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EXPONENTIAL STABILITY OF SECOND ORDER NEUTRAL STOCHASTIC EVOLUTION EQUATIONS WITH INFINITE DELAYS

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ABSTRACT

This paper is concerned with the exponential stability of second order neutral stochastic evolution equation with infinite delays. By applying fixed point principle authors present sufficient conditions to ensure that, the mild solutions are exponentially stable in p-th moment. An example is provided to illustrate the effectiveness of the proposed result.

***Keywords**: Exponential stability, Infinite delay, Cosine family, Fixed point theorem.*

***AMS MSC 2010**: 93E15, 35R60, 60H15.*

1. INTRODUCTION

Neutral stochastic partial differential equations with delay are well known to describe many sophisticated dynamical systems in physical, biological, medical, chemical engineering, aero-elasticity and social sciences. The existence, uniqueness and asymptotic behavior of mild solutions for first order non linear stochastic evolution equations have recently received a lot of attentions [1, 3, 14] and the references therein. However as for neutral SPDEs with infinite delay, as far as we know, there exist only a few results about the existence and asymptotic behavior of mild solutions. Ren and Sun [13], Li and Liu [10] considering the existence of solutions of second order stochastic evolution equations and neutral stochastic differential inclusions with infinite delay respectively. Cui and Yan [5] investigated that existence and long time behavior of mild solutions for a class of neutral stochastic partial differential equations with infinite delay in distribution. A difficulty is that mild solution does not have stochastic differentials. In [11] Luo and Taniguchi have analyzed the asymptotical stability for mild solution to neutral stochastic partial differential equations with infinite delay by using the fixed point theorem. Inspired by the idea proposed in Luo and Taniguchi in [11], Cui et al [6] have discussed the exponential stability for mild solution of neutral stochastic partial differential equations with delays and poisson jumps and Sakthivel and Ren [15] has studied the exponential stability for mild solution of second order stochastic evolution equations with poisson jumps respectively. By employing the integral inequality established in Chen [4], Boufoussi and Hajji [2] have obtained some sufficient conditions ensuring the exponential stability for neutral stochastic partial delayed differential equations driven by a fractional Brownian motion. Ren and Sakthivel [12] have considered the existence, uniqueness and stability of mild solution for second order neutral stochastic evolution equations with infinite delay and poisson jumps by employing the generalized Bihari's inequality. In [9] the authors studied the Stability behavior of second order neutral impulsive stochastic differential equations with delay. In this work the exponential stability of non linear second order stochastic evolution equations with infinite delay are studied by using fixed point theorem.

2. PRELIMINARIES

Let X and E be two real separable Hibert spaces with the norms $\|\cdot\|_X$ and $\|\cdot\|_E$ respectively. Let $(\Omega, \Gamma, \{\Gamma_t\}_{t\geq 0}, P)$ be a complete probability space equipped with a normal filtration $\{\Gamma_t\}_{t\geq 0}$ satisfying the usual conditions, that is filtration is right continuous and Γ_0 contains all P null sets. Let $L(E, X)$ denotes the space of bounded linear operators from E in to X. Let Q be a nuclear operator from E to E. $L_2^0 = L_2(E; X)$ be the space of all Q–Hilbert–Schmidt operators from E to X with the norm $\|\psi\|_{L_2^0} = tr(\psi Q \psi^*) < \infty, \psi \in L(E, X)$.

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Let $\beta_n(t)$ $(n = 1, 2, 3, \dots)$ be a sequence of real value one dimensional standard Brownian motions mutually independent on $(\Omega, \Gamma, \{\Gamma_t\}_{t\geq 0}, P)$. Now we define a Q - Wiener process $w(t)$ by $w(t) = \sum_{n=1}^{\infty} \sqrt{\lambda_n}\, \beta_n(t) e_n$, $t \geq 0, e \in E$, Here $\lambda_n \geq 0$ $(n = 1, 2, 3, \dots)$ are non negative real numbers and $\{e_n\}$ $(n = 1, 2, 3, \dots)$ is a complete orthonormal basis in E such that $Qe_n = \lambda_n e_n$, $n = 1, 2, 3, \dots$.

The main purpose of this paper is to establish the exponential stability of mild solutions for a class of second order neutral stochastic differential equations with infinite delays of the form

$$d\left[x'(t) - f_1\left(t, x\left(t - \rho(t)\right)\right)\right] = \left[Ax(t) + f_2\left(t, \int_{-\infty}^{0} g\left(\theta, x(t+\theta)\right) d\theta\right)\right] dt + h\left(t, \int_{-\infty}^{0} \sigma\left(\theta, x(t+\theta)\right) d\theta\right) dw(t) \qquad (1)$$

$$x(s) = \varphi(s), -\infty < s \leq 0, x'(0) = x_1 \qquad (2)$$

where $A: D(A) \subset X \to X$ is the infinitesimal generator of a strongly continuous cosine family on X. The mappings $f_1, f_2: [0, \infty) \times X \to X$, $h: [0, \infty) \times X \to L_2^0(E, X)$, $g, \sigma: (-\infty, 0] \times X \to X$ are measurable functions. $\varphi: (-\infty, 0] \to X$ is a cadlag stochastic process with $E\left(\sup_{-\infty < s \leq 0} \|\varphi(s)\|_X^p\right) < \infty$ and x_1 is a Γ_0 measurable X valued random variable independent of w with finite second moment.

In this section, let us recall some basic concepts about cosine families of operators [8, 16]. The one parameter family $\{C(t): t \in R\} \subset BL(X, X)$ satisfying that

(i) $C(0) = I$,

(ii) $C(t)x$ is continuous in t on R, for all $x \in X$,

(iii) $C(t+s) + C(t-s) = 2C(t)C(s)$ for all $t, s \in R$ is called a strongly continuous cosine family.

The corresponding strongly continuous sine family $\{S(t): t \in R\} \subset BL(X, X)$ is defined by $(t)x = \int_0^t C(s)x ds$, $t \in R, x \in X$. The generator $A : X \to X$ of $\{C(t): t \in R\}$ is given by $Ax = \frac{d^2}{dt^2} C(t)x\Big|_{t=0}$ for all $x \in D(A) = \{x \in X: C(\cdot)x \in$ C2R;X.

It is well known that the infinitesimal generator A is a closed, densely defined operator on X. Such cosine and corresponding sine families and their generators satisfy the following properties.

Lemma 2.1: [8] Suppose that A is the infinitesimal generator of a cosine family of operators $\{C(t): t \in R\}$. Then the following properties hold.

(i) There exists $N^* \geq 1$ and $w \geq 0$ such that $\|C(t)\| \leq N^* e^{w|t|}$ and hence $\|S(t)\| \leq N^* e^{w|t|}$.

(ii) $A \int_s^{\hat{r}} S(u)\, x du = [C(\hat{r}) - C(s)]x$ for all $0 \leq s \leq \hat{r} < \infty$.

(iii) There exists $\widehat{N}^* \geq 1$ such that $\|S(s) - S(\hat{r})\| \leq \widehat{N}^* \left|\int_s^{\hat{r}} e^{w|s|} ds\right|$ for all $0 \leq S \leq \hat{r} < \infty$.

In order to state our main results we impose the following assumptions on the functions $f_i, g, h, \sigma (i = 1,2)$.

(H_1) The cosine family of operators $\{C(t): t \geq 0\}$ on X and the corresponding sine family $\{S(t): t \geq 0\}$ satisfy the conditions $\|C(t)\|_X \leq Me^{-\beta t}$, $\|S(t)\|_X \leq Me^{-\gamma t}$, $t \geq 0$ for some constants $M \geq 1$ and $\beta, \gamma > 0$.

(H_2) For any $x, y \in X$, $\|f_1(t, 0)\|_X^p = 0$; $\|f_2(t, 0)\|_X^p = 0$; and

$$\|f_i(t, x_1) - f_i(t, x_2)\|_X^p \leq k_i \|x_1 - x_2\|_X^p, k_i > 0, i = 1,2.$$

$$\|h(t, x) - h(t, y)\|_X^p \leq k_3 \|x_1 - x_2\|_X^p, k_3 > 0.$$

(H_3) The function g, σ satisfies that $\|g(t, 0)\|_X = 0$ and $\|\sigma(t, 0)\|_X = 0$ and

$$\|g(t, x_1) - g(t, x_2)\|_X \leq \eta_1(t) \|x_1 - x_2\|_X,$$

$$\|\sigma(t, x_1) - \sigma(t, x_2)\|_X \leq \eta_2(t) \|x_1 - x_2\|_X,$$

where $0 < \eta_1(t) \leq d_1 e^{-\xi\|t\|}, d_1 > 0, \xi > 0$ and $0 < \eta_2(t) \leq d_2 e^{-\xi\|t\|}, d_2 > 0, \xi > 0$.

(H_4) $k = 3^{p-1}\left[M^p K_1 \beta^{1-p} + M^p K_2 \gamma^{1-p} {d_1}^p \xi^{1-p+} C_p M^p K_3\, d_2^p\, \xi^{-p} (2\gamma)^{-p/2}\right]$ be such that $(0 \leq k \leq 1)$.

Definition 2.2: A stochastic process $\{x(t), -\infty < t \le b\}(0 < b \le \infty)$ is called a mild solution of Equations (1) and (2) if

(i) $x(t)$ is measurable and Γ_t - adapted càdlàg process with $E\int_0^b \|x(t)\|_X^p\, dt < \infty$,

(ii) $x(t)$ satisfies the following integral equation

$$x(t) = C(t)\varphi(0) + S(t)\big(x_1 - f_1(0,\varphi)\big) + \int_0^t C(t-s)\, f_1\Big(s, x\big(s-\rho(s)\big)\Big)ds + \int_0^t S(t-s)\, f_2\Big(s, \int_{-\infty}^0 g\big(\theta, x(s+\theta)\big)d\theta\Big)ds + \int_0^t S(t-s)\, h\Big(s, \int_{-\infty}^0 \sigma\big(\theta, x(s+\theta)\big)d\theta\Big)dw(s). \quad (3)$$

Definition 2.3: The equation (3) is said to be exponentially stable in p-th moment if there exist some constants $M^* \ge 1$, $\eta > 0$ such that $E\|x(t)\|_X^p \le M^* E\|x(0)\|_X^p e^{-\eta t}, t \ge 0$.

3. EXPONENTIAL STABILITY RESULT

In this section, we establish existence of mild solutions to equations (1) and (2) and prove the exponential stability result.

Let the space H denote Banach space of all Γ_t adapted càdlàg process $x(t)$ such that there exist two constants $M^* \ge 1$ and $\eta > 0$ satisfying the inequality $E\|x(t)\|_X^p \le M^* E\|x(0)\|_X^p e^{-\eta t}, t \ge 0$ with the norm $\|X\|_H = \sup_{t\ge0} E\|x(t)\|_X^p$. Further let the delay $\rho(t)$ be finite, that is there exist a constant $r > 0$ and $0 \le \rho(t) \le r$. Now let us prove the required result by using a fixed point argument.

Lemma 3.1:[7] For any $r \ge 1$ and for arbitrary L_2^0 valued predictable process $\varphi(\cdot)$,

$$\sup_{s\in[0,t]} E\left\|\int_0^s \varphi(u)\, dw(u)\right\|_X^{2r} \le \big(r(2r-1)\big)^r \left(\int_0^t \left(E\|\varphi(s)\|_{L_2^0}^{2r}\right)^{\frac{1}{r}} ds\right)^r$$

Theorem 3.2: Let $p \ge 2$ be an integer. Suppose that (H_1) - (H_4) are satisfied the initial condition $\varphi(s)$ satisfies the condition $E\|\varphi(s)\|_X^p \le M_1^* E\|\varphi(0)\|_X^p e^{-\mu s}$, $s \le 0$, $0 < \mu < \eta$, here $M^* \ge 1$, then the mild solution to the second order stochastic evolution equations (1) and (2) exists and it is exponentially stable in p-th moment.

Proof: Define a nonlinear operator $F: H \to H$ by

$$F(x)(t) = C(t)\varphi(0) + S(t)\big(x_1 - f_1(0,\varphi)\big) + \int_0^t C(t-s)\, f_1\Big(s, x\big(s-\rho(s)\big)\Big)ds + \int_0^t S(t-s)\, f_2\Big(s, \int_{-\infty}^0 g\big(\theta, x(s+\theta)\big)d\theta\Big)ds + \int_0^t S(t-s)\, h\Big(s, \int_{-\infty}^0 \sigma\big(\theta, x(s+\theta)\big)d\theta\Big)dw(s) \quad (4)$$

In order to prove the exponential stability, it is enough to show that the operator F has a fixed point in H.

$$\begin{aligned} E\|F(x)(t)\| &\le 5^{p-1}E\|C(t)\varphi(0)\|_X^p + 5^{p-1}E\big\|S(t)\big(x_1 - f_1(0,\varphi)\big)\big\|_X^p \\ &\quad + 5^{p-1}E\left\|\int_0^t C(t-s)\, f_1\Big(s, x\big(s-\rho(s)\big)\Big)ds\right\|_X^p \\ &\quad + 5^{p-1}E\left\|\int_0^t S(t-s)\, f_2\Big(s, \int_{-\infty}^0 g\big(\theta, x(s+\theta)\big)d\theta\Big)ds\right\|_X^p \\ &\quad + 5^{p-1}E\left\|\int_0^t S(t-s)\, h\Big(s, \int_{-\infty}^0 \sigma\big(\theta, x(s+\theta)\big)d\theta\Big)dw(s)\right\|_X^p \\ &= 5^{p-1}(I_1 + I_2 + I_3 + I_4 + I_5) \end{aligned} \quad (5)$$

First verify the continuity of $F(x)(t)$ on $t \ge 0$. Let $x \in X$, $t \ge 0$ and $|r| > 0$ be sufficiently small then

$$\|(Fx)(t_1 + r) - (Fx)(t_1)\|_X^p \le 5^{p-1}\sum_{i=1}^5 \|I_i(t_1 + r) - I_i(t_1)\|_X^p.$$

Moreover, by using the lemma 3.1, we obtain

$$\begin{aligned} E\|I_5(t_1 + r) - I_5(t_1)\|_X^p &\le 2^{p-1}C_p\left[\int_0^{t_1}\left(E\left\|\big(S(t_1 + r - s) - S(t_1 - s)\big)h\Big(s, \int_{-\infty}^0 \sigma\big(\theta, x(s+\theta)\big)d\theta\Big)\right\|_{L_2^0}^p\right)^{2/p} ds\right]^{p/2} \\ &\quad + 2^{p-1}C_p\left[\int_{t_1}^{t_1+r}\left(E\left\|S(t_1 + r - s)h\Big(s, \int_{-\infty}^0 \sigma\big(\theta, x(s+\theta)\big)d\theta\Big)\right\|_{L_2^0}^p\right)^{2/p} ds\right]^{p/2} \\ &\to 0 \text{ as } r \to 0. \end{aligned}$$

Similarly we can verify that

$E\|I_i(t_1 + r) - I_i(t_1)\|_X^p \to 0$ as $\to 0$, $i = 1,2,3,4$.

First we show that $F(H) \subset H$. Let $x(t) \subset H$, without loss of generality, we assume that $0 < \eta < \xi$. From the condition(H_2), we obtain

$$
\begin{aligned}
I_3 &\le M^p K_1 \left[\int_0^t e^{-\beta(t-s)}\right]^{p-1} \int_0^t e^{-\beta(t-s)}\ E\|x(s-\rho(s))\|ds \\
&\le M^p K_1\, \beta^{1-p} \int_0^t e^{-\beta(t-s)}\ \left(M^* E\|x(0)\|_X^p e^{-\eta(s-\rho(s))} ds + e^{\mu r} M_1^* E\|\varphi(0)\|_X^p e^{-\mu s} ds\right) \\
&\le \frac{M^p K_1\, \beta^{1-p}\ M^* E\|x(0)\|_X^p e^{-\eta t} e^{\eta r}}{\beta-\eta} + \frac{M^p K_1\, \beta^{1-p} M_1^* E\|\varphi(0)\|_X^p e^{\mu(r-t)}}{\beta-\eta} \qquad (6)
\end{aligned}
$$

$$
\begin{aligned}
I_4 &\le E\left[\int_0^t \left\|S(t-s) f_2\left(s, \int_{-\infty}^0 g(\theta, x(s+\theta))d\theta\right)\right\|_X ds\right]^p \\
&\le M^p K_2 \left[\int_0^t e^{-\gamma(t-s)}\right]^{p-1} E\left[\int_0^t \left\| e^{\frac{-\gamma(t-s)}{p}} \int_{-\infty}^0 g\left(\theta, x(s+\theta)\right)d\theta\right\|_X^p\right] \\
&\le M^p K_2\, \gamma^{1-p} {d_1}^p \int_0^t \left[\left(\int_{-\infty}^s e^{\xi(\tau-s)} d\tau\right)^{p-1} \times \int_{-\infty}^s e^{-\gamma(\tau-s)} e^{\xi(\tau-s)} E\|x(\tau)\|_X^p d\tau\right] ds \\
&\le M^p K_2\, \gamma^{1-p} {d_1}^p \xi^{1-p} \int_0^t \left[\int_{-\infty}^0 e^{-\gamma(t-s)} e^{\xi(\tau-s)} M_1^* E\|\varphi(0)\|_X^p e^{-\mu\tau} d\tau + \int_0^s e^{-\gamma(t-s)} e^{\xi(\tau-s)} M^* E\|x(0)\|_X^p\, d\tau\right] ds \\
&\le M^p K_2\, \gamma^{1-p} {d_1}^p \xi^{1-p} \left[\frac{M^*}{(\xi-\eta)(\gamma-\eta)} E\|x(0)\|_X^p e^{-\xi t} + \frac{M_1^*}{(\xi-\mu)(\gamma-\xi)} E\|\varphi(0)\|_X^p e^{-\mu t}\right] \qquad (7)
\end{aligned}
$$

$$
\begin{aligned}
I_5 &= E\left\|\int_0^t S(t-s)\, h\left(s,\ \int_{-\infty}^0 \sigma\left(\theta, x(s+\theta)\right)d\theta\right) dw(s)\right\|_X^p \\
&\le c_p \left\{\int_0^t \left[E\left\|S(t-s) h\left(s,\ \int_{-\infty}^0 \sigma\left(\theta, x(s+\theta)\right)d\theta\right)\right\|_{L_2^0}^p\right]^{2/p} ds\right\}^{p/2} \quad \text{where } c_p = \frac{(p(p-1))}{2} \\
&\le k_3 c_p {d_2}^p \left\{\int_0^t e^{-2\gamma(t-s)} \left[E\left(\int_{-\infty}^s e^{\xi(\tau-s)} \|x(\tau)\|_X\right)^p d\tau\right]^{\frac{2}{p}} ds\right\}^{p/2} \\
&\le k_3 c_p {d_2}^p \left\{\int_0^t e^{-2\gamma(t-s)} \left[\left(\int_{-\infty}^s e^{\xi(\tau-s)} d\tau\right)^{p-1} \times \int_{-\infty}^s e^{\xi(\tau-s)} E\|x(\tau)\|_X^p d\tau\right]^{2/p} ds\right\}^{p/2} \\
&\le k_3 c_p {d_2}^p \xi^{1-p} \left\{\int_0^t e^{-2\gamma(t-s)} \left[\int_{-\infty}^s e^{\xi(\tau-s)} E\|x(\tau)\|_X^p d\tau\right]^{2/p} ds\right\}^{p/2} \\
&\le k_3 c_p {d_2}^p \xi^{1-p} \left\{\int_0^t e^{-2\gamma(t-s)} \left[\int_{-\infty}^0 e^{\xi(\tau-s)}\ M_1^* E\|\varphi(0)\|_X^p e^{-\mu\tau} d\tau + \int_0^s e^{\xi(\tau-s)} M^* E\|x(0)\|_X^p e^{-\eta\tau} d\tau\ \right]^{2/p} ds\right\}^{p/2} \\
&\le k_3 c_p {d_2}^p \xi^{1-p} \left\{\int_0^t e^{-2\gamma(t-s)} \left[\frac{M^* E\|x(0)\|_X^p e^{-\xi s}}{\xi-\eta} + \frac{M_1^* E\|\varphi(0)\|_{X e^{-\mu s}}^p}{\xi-\mu}\right]^{2/p} ds\right\}^{p/2} \\
&\le k_3 c_p {d_2}^p \left[\frac{M^* E\|x(0)\|_X^p}{\xi-\eta}\left(\frac{p}{2p\gamma-2\eta}\right)^{p/2} e^{-\eta t} + \frac{M_1^* E\|\varphi(0)\|_X^p}{\xi-\mu}\left(\frac{p}{2p\gamma-2\xi}\right)^{p/2} e^{-\xi t}\right] \qquad (8)
\end{aligned}
$$

From the above equations $(5)-(8)$, one can see that there exist $k \ge 1$ and $\eta \ge 0$ such that

$$E\|(Fx)(t)\|_X^p \ \le\ FE\|F(x)(0)\|_X^p e^{-\eta t}.$$

Thus we obtain$F(H) \subset H$. Next, we prove that F is a contraction mapping. To see this, let $x, y \in H$, we have

$$
\begin{aligned}
E\|(Fx)(t)-(Fy)(t)\|_X^p \le\ & 3^{p-1} E\left\|\int_0^t C(t-s)\ \left(f_1\left(s, x(s-\rho(s))\right) - f_1\left(s, y(s-\rho(s))\right)\right) ds\right\|_X^p \\
&+ 3^{p-1} E\left\|\int_0^t S(t-s)\left(f_2\left(s, \int_{-\infty}^0 g\left(\theta, x(s+\theta)\right)d\theta\right) ds - f_2\left(s, \int_{-\infty}^0 g\left(\theta, y(s+\theta)\right)d\theta\right) ds\right)\right\|_X^p \\
&+ 3^{p-1} E\left\|\int_0^t S(t-s)\left(h\left(s, \int_{-\infty}^0 \sigma\left(\theta, x(s+\theta)\right) d\theta\right) - h\left(s, \int_{-\infty}^0 \sigma\left(\theta, y(s+\theta)\right) d\theta\right)\right) dw(s)\right\|_X^p \qquad (9)
\end{aligned}
$$

Now we estimate for each term in equation(9). We have

$$
\begin{aligned}
E&\left\|\int_0^t C(t-s)\ \left(f_1\left(s, x(s-\rho(s))\right) - f_1\left(s, y(s-\rho(s))\right)\right) ds\right\|_X^p \\
&\le E\left[\int_0^t \left\|C(t-s)\left(f_1\left(s, x(s-\rho(s))\right) - f_1\left(s, y(s-\rho(s))\right)\right)\right\|_X\right]^p \\
&\le M^p K_1\, E\left[\int_0^t e^{-\beta(t-s)} \left\|x(s-\rho(s)) - y(s-\rho(s))\right\|_X ds\right]^p
\end{aligned}
$$

$$\leq M^p K_1\, \beta^{1-p} \int_0^t e^{-\beta(t-s)} E\left\|x\big(s-\rho(s)\big)-y\big(s-\rho(s)\big)\right\|_X^p ds$$
$$\leq M^p K_1\, \beta^{1-p} \sup_{s\geq 0} E\|x(s)-y(s)\|_X^p$$

and

$$E\left\|\int_0^t S(t-s)\left(f_2\left(s,\ \int_{-\infty}^0 g\big(\theta,x(s+\theta)\big)d\theta\right)-f_2\left(s,\ \int_{-\infty}^0 g\big(\theta,y(s+\theta)\big)d\theta\right)ds\right)\right\|_X^p$$
$$\leq M^p K_2\left[\int_0^t e^{-\gamma(t-s)}\left\|\int_{-\infty}^0 g\big(\theta,x(s+\theta)\big)d\theta-\int_{-\infty}^0 g\big(\theta,y(s+\theta)\big)d\theta\right\|_X ds\right]^p$$
$$\leq M^p K_2\, E\left[\left(\int_0^t e^{-\gamma(t-s)}ds\right)^{p-1}\int_0^t\left(\int_{-\infty}^0\left\|e^{\frac{-\gamma(t-s)}{p}}\Big(g\big(\theta,x(s+\theta)\big)-g\big(\theta,x(s+\theta)\big)\Big)\right\|_X d\theta\right)^p ds\right]$$
$$\leq M^p K_2\, \gamma^{1-p} d_1^p\left[\int_0^t\left(\int_{-\infty}^s e^{\xi(\tau-s)}d\tau\right)^{p-1}\left(\int_{-\infty}^s e^{-\gamma(\tau-s)}e^{\xi(\tau-s)}E\|x(\tau)-y(\tau)\|_X^p d\tau\right)ds\right]$$
$$\leq M^p K_2\, \gamma^{1-p} d_1^p \xi^{-p} \sup_{s\geq 0} E\|x(s)-y(s)\|_X^p$$

Also

$$E\left\|\int_0^t S(t-s)\left(h\left(s,\int_{-\infty}^0\sigma\big(\theta,x(s+\theta)\big)\,d\theta\right)-h\left(s,\int_{-\infty}^0\sigma\big(\theta,y(s+\theta)\big)\,d\theta\right)\right)dw(s)\right\|_X^p$$
$$\leq C_p M^p K_3\left\{\int_0^t e^{-2\gamma(t-s)}\left[E\left\|\int_{-\infty}^0\Big(\sigma\big(\theta,x(s+\theta)\big)-\sigma\big(\theta,y(s+\theta)\big)\Big)d\theta\right\|_X^p\right]^{2/p}ds\right\}^{p/2}$$
$$\leq C_p M^p K_3\, d_2^p\left\{\int_0^t e^{-2\gamma(t-s)}\left[E\left(\int_{-\infty}^s e^{\xi(\tau-s)}\|x(\tau)-y(\tau)\|_X d\tau\right)^p\right]^{2/p}ds\right\}^{p/2}$$
$$\leq C_p M^p K_3\, d_2^p\left\{\int_0^t e^{-2\gamma(t-s)}\left[\left(\int_{-\infty}^s e^{\xi(\tau-s)}d\tau\right)^{p-1}\int_{-\infty}^s e^{\xi(\tau-s)}E\|x(\tau)-y(\tau)\|d\tau\right]^{2/p}\right\}^{p/2}$$
$$\leq C_p M^p K_3\, d_2^p\, \xi^{-p}(2\gamma)^{-p/2}\sup_{s\geq 0} E\|x(s)-y(s)\|_X^p$$

Consequently we have

$$\sup_{s\geq 0} E\|(Fx)(t)-(Fy)(t)\|_X^p \leq k \sup_{s\geq 0} E\|x(s)-y(s)\|_X^p$$

where $k = 3^{p-1}\left[M^p K_1\,\beta^{1-p}+M^p K_2\,\gamma^{1-p}{d_1}^p\xi^{1-p+}C_p M^p K_3\, d_2^p\,\xi^{-p}(2\gamma)^{-p/2}\right]$

Since $0\leq k<1$, then F is a contraction mapping.Thus by the contraction mapping the operator F has a unique fixed point $x(t)$ in H which is a solution of equations (1) and (2) with $x(s)=\varphi(s)$, $x'(0)=x_1$ and $x(t)$ is exponentially stable in p -th moment.

Corollary 3.3: Under the conditions of theorem 3.2 with $p=2$, the mild solution of (2.1) exists uniquely which is exponentially stable in mean square.

4. EXAMPLE

In this section we present an example for illustrating the main theorem. Let $X=E=L^2(0,\ \pi)$ and $e_n=\sqrt{\frac{2}{\pi}}\sin(nx)$. Then $\{e_n\}$ be a complete orthonormal basis in X. Let $w(t)=\sum_{n=1}^{\infty}\sqrt{\lambda_n}\,\beta_n(t)e_n$, $\lambda_n>0$, where $\beta_n(t)$ are one dimensional Brownian motions mutually independent on a usual complete probability space. Define the operator $Q: X\to X$ by setting $Qe_n=\lambda_n e_n$, $(n=1,2,3,\dots)$ and assume that $trace(Q)=\sum_{n=1}^{\infty}\lambda_n<\infty$. Let $A=\frac{-\partial^2}{\partial x^2}$ with domain $D(A)=H_0^1(0,\pi)\cap H^2(0,\ \pi)$. Here $H_0^1(0,\pi)=\left\{w\in L^2(0,\ \pi): \frac{\partial w}{\partial z}\in L^2(0,\ \pi), w(0)=w(\pi)=0\right\}$ and $H^2(0,\pi)=\left\{w\in L^2(0,\ \pi): \frac{\partial w}{\partial z}, \frac{\partial^2 w}{\partial z^2}\in L^2(0,\ \pi)\right\}$.

Consider the following stochastic partial differential equation.

$$\partial\left[\frac{\partial x(t,\xi)}{\partial t}\right]=\frac{\partial^2 x(t,\xi)}{\partial \xi^2}\partial t+f_1\left(t,\int_{-\infty}^0 ae^{\eta\theta}\,x(t+\theta,\xi)\,d\theta\right)dt+f_2\left(t,\int_{-\infty}^0 be^{\eta\theta}\,x(t+\theta,\xi)\,d\theta\right)dw(t);\ t\geq 0 \qquad (10)$$

$x(t,\xi)=\varphi(t,\xi)=0\ ;\ \xi\in[0,\pi]\ ;\ t\leq 0$

$x(t,0)=x(t,\pi)=0.$

$$\frac{\partial}{\partial t}x(0,\xi)=x_1(\xi);\ 0<\xi<\pi; \qquad (11)$$

where $a, b \geq 0; \eta > 0$ and assume that $E\|\varphi(s)\|_X^2 \leq ME\|\varphi(0)\|_X^2 e^{-\mu s}$ for $s \leq 0$ where $M \geq 1; \mu > 0$.

Take $M = 1, \gamma = 1$ and by theorem 3.2 we obtain the inequality $4a^2 + 2\text{trace}(Q)b^2 < \xi^2$.Therefore by theorem 3.2, the mild solution to equations (10) and (11) exists and also it is exponentially stable in mean square.

REFERENCES

1. Balasubramaniam P, Ntouyas S.K,Vinayagam D. Existence of solutions of semilinear stochastic delay evolution inclusions in a Hilbert space, J. Math. Anal. Appl. 305, (2005), 438-451.
2. Boufoussi B, Hajji S. Neutral stochastic functional differential equations driven by a fractional Brownian motion in a Hilbert space, Statist. Probab. Lett. 82, (2012), 1549-1558.
3. Chang Y-K, Zhao Z-H, N'Guerrekata GM. Square-mean almost automorphic mild solutions to non-autonomous stochastic differential equations in Hilbert spaces, Comput. Math. Appl. 61, (2011), 384-391.
4. Chen H. Integral inequality and exponential stability for neutral stochastic partial differential equations with delays, J. Inequal. Appl. 2009, (2009), 1-15.
5. Cui J, Yan L. Asymptotic behavior for neutral stochastic partial differential equations with infinite delays, Electron. Comm. Probab. 18, (2013), 1-12.
6. Cui J, Yan L .T, Sun X.C. Exponential stability for neutral stochastic partial differential equations with delays, Statist. Probab. Lett. 81, (2011), 1970-1977.
7. Da Prato G., Zabczyk. J. Stochastic equations in infinite dimensions. Cambridge University Press, Cambridge, 1992.
8. Fattorini. H. O. Second order linear differential equations in Banach spaces. North Holland Mathematics studies 108, Elsevier Science, North Holland, 1985.
9. Karunanithi S, Maheswari R. Stability behavior of second order neutral impulsive stochastic differential equations with delay, IJCATR, 2, (2013), 415 – 421.
10. Li Y and Liu B, Existence of solution of nonlinear neutral stochastic differential inclusions with infinite delays, Stochastic Anal. Appl. 25, (2007), 397-415.
11. Luo J, Taniguchi T. Fixed point and stability of stochastic neutral partial differential equations with infinite delays, Stochastic Anal. Appl. 27, (2009), 1163-1173.
12. Ren Y, Sakthivel R. Existence and uniqueness, and stability of mild solutions for second order neutral stochastic evolution equations with infinite delay and Poisson Jumps, J. Math. Phys. 53, (2012), 073517-073531.
13. Ren Y, Sun D.D. Second order neutral stochastic evolution equations with infinite delay under Caratheodory Conditions, J. Optim. Theory Appl.147, (2010), 569-582.
14. Sakthivel R, Luo J. Asymptotic stability of impulsive stochastic partial differential equations with infinite delays, J. Math. Anal. Appl. 356, (2009), 1-6.
15. Sakthivel R, Ren Y. Exponential stability of second order stochastic evolution equations with poisson jumps, Commun. Nonlinear Sci. Numer. Simul. 17, (2012), 4517-4523.
16. Travis. C. C, Webb. G. F. Cosine families and abstract nonlinear second order differential equations, Acta Math. Acad. Sci. Hungaricae. 32, (1978), 76-96.

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