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EFFECT OF POROSITY ON THE FLOW RATE OF AN INCOMPRESSIBLE FLUID OF SECOND ORDER TYPE BY CREATING SINUSOIDAL DISTURBANCES

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ABSTRACT

In this paper, the effect of various flow entities on the nature of flow rate has been investigated in detail. As the porosity increases, the flow rate decreases. It is observed that, as the visco elasticity of the fluid increases, the flow rate is found to be decreasing. Further, it is noticed that, when the porosity of the fluid bed is held constant and the frequency of excitation increases, the flow rate is noticed to be decreasing. Also, as the time increases, the flow rate decreases when all the other flow entities are held constant. All such observations have been illustrated graphically.

Keywords: *Second order fluid, Porous media, Visco elasticity, Angle of inclination, Frequency of excitation and Flow rate.*

NOMENCLATURE

A_i	:	Acceleration component in i th direction
$A_{i,j}$	:	Acceleration tensor
a_i	:	Non dimensional acceleration in i th direction
$E_{ij}^{(1)}, E_{ij}^{(2)}$	:	Strain tensor in the dimensional form
$e_{ij}^{(1)}, e_{ij}^{(2)}$	:	Strain tensor in the non-dimensional form
F	:	Non dimensional flow rate
F_X, F_Y, F_Z	:	External forces applied along X, Y and Z directions
K	:	Non dimensional permeability of the porous bed
k	:	Dimensionalised porosity factory
L	:	Characteristic length
P	:	Indeterminate hydrostatic pressure
p	:	Non dimensional indeterminate pressure
r	:	Polar coordinate
S_{ij}	:	Dimensional stress tensor
s_{ij}	:	Non dimensional stress tensor
T	:	Dimensional time parameter
t	:	Non dimensional time parameter
U_i	:	Dimensional velocity component in i th direction
$U_{i,j}$	:	Dimensional velocity tensor

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u	:	Non dimensional velocity
u_i	:	Non dimensional velocity component along the i^{th} coordinate
X_i, Y_i	:	Co-ordinate axes (dimensional form)
x_i, y_i	:	Co-ordinate axes (non-dimensional form)

GREEK SYMBOLS

α	:	Angle of inclination with respect to horizontal line
β	:	Visco elasticity parameter
ε	:	Polar coordinate
ϕ_1	:	Coefficient of viscosity
ϕ_2	:	Coefficient of elastico viscosity
ϕ_3	:	Coefficient of cross viscosity
μ	:	Viscosity of the fluid
υ_c	:	Non dimensionalised cross viscosity parameter
ρ	:	Density of the fluid
σ	:	Frequency of excitation

1. INTRODUCTION

Flow through porous media has been the subject of considerable research activity in recent years because of its several important applications notably in the flow of oil through porous rock, the extraction of geothermal energy from the deep interior of the earth to the shallow layers, the evaluation of the capability of heat removal from particulate nuclear fuel debris that may result from a hypothetical accident in a nuclear reactor, the filtration of solids from liquids, flow of liquids through ion-exchange beds, drug permeation through human skin, chemical reactor for economical separation or purification of mixtures and so on.

In many chemical processing industries, slurry adheres to the reactor vessels and gets consolidated. As a result of this, the chemical compounds within the reactor vessel percolates through the boundaries causing loss of production and then consuming more reaction time. In view of such technological and industrial importance wherein the heat and mass transfer takes place in the chemical industry, the problem by considering the permeability of the bounding surfaces in the reactors attracted the attention of several investigators.

An important application is in the petroleum industry, where crude oil is tapped from natural underground reservoirs in which oil is entrapped. Since the flow behaviour of fluids in petroleum reservoir rock depends, to a large extent, on the properties of the rock, techniques that yield new or additional information on the characteristics of the rock would enhance the performance of the petroleum reservoirs. A related biomechanical application is the flow of fluids in the lungs, blood vessels, arteries and so on, where the fluid is bounded by two layers which are held together by a set of fairly regularly spaced tissues.

Viscous fluid flow over wavy wall had attracted the attention of relatively few researchers although the analysis of such flows finds application in different areas, such as transpiration cooling of re-entry vehicles and rocket boosters, cross hatching on ablative surfaces and film vaporization in combustion chambers. Especially, where the heat and mass transfer takes place in the chemical processing industry, the problem by considering the permeability of the bounding surface in the reactors assumes greater significance.

Many materials such as drilling muds, clay coatings and other suspensions, certain oils and greases, polymer melts, elastomers and many emulsions have been treated as non-Newtonian fluids. Because of the difficulty to suggest a single model, which exhibits all properties of non-Newtonian fluids, they cannot be described simply as Newtonian fluids and there has been much confusion over the classification of non-Newtonian fluids. However, non-Newtonian fluids may be classified as (i) fluids for which the shear stress depends only on the rate of shear; (ii) fluids for which the relation between shear stress and shear rate depends on time; (iii) the visco-elastic fluids, which possess both elastic and viscous properties.

Because of the great diversity in the physical structure of non-Newtonian fluids, it is not possible to recommend a single constitutive equation as the equation for use in the cases described in (i)—(iii). For this reason, many non-Newtonian models or constitutive equations have been proposed and most of them are empirical or semi-empirical. For more general three-dimensional representation, the method of continuum mechanics is needed [1]. Although many constitutive equations have been suggested, many questions are still unsolved. Some of the continuum models do not give satisfactory results in accordance with the available experimental data. For this reason, in many practical applications, empirical or semi-empirical equations have been used.

It has been shown that, for many types of problems in which the flow is slow enough in the visco-elastic sense, the results given by Olroyd's constitutive equations will be substantially equal to those of the second or third-order Rivlin–Ericksen constitutive equations [2]. Thus, if this is the sense in which the solutions to which problems are to be interpreted, it would seem reasonable to use the second- or third-order constitutive equations in carrying out the calculations. This is particularly so in view of the fact that, the calculation will generally be still simpler. For this reason, in this paper, the second-order fluid model is used. The constitutive equation for the fluids of second grade (or second-order fluids) is a linear relationship between the stress, the first Rivlin–Ericksen tensor, its square and the second Rivlin–Ericksen tensor [1]. The constitutive equation has three coefficients. There are some restrictions on these coefficients due to the Clausius–Duhem inequality and the assumption that the Helmholtz free energy is a minimum in equilibrium. A comprehensive discussion on the restrictions for these coefficients has been given in [3] and [4]. One of these coefficients represents the viscosity coefficient in a way similar to that of a Newtonian fluid and the constitutive equation reduces to that of a Newtonian fluid in the absence of the other two coefficients. The restrictions on these two coefficients have not been confirmed by experiments and the sign of these material moduli is the subject of much controversy [5].

The equation of motion of incompressible second grade fluids, in general, is of higher order than the Navier–Stokes equation. The Navier–Stokes equation is a second-order partial differential equation, but the equation of motion of a second-order fluid is a third-order partial differential equation. A marked difference between the case of the Navier–Stokes theory and that for fluids of second grade is that, ignoring the nonlinearity in the Navier–Stokes equation does not lower the order of the equation, however, ignoring the higher order non-linearities in the case of the second grade fluid, reduces the order of the equation.

In view of several industrial and technological importances, Pattabhi Ramacharyulu [6] studied the problem of the exact solutions of two dimensional flows of a second order incompressible fluid by considering the rigid boundaries. Later, Lekoudis *et.al* [7] presented a linear analysis of the compressible boundary layer flow over a wall. Subsequently, Shankar and Sinha [8] studied the problem of Rayleigh for wavy wall. The effect of small amplitude wall waviness upon the stability of the laminar boundary layer had been studied by Lessen and Gangwani [9]. Ramana Murthy *et.al* [10] discussed the flow of an elastico viscous fluid past an infinite plate with variable suction wherein the effects of various flow entities have been discussed. Further, the problem of free convective heat transfer in a viscous incompressible fluid confined between vertical wavy wall and a vertical flat wall was examined by Vajravelu and Shastri [11] and thereafter by Das and Ahmed [12]. The free convective flow of a viscous incompressible fluid in porous medium between two long vertical wavy walls was investigated by Patidar and Purohit [13]. Rajeev Taneja and Jain [14] had examined the problem of MHD flow with slip effects and temperature dependent heat source in a viscous incompressible fluid confined between a long vertical wall and a parallel flat plate. Recently, Ramana Murthy *et.al* [15] studied on the class of exact solutions of an incompressible second order fluid flow by creating sinusoidal disturbances, where different situations and effects have been examined.

In all of the above situations, the investigators main aim was to examine the parameters that influence the velocity component. Not much of attention has been paid on the flow rate and the factors influencing it. Therefore, an attempt has been made to study the influence of various critical parameters on the flow rate. In all above investigations, the fluid under consideration was viscous incompressible fluid of second order type. The aim of the present analysis is to examine the nature of the flow rate by considering an additional property namely elastico viscosity of the fluid and also by creating sinusoidal disturbance at the bottom while the fluid is resting on the plate. This paper is aimed to investigate flow rate of the fluid by also taking into account the porosity factor of the bounding surface. The results are expressed in terms of a non-dimensional parameter K, which depends on the non-Newtonian coefficient ϕ_2 and the frequency of excitation σ. It is noticed that, the flow properties are identical with those of in the Newtonian case ($K = \infty$).

2. FORMULATION OF THE PROBLEM

Since the stress at any point in the fluid is an expression of the mutual reaction of adjacent points of fluid near that point, it is natural to consider the connection between the stress and the local properties of the fluid. In the fluid at rest, the stress is determined wholly by the static pressure. In the case of a fluid in relative motion, the connection between the stress and local properties of the fluid is more complicated. However, some modifications may be made such as allowing the stress to depend only on the instantaneous distribution of fluid velocity in the neighborhood of the element.

Because of the difficulty to suggest a single model which exhibits all properties of non-Newtonian fluids, they cannot simply be described as a Newtonian fluid. For this reason, many non-Newtonian models or constitutive equations have been proposed and most of them are empirical or semi-empirical. For more general three-dimensional representation, the method of continuum mechanics is needed. One of the most popular models for non-Newtonian fluids is the model that is called the second-order fluid (or fluid of second grade). The constitutive assumption for the fluid of second grade is in the following form [1].

$$S_{ij} = -PI + \phi_1 E_{ij}^{(1)} + \phi_2 E_{ij}^{(2)} + \phi_3 E_{ij}^{(1)^2} \tag{1}$$

where
$$E_{ij}^{(1)} = U_{i,j} + U_{j,i} \tag{2}$$

and
$$E_{ij}^{(2)} = A_{i,j} + A_{j,i} + 2U_{m,i}U_{m,j} \tag{3}$$

where ϕ_1, ϕ_2 and ϕ_3 are material moduli.

The Clausius–Duhem inequality and the assumption that the Helmholtz free energy is minimum in equilibrium provide the following restrictions [3].

$$\phi_1 \geq 0, \quad \phi_2 \geq 0, \; \phi_2 + \phi_3 = 0 \tag{4}$$

The condition $\phi_2 + \phi_3 = 0$ is a consequence of the Clausius–Duhem inequality and the condition $\phi_2 \geq 0$ follows the requirement that the Helmholtz free energy is a minimum in equilibrium. A comprehensive discussion on the restrictions for ϕ_1, ϕ_2 and ϕ_3 can be found in the work by Dunn and Rajagopal [4]. The sign of the material moduli ϕ_1 and ϕ_2 is the subject of much controversy [5]. In the experiments on several non-Newtonian fluids, the experimentalists have not confirmed these restrictions on ϕ_1 and ϕ_2.

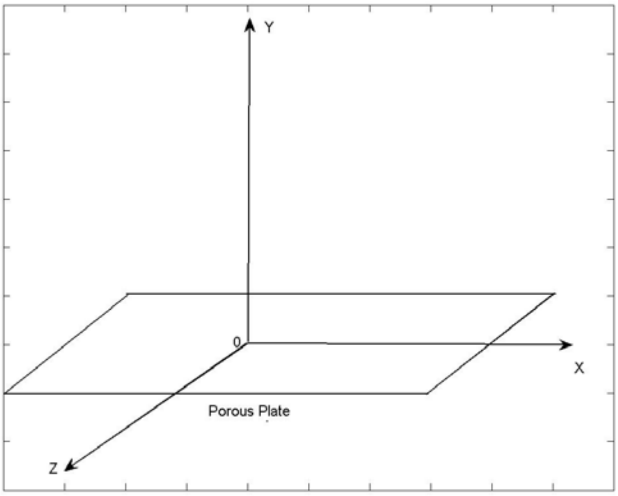

Figure-1: Geometry of the fluid over porous bed.

In general, the equations (in the dimensional form) of motions in the X, Y and Z directions and when the bounding surface is porous are given by

$$\rho \frac{DU_1}{DT} = \rho F_X + \frac{\partial S_{XX}}{\partial X} + \frac{\partial S_{XY}}{\partial Y} + \frac{\partial S_{XZ}}{\partial Z} - \frac{\mu}{k} U_1 \tag{5}$$

$$\rho \frac{DU_2}{DT} = \rho F_Y + \frac{\partial S_{YX}}{\partial X} + \frac{\partial S_{YY}}{\partial Y} + \frac{\partial S_{YZ}}{\partial Z} - \frac{\mu}{k} U_2 \tag{6}$$

$$\rho \frac{DU_3}{DT} = \rho F_Z + \frac{\partial S_{ZX}}{\partial X} + \frac{\partial S_{ZY}}{\partial Y} + \frac{\partial S_{ZZ}}{\partial Z} - \frac{\mu}{k} U_3 \tag{7}$$

Introducing the following non-dimensional variables as:

$$U_i = \frac{\phi_1 u_i}{\rho L} \quad T = \frac{\rho L^2 t}{\phi_1} \quad \phi_2 = \rho L^2 \beta \quad P = \frac{\phi_1^2 p}{\rho L^2}$$

$$\frac{X_i}{L} = x_i \quad \frac{Y_i}{L} = y_i \quad \phi_3 = \rho L^2 \upsilon_c \quad A_i = \frac{\phi_1^2 a_i}{\rho^2 L^3}$$

$$S_{ij} = \frac{\phi_1^2 s_{ij}}{\rho L^2} \quad E_{ij}^{(1)} = \frac{\phi_1 e_{ij}^{(1)}}{\rho L^2} \quad E_{ij}^{(2)} = \frac{\phi_1^2 e_{ij}^{(2)}}{\rho^2 L^4} \quad k = \frac{\rho L^3}{\phi_1^2 K}$$

where T is the (dimensional) time variable, ρ is the mass density and L is a characteristic length. We consider a class of plane flows given by the velocity components

$$u_1 = u(y,t) \text{ and } u_2 = 0 \text{ while } u_3 = 0 \tag{8}$$

in the directions of rectangular Cartesian coordinates x and y. The velocity field given by Equation (8) identically satisfies the incompressibility condition.

The stresses in the non dimensional form are

$$S_{xx} = -p + \upsilon_c \left(\frac{\partial u}{\partial y} \right)^2 \tag{9}$$

$$S_{yy} = -p + \left(\upsilon_c + 2\beta \right) \left(\frac{\partial u}{\partial y} \right)^2 \tag{10}$$

$$S_{xy} = \frac{\partial u}{\partial y} + \beta \frac{\partial}{\partial y} \left(\frac{\partial u}{\partial t} \right) \tag{11}$$

In view of the above, the equations of motion will be transformed to

$$\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} + \beta \frac{\partial}{\partial t} \left(\frac{\partial^2 u}{\partial y^2} \right) - \frac{1}{K} u \tag{12}$$

and

$$0 = -\frac{\partial p}{\partial y} + \left(2\beta + \upsilon_c \right) \frac{\partial}{\partial y} \left(\frac{\partial u}{\partial y} \right)^2 \tag{13}$$

Equation (12) shows that $-\frac{\partial p}{\partial x}$ must be independent of space variables and hence may be taken as $\varsigma\left(t\right)$. Equation (13) now yields

$$p = p_0\left(t\right) - \varsigma\left(t\right) x + \left(\upsilon_c + 2\beta \right) \left(\frac{\partial u}{\partial y} \right)^2 \tag{14}$$

Considering $\varsigma\left(t\right) = 0$, the flow characterized by the velocity is given by

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + \beta \frac{\partial}{\partial t} \left(\frac{\partial^2 u}{\partial y^2} \right) - \frac{1}{K} u \tag{15}$$

where K is the non-dimensional porosity constant. It may be noted that, the presence of β changes the order of differential from two to three.

3. SOLUTION OF THE PROBLEM

The oscillations of a classical viscous liquid on the upper half of the plane $y \geq 0$ with the bottom oscillating with a velocity $\alpha e^{i\sigma t}$ then

$$u\left(0,t\right) = \alpha e^{i\sigma t} \text{ while } u\left(\infty,t\right) = 0 \tag{16}$$

Assuming the trial solution as

$$u\left(y,t\right) = \alpha e^{i\sigma t} f\left(y\right), \text{ then } f''\left(y\right) = p^2 f\left(y\right) \tag{17}$$

where

$$p^2 = \frac{\frac{1}{K} + i\sigma}{1 + i\beta\sigma} = \frac{\left(\beta\sigma^2 + \frac{1}{K} \right) + i\left(\sigma - \frac{\beta\sigma}{K} \right)}{\left(1 + \beta^2\sigma^2 \right)} \tag{18}$$

When expressed in the polar form

$$p = r\left(\cos\left(\frac{\pi}{4}-\frac{\varepsilon}{2}\right)+i\sin\left(\frac{\pi}{4}-\frac{\varepsilon}{2}\right)\right) \tag{19}$$

where $$r = \frac{\left[\left(\beta\sigma^2+\frac{1}{K}\right)^2+\left(\sigma-\frac{\beta\sigma}{K}\right)^2\right]^{\frac{1}{4}}}{\sqrt{\left(1+\beta^2\sigma^2\right)}}, \quad \varepsilon = \tan^{-1}\left(Q\right) \; and \; Q = \frac{\left(\beta\sigma^2+\frac{1}{K}\right)}{\left(\sigma-\frac{\beta\sigma}{K}\right)}$$

Also the conditions satisfied are:

$$f\left(0\right)=1, \quad f\left(\infty\right)=0 \tag{20}$$

This yields the solution

$$u\left(y,t\right) = \alpha e^{\left(i\sigma t - yr\left(\cos\left(\frac{\pi}{4}-\frac{\varepsilon}{2}\right)+i\sin\left(\frac{\pi}{4}-\frac{\varepsilon}{2}\right)\right)\right)} \tag{21}$$

The flow is thus represented by standing transverse wave with its amplitude rapidly diminishing with increasing distance from the plane. The flow rate F is given by

$$F = \int_0^1 u\,dy = \alpha\int_0^1 e^{i\sigma t - yr\alpha_1}dy = \frac{\alpha e^{i\sigma t}}{r\alpha_1}\left(1-e^{-r\alpha_1}\right)$$

where $$\alpha_1 = \cos\left(\frac{\pi}{4}-\frac{\varepsilon}{2}\right)+i\sin\left(\frac{\pi}{4}-\frac{\varepsilon}{2}\right)$$

4. RESULTS AND DISCUSSIONS

1. Figure 2 shows the effect of porosity $\left(K\right)$ on the flow rate $\left(F\right)$. In general it is found that, as the porosity $\left(K\right)$ of the fluid bed increases, flow rate $\left(F\right)$ decreases. In addition to the above, it also seen that, for a fixed porosity $\left(K\right)$ of the fluid bed, as the angle of inclination $\left(\alpha\right)$ of the plate increases, the same effect as noted above is seen.

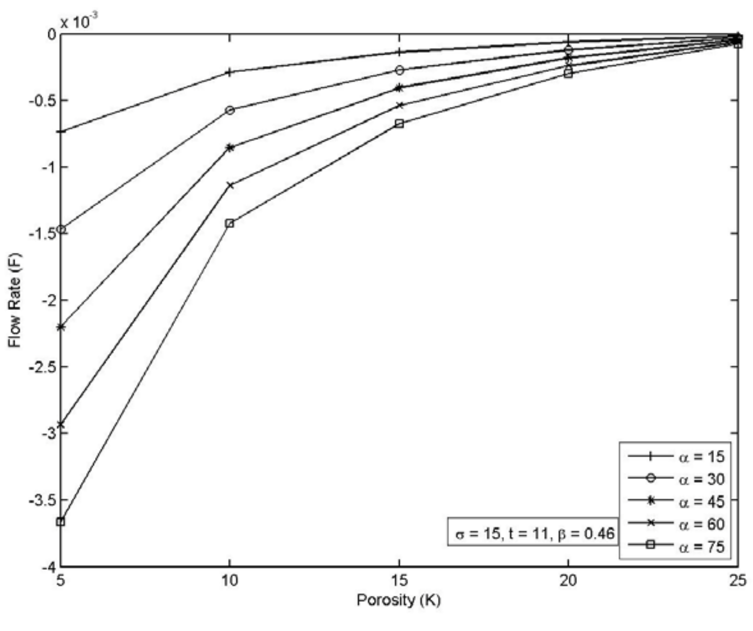

Figure-2: Effect of angle of inclination $\left(\alpha\right)$ on the flow rate $\left(F\right)$.

2. Figure 3, Figure 4 and Figure 5 show the behaviour of flow rate (F) with respect to the porosity parameter (K). In general it is seen that, as the porosity (K) of the fluid bed increases, the flow rate (F) also increases. This is partly due to the flow over the bed and partly due to the fluid percolation through the fluid bed. Also, for a fixed porosity parameter (K) as the visco elasticity (β) of the fluid increases, the flow rate (F) decreases. Figure 4 and Figure 5 shows that the effect of the frequency of excitation (σ) and time (t) has profound effect on the flow rate (F).

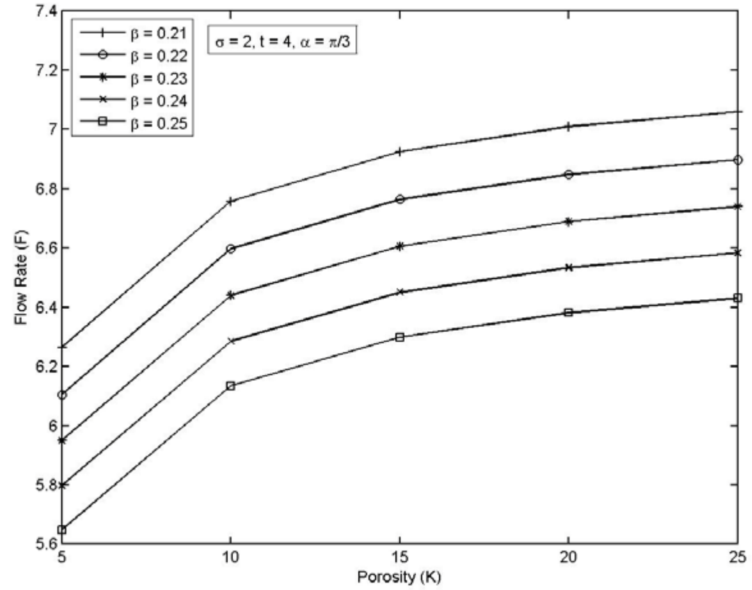

Figure-3: Effect of visco elasticity (β) on the flow rate (F).

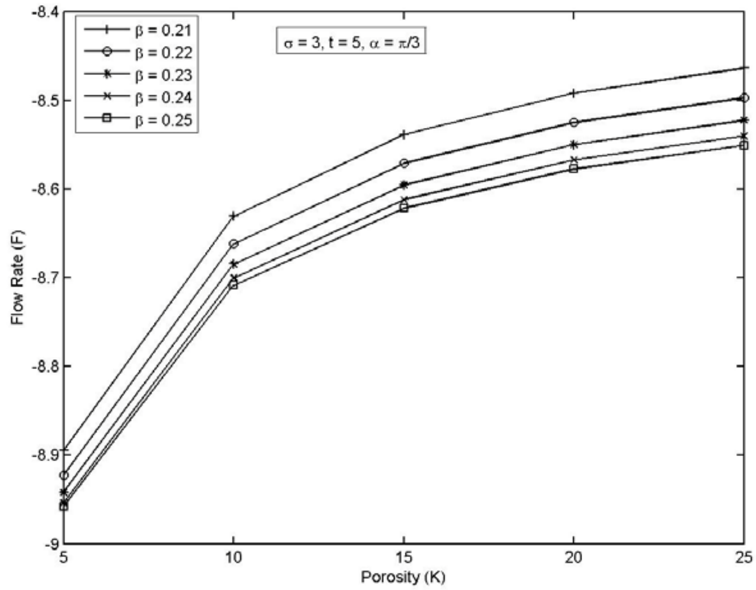

Figure-4: Effect of visco elasticity (β) on the flow rate (F).

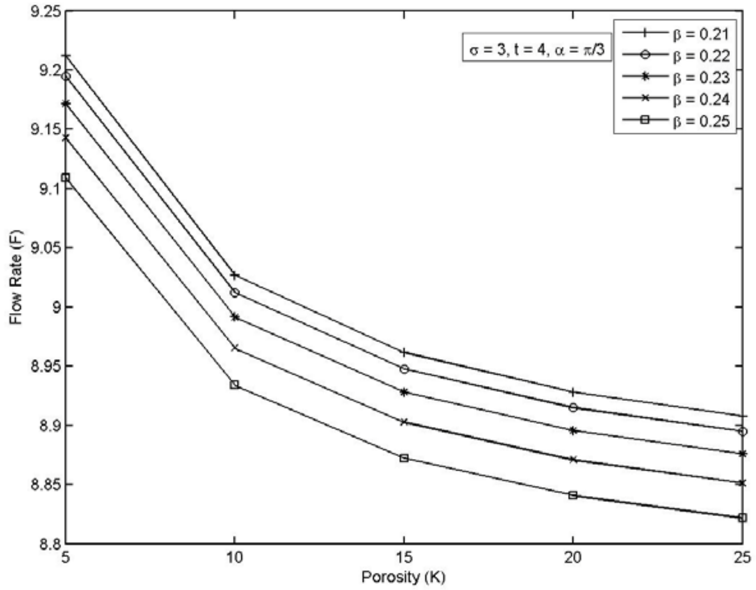

Figure-5: Effect of visco elasticity (β) on the flow rate (F).

3. Figure 6 shows the nature of flow rate (F) as the frequency of excitation (σ) is varied. It is noticed that, as the frequency of excitation (σ) is increased, the flow rate (F) decreases. Further, it is observed that, for a constant porosity (K) of the fluid bed, as frequency of excitation (σ) increases, the flow rate (F) also decreases which is in confirmation with the natural law of fluid mechanics.

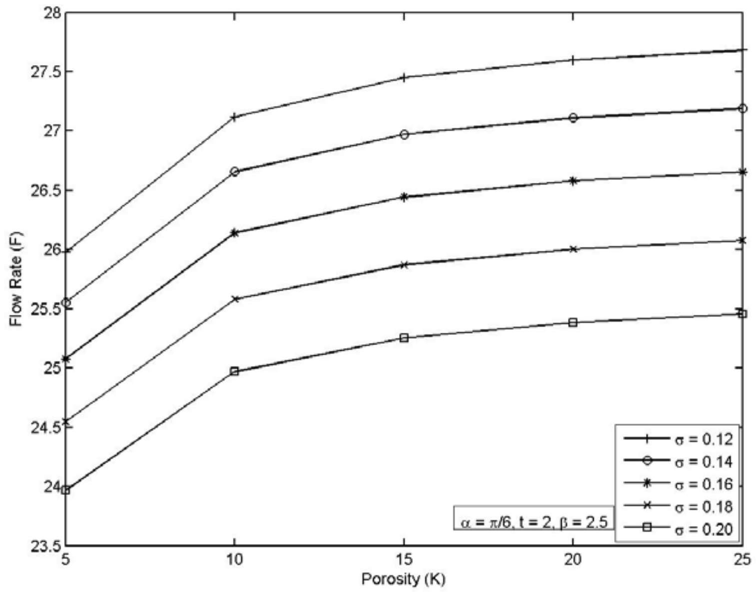

Figure-6: Effect of frequency of excitation (σ) on flow rate (F).

4. The effect of porosity (K) on the flow rate (F) has been analyzed in Figure 7. It is noticed that, as the porosity (K) of the fluid bed increases, irrespective of time (t), the flow rate (F) remains constant. However, it seen that, at time (t) increases the flow rate (F) decreases when all other flow entities are held constant.

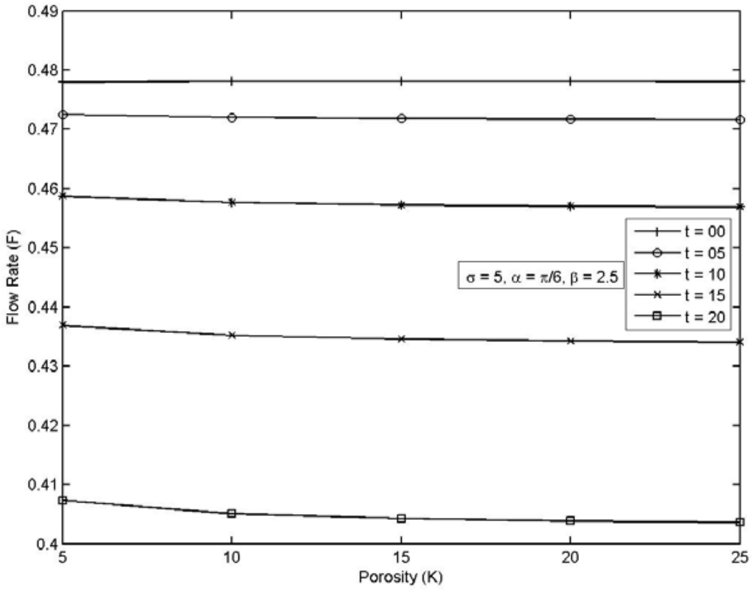

Figure-7: Effect of time (t) on the flow rate (F).

5. CONCLUSIONS

As the porosity (K) of the fluid bed increases, it is noticed that, the flow rate (F) decreases. Also, when the visco elasticity (β) of the fluid increases, the flow rate (F) is found to be decreasing. As the frequency of excitation (σ) of the fluid bed is increased, the flow rate (F) decreases. Irrespective of the porosity (K) of the bounding surface, as time (t) increases, the flow rate (F) is found to be constant.

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