{\lambda}+\sqrt{at}\right)+e^{-2\eta\sqrt{\lambda at}}erfc\left(\eta\sqrt{\lambda}-\sqrt{at}\right)\right]-\frac{2erfc(\eta\sqrt{\lambda})}{e^{at}}\right\}
\end{aligned}$$

$$-e^{dt}\left[e^{2\eta\sqrt{dt}}erfc\left(\eta+\sqrt{dt}\right)+e^{-2\eta\sqrt{dt}}erfc\left(\eta-\sqrt{dt}\right)\right]\Bigg]$$
$$-\left[e^{2\eta\sqrt{ScKt}}erfc\left(\eta\sqrt{Sc}+\sqrt{Kt}\right)+e^{-2\eta\sqrt{ScKt}}erfc\left(\eta\sqrt{Sc}-\sqrt{Kt}\right)\right]$$
$$-dt\left[e^{2\eta\sqrt{ScKt}}erfc\left(\eta\sqrt{Sc}+\sqrt{Kt}\right)+e^{-2\eta\sqrt{ScKt}}erfc\left(\eta\sqrt{Sc}-\sqrt{Kt}\right)\right.$$
$$\left.-\frac{\eta\sqrt{tSc}}{2\sqrt{K}}\left[e^{-2\eta\sqrt{ScKt}}erfc\left(\eta\sqrt{Sc}-\sqrt{Kt}\right)-e^{2\eta\sqrt{Sckt}}erfc\left(\eta\sqrt{Sc}+\sqrt{Kt}\right)\right]\right]$$
$$-\frac{\eta\sqrt{tSc}}{2\sqrt{K}}\left[e^{-2\eta\sqrt{ScKt}}erfc\left(\eta\sqrt{Sc}-\sqrt{Kt}\right)-e^{2\eta\sqrt{Sckt}}erfc\left(\eta\sqrt{Sc}+\sqrt{Kt}\right)\right]\Bigg] \tag{15}$$

where $\eta=\frac{y}{\sqrt{2t}},\ d=\frac{kSc}{1-Sc}\ \ and\ \ b=d+K$

4. RESULTS AND DISCUSSION

Radiative and chemical effects over exponentially accelerated infinite vertical plate and temperature with constant mass flux has been formulated, analysed and solved analytically. In order to point out the effects of physical parameters namely; Accelerating parameter a, thermal Grashof number Gr, mass Grashof number Gc, Prandtl number Pr, Schmidt number Sc, time t, radiation parameter R, and chemical reaction parameter K. on the flow patterns, the computation of the flow fields are carried out. The value of the Prandtl number Pr is chosen to represent air (Pr = 0.71). The value of Schmidt number is chosen to represent water vapour (Sc = 0.6). The values of velocity, temperature and concentration are obtained for the physical parameters as mention.

The velocity profiles has been studied and presented in figure 1 to 7. The effect of velocity for different values of Schmidt number (Sc = 0.16, 0.3, 0.6) is presented in figure 1. The trend shows that the velocity increases with increasing Schmidt number. The effect of velocity profiles again have been studied for different values of thermal Grashof number (Gr = 0.05, 0.06, 0.1, 0.2) and mass Grashof number (Gc = 0.05, 0.1, 0.2, 0.3) is studied and then presented in figure 2 and 3 respectively. The results are here observed that the increase in the values of velocity increases with increasing values of Gr and decreasing Gc.

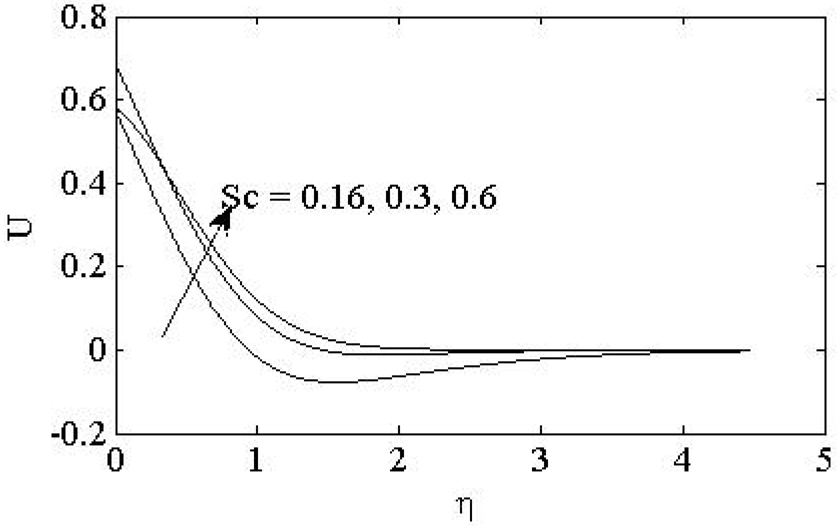

Figure 1. Velocity profiles for different values of Sc

The velocity profiles for different values of thermal Grashof number (Gc = 0.05, 0.1, 0.2, 0.3) is seen in Figure 2 It is observed that velocity increases with increasing Gc. The velocity profiles for different values of mass Grashof number (Gr = 0.05, 0.06, 0.1, 0.2) is presented in Figure 3. It is observed that velocity increases with decreasing Gr.

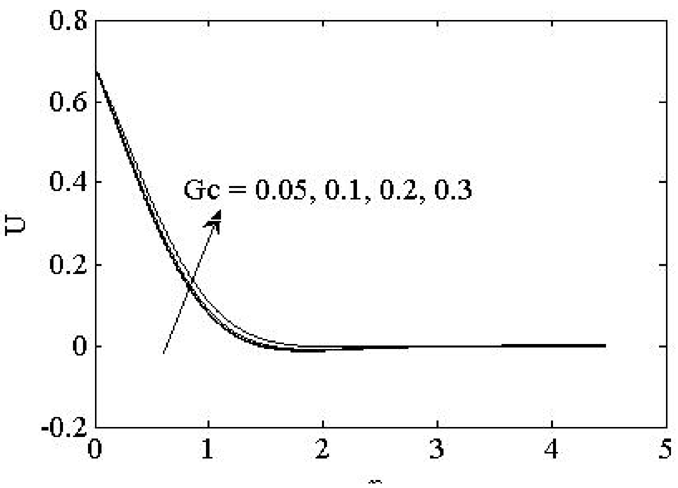

Figure 2. Velocity profiles for different values of Gc

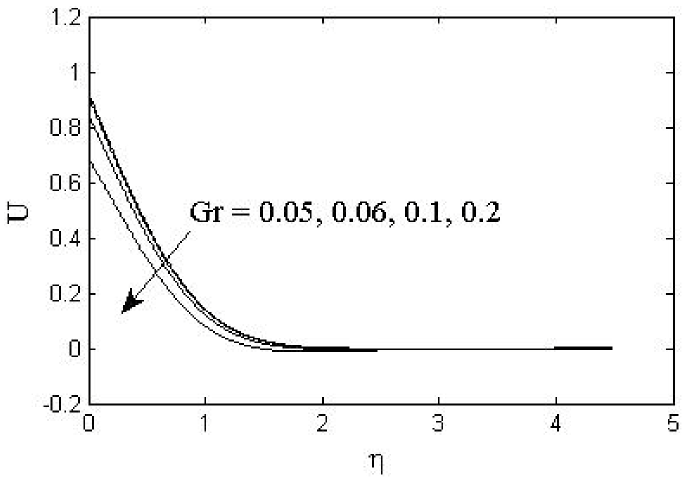

Figure 3. Velocity profiles for different values of Gr

The velocity profiles for different values of accelerating parameter (a = 0.01, 0.05, 0.1, 0.5) is seen in Figure 4. It is observed that velocity increases with increasing a. The velocity profiles for different values of radiation parameter (R =2, 5, 10, 20) is presented in Figure 5. It is observed that velocity increases with increasing R

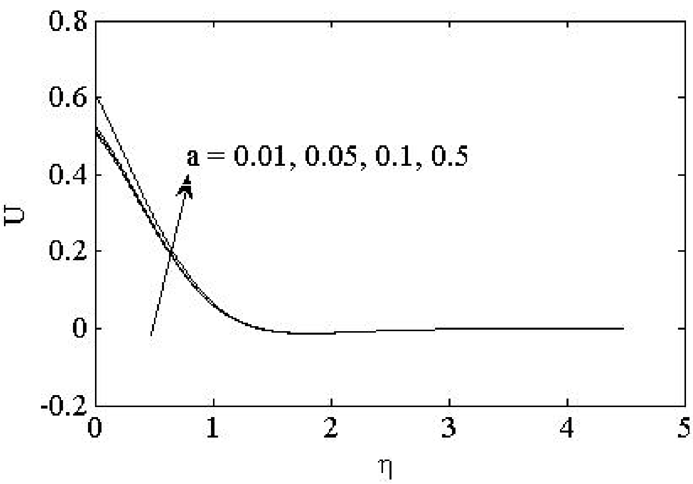

Figure 4. Velocity profiles for different values of a

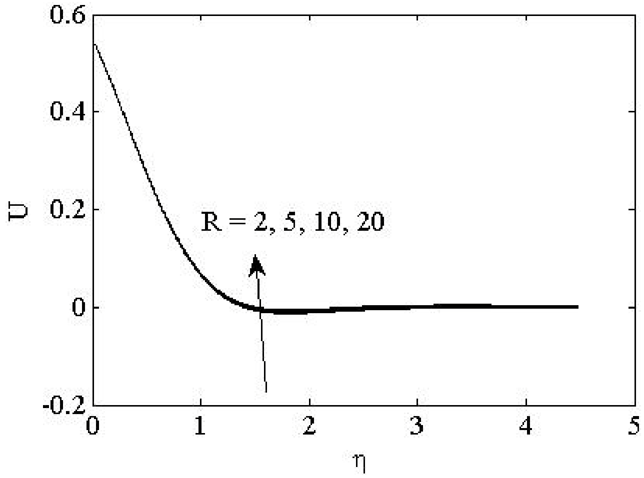

Figure 5. Velocity profiles for different values of R

The velocity profiles for different values of Prandtl number (Pr = 0.71, 1, 7, 10) is seen in Figure 6. It is observed that velocity increases with decreasing Pr. The velocity profiles for different values of chemical reaction parameter (K= 1, 2, 4, 5) is presented in Figure 7. It is observed that velocity increases with decreasing K

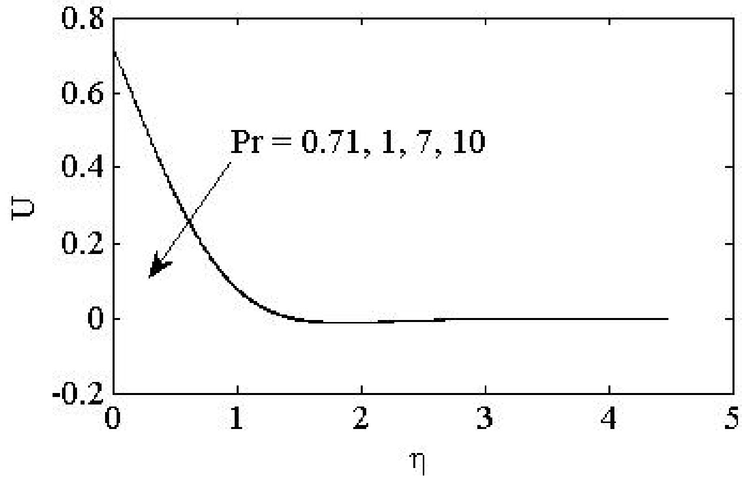

Figure 6. Velocity profiles for different values of Pr

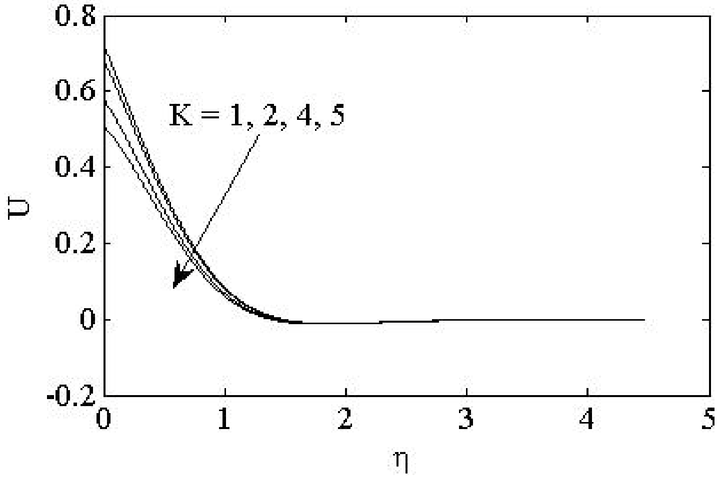

Figure 7. Velocity profiles for different values of K

The temperature profiles has been studied and presented in figure 8 to 10. The temperature profiles for different values Prandtl number (Pr = 0.71, 0.85, 1, 7) is presented in figure 8. It is observed that increases in Prandtl number Pr decreases temperature.

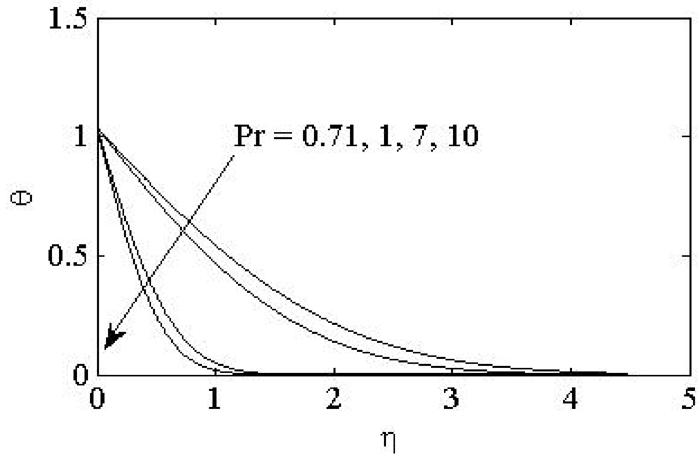

Figure 8. Temperature profiles for different values of Pr

The temperature profiles for different values of accelerating parameter (a = 0.2, 0.5, 0.8, 2) is presented in Figure 9 It shows that temperature rises with increasing a. The temperature profiles for different values of radiation parameter (R = 2, 5, 10, 20) is presented in figure 10 It is observed that temperature increases with increasing R

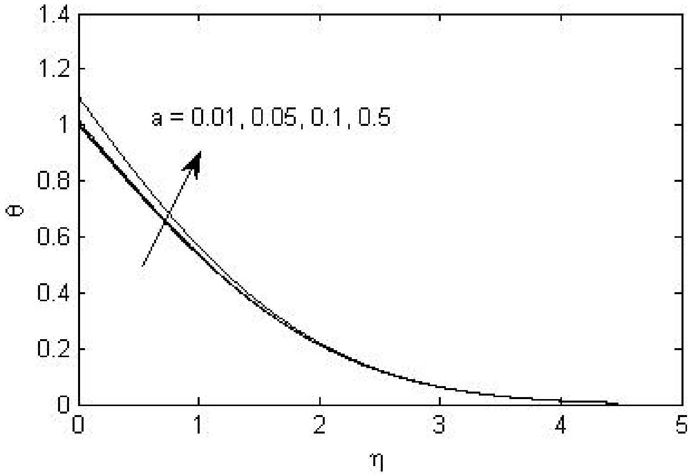

Figure 9. Temperature profiles for different values of a

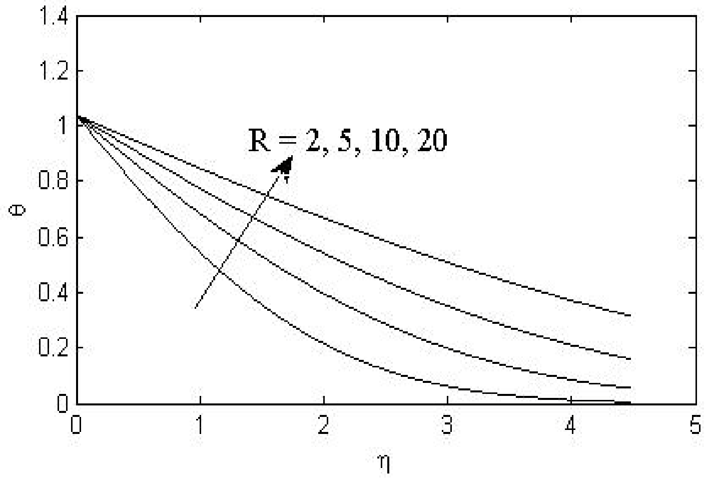

Figure 10. Temperature profiles for different values of R

The concentration profiles has been studied and presented in figure 13 to 15

The concentration profiles for different values of Schmidt number (Sc = 0.16, 0.22, 0.3, 0.6) is presented in Figure 11. It is observed that the concentration increase with decreasing Sc. The effect of concentration for different values of chemical reaction parameter (K=0.2, 0.5, 2, 5) is presented in Figure 12. It is observed that the concentration increases with a decrease in the values of K.

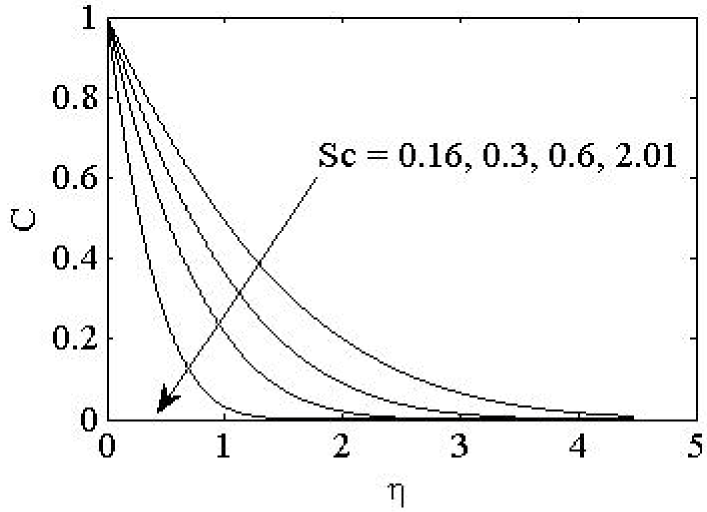

Figure 11. Concentration profiles for different values of Sc

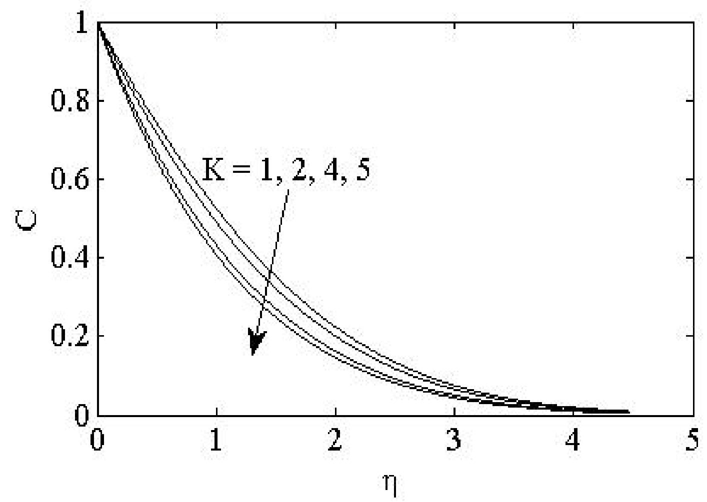

Figure 12. Concentration profiles for different values of K

5. CONCLUSION

Radiative and chemical effects over exponentially accelerated infinite vertical plate and temperature with constant mass flux has been studied. The dimensional governing equations are solved by Laplace transform technique. The effect of different parameters like accelerating parameter, radiation parameter, chemical reaction parameter, Schmidt number, Prandtl number, mass Grashof number, thermal Grashof number, and time are presented graphically. It is observed that velocity profile increases with increasing parameter namely R, Sc, Gc and a while Gr, Pr and K decreases with increasing velocity. It also observed that temperature rise with increasing a, R while Pr decreases with increasing temperature. The concentration profiles increases with decreasing Sc and K.

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