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MEANNESS OF SPECIAL CLASS OF GRAPHS

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ABSTRACT

Let G = (V, E) be a simple graph. G is said to be a mean graph if f : V(G)→{0,1,2….q} such that for each edge uv, the induced map f defined by f*(uv) =* $\left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ *is injective where* $\lceil x \rceil$ *denote the least integer which is greater than or equal to x and f*(E(G)) = {1,2,3… q}*

The graph that admits a mean labeling is called a mean graph.

In this paper, we proved that $D_2[K_{1,n}]$, $C_2[P_n]$, $T_2[C_3]$, *Gl (n)*, $D_2(K_{1,n})$, $C_2(P_n)$, $C_3{*}C_n$, *Ladder* $P_n \times P_2$ *and* $Z\text{-}(P_n)$ *are mean graphs.*

Key words: *Mean labeling, Mean graph.*

2000 Mathematics Subject Classification: *05C78.*

1. INTRODUCTION

A graph G is a finite non-empty set of objects called vertices together with a set of unordered pairs of distinct vertices of G which is called edges. Each e = {uv} of vertices in E is called an edge or a line of G. For Graph theory terminology we follow [2].

2. PRELIMINARIES:

A graph G is a finite non-empty set of objects called vertices together with a set of unordered pairs of distinct vertices of G which is called edges. Each pair e = {u, v} of vertices in E is called an edge or a line of G in which e is said to join u and v. We write e = uv and say that u and v are adjacent vertices (sometimes denoted as u adj v); vertex u and the edge e are incident with each other, as are v and e. If two distinct edges e_1 and e_2 are incident with a common vertex, then they are called adjacent edges. A graph with p vertices and q edges is called (p, q) - graph, we proved that $D_2[K_{1,n}]$, $C_2[P_n]$, $T_2[C_3]$, Gl (n), $D_2(K_{1,n})$, $C_2(P_n)$, $C_3{*}C_n$,Ladder $P_n \times P_2$ and $Z\text{-}(P_n)$ are mean graphs

Definition: 2.1 (DOUBLE QUADRILATERAL) It is a graph obtained from the path P_n by having two quadrilaterals on each edge of the path with one side of the quadrilateral as an edge of the path.It is denoted by **DQ(n)**.

Definition: 2.2 (TRIPLE QUADRILATERAL) It is a graph obtained from the path P_n by having three quadrilaterals on each edge of the path with one side of the quadrilateral as an edge of the path. it is denoted by **TQ(n)**.

Definition: 2.3 (GLOBE) Two isolated vertex are joined by n paths of length 2.it is denoted by **GL(n)**.

Definition: 2.4 (Shadow Graph) Let G be a connected Graph. A Graph, constructed by taking two copies of G say G_1 and G_2 and joining each vertex u in G_1 to the neighbours of the corresponding vertex v in G_2 ,that is for every vertex u in G_1 there exists v in G_2 such that N (u) = N (v). The resulting Graph is known as shadow Graph and it is denoted by $D_2(G)$.

Definition: 2.5 ($C_2(P_n)$) Graph obtained from a path P_n by joining each end vertices of an edge with two isolated vertex.it is denoted by $C_2(P_n)$.

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Definition: 2.6 (C_3*C_n) A graph obtained by joining one vertex of the cycle C_n with one of the vertex of cycle C3.it is denoted by **C_3*C_n**.

Definition: 2.7 To define the ***product*** G_1 x G_2, consider any two vertices u = (u_1, u_2) and v = (v_1, v_2) in $V_1\times V_2$. Then u and v are adjacent in $G_1\times G_2$ whenever ($u_1 = v_1$ and u_2 adj v_2) or ($u_2 = v_2$ and u_1 adj v_1).
The product $P_n\times P_2$ is called ***Laddar***.

Definition: 2.8(Z-P_n) In a pair of path Pn, i+1th vertex of a path p_1 is joined with i+1th vertex of a path P_2.it is denoted by **Z-P_n**.

3. MAIN RESULTS

Theorem 3.1: DQ (n) is a mean graph.

Proof: Let G = DQ (n).

Let V (G) = {[u_i:1≤i≤n],[(u_{ij},v_{ij}):1≤i≤n-1,1≤j≤2]}.

Let E(G) = {[(u_iu_{i+1}) ∪ (u_iu_{i1}) ∪ ($u_{i1}u_{i2}$) ∪ ($u_{i2}u_{i+1}$) ∪ (u_iv_{i1}) ∪ ($v_{i1}v_{i2}$) ∪ ($v_{i2}u_{i+1}$):1≤i≤n-1]}.

Define f: V (G) -> {0, 1, 2…….q}

The vertex labellings are

$f(u_i) = 7(i-1)$ 1≤i≤n
$f(u_{i1}) = 7i-2$ 1≤i≤n-1
$f(u_{i2}) = 7i-1$ 1≤i≤n-1
$f(v_{i1}) = 7i-6$ 1≤i≤n-1
$f(v_{i2}) = 7i-5$ 1≤i≤n-1

The induced edge labellings are

$f^*(u_iu_{i+1}) = 7i-3$ 1≤i≤n-1
$f^*(u_iu_{i1}) = 7i-4$ 1≤i≤n-1
$f^*(u_{i1}u_{i2}) = 7i-1$ 1≤i≤n-1
$f^*(u_{i2}u_{i+1}) = 7i$ 1≤i≤n-1
$f^*(u_iv_{i1}) = 7i-6$ 1≤i≤n-1
$f^*(v_{i1}v_{i2}) = 7i-5$ 1≤i≤n-1
$f^*(v_{i2}u_{i+1}) = 7i-2$ 1≤i≤n-1

Clearly, E (G) = {1, 2 ….q}

Hence, DQ (n) is a mean graph.

For example, DQ (3) is a mean graph as shown in figure 3.2.

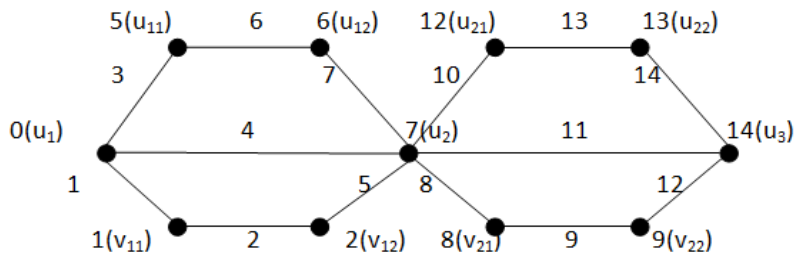

Figure 3.2

Theorem 3.3: TQ(n) is a mean graph.

Proof: Let G = TQ (n).

Let V (G) = { [u_i:1≤i≤n],[(u_{ij},v_{ij},w_{ij}):1≤i≤n-1,1≤j≤2]}.

Let
$E(G) = \{[(u_iu_{i+1}) \cup (u_iu_{i1}) \cup (u_{i1}u_{i2}) \cup (u_{i2}u_{i+1}) \cup (u_iv_{i1}) \cup (v_{i1}v_{i2}) \cup (v_{i2}u_{i+1}) \cup (u_iw_{i1}) \cup (w_{i1}w_{i2}) \cup (w_{i2}u_{i+1}): 1 \le i \le n-1]\}$.

Define f: V (G) -> {0, 1, 2…….q}

The vertex labellings are
$f(u_i) = 10(i-1)$ $1 \le i \le n$
$f(u_{i1}) = 10i-3$ $1 \le i \le n-1$
$f(u_{i2}) = 10i-1$ $1 \le i \le n-1$
$f(v_{i1}) = 10i-9$ $1 \le i \le n-1$
$f(v_{i2}) = 10i-8$ $1 \le i \le n-1$
$f(w_{i1}) = 10i-5$ $1 \le i \le n-1$
$f(w_{i2}) = 10i-2$ $1 \le i \le n-1$

The induced edge labeling are
$f^*(u_iu_{i+1}) = 10i-5$ $1 \le i \le n-1$
$f^*(u_iu_{i1}) = 10i-6$ $1 \le i \le n-1$
$f^*(u_{i1}u_{i2}) = 10i-2$ $1 \le i \le n-1$
$f^*(u_{i2}u_{i+1}) = 10i$ $1 \le i \le n-1$
$f^*(u_iv_{i1}) = 10i-9$ $1 \le i \le n-1$
$f^*(v_{i1}v_{i2}) = 10i-8$ $1 \le i \le n-1$
$f^*(v_{i2}u_{i+1}) = 10i-4$ $1 \le i \le n-1$
$f^*(u_iw_{i1}) = 10i-7$ $1 \le i \le n-1$
$f^*(w_{i1}w_{i2}) = 10i-3$ $1 \le i \le n-1$
$f^*(w_{i2}u_{i+1}) = 10i-1$ $1 \le i \le n-1$

Clearly, E (G) = {1, 2 ….q}

Hence, TQ (n) is a mean graph.

For example, TQ (3) is a mean graph as shown in figure 3.4.

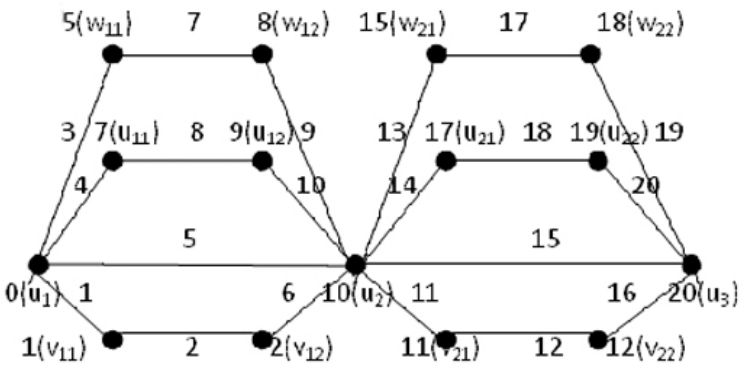

Figure: 3.4 (TQ (3))

Theorem 3.5: Prove that Gl (n) is a mean graph.

Proof: Let G = Gl (n)

Let $V(G) = \{u,v,(w_i:1 \le i \le n)\}$.

Let $E(G) = \{[(uw_i) \cup [(vw_i):1 \le i \le n]\}$.

Define f: V (G) -> {0, 1, 2….q}

The vertex labellings are
$f(u) = 0$ $1 \le i \le n$
$f(w_i) = 2i-1$ $1 \le i \le n$
$f(v) = 2n$

The induced edge labellings are
$f^*(uw_i) = i$ $1 \le i \le n$
$f^*(w_iv) = n+i$ $1 \le i \le n$

Clearly, E (G) = {1, 2 ….q}

Hence, Gl (n) is a mean graph.

For example, Gl (4) is a mean graph as shown in figure 3.6.

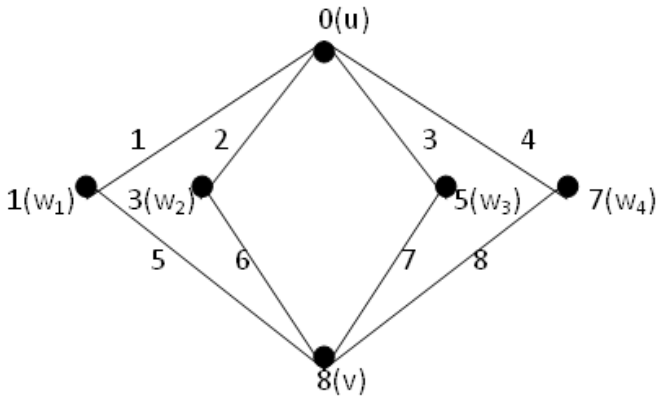

Figure: 3.6

Theorem 3.7: D_2 ($K_{1,n}$) is a mean graph.

Proof: Let G = D_2 ($K_{1,n}$).

Let V (G) = {u, v, (u_i, v_i: $1 \le i \le n$)}.

Let E (G) = {(uu_i) ∪ (uv_i) ∪ (vu_i) ∪ (vv_i):$1 \le i \le n$}.

Let f: V (G) -> {0, 1, 2……4n} defined by

f (u) = 4n
f (v) = 0
f (u_i)=2i-1 $1 \le i \le n$
f (v_i)=2n+2i-1 $1 \le i \le n$

The induced edge labellings are

f* (vu_i) = i $1 \le i \le n$
f* (vv_i) = n+i $1 \le i \le n$
f* (uu_i) = 2n+i $1 \le i \le n$
f* (uv_i) = 3n+i $1 \le i \le n$

Clearly, E (G) = {1, 2, 3....q}

Hence, D_2 ($K_{1,n}$) is a mean graph.

For example, D_2 ($K_{1,3}$) is a mean graph as shown in figure 3.8.

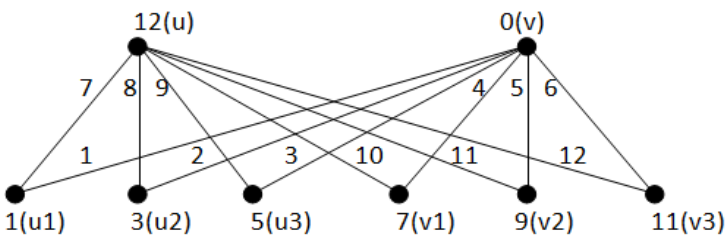

Figure: 3.8

Theorem 3.9: C_2 (P_n) is a mean graph.

Proof: Let G = C_2 (P_n)

Let V(G) = { (u_i:$1 \le i \le n$) , (v_i:$1 \le i \le n-1$) , (w_i:$1 \le i \le n-1$) }.

Let E (G) = { (u_iv_i) ∪ (v_iu_{i+1}) ∪ (u_iw_i) ∪ (w_iu_{i+1}) ∪ (u_iu_{i+1}):$1 \le i \le n-1$}.

Define f: V (G) -> {0, 1, 2….q}

The vertex labellings are

$f(u_i) = 5(i-1)$ $\quad 1 \le i \le n$
$f(v_i) = 5i-3$ $\quad 1 \le i \le n-1$
$f(w_i) = 5i-1$ $\quad 1 \le i \le n-1$

The induced edge labeling are

$f^*(u_iv_i) = 5i-4$ $\quad 1 \le i \le n-1$
$f^*(v_iu_{i+1}) = 5i-1$ $\quad 1 \le i \le n-1$
$f^*(u_iw_i) = 5i-3$ $\quad 1 \le i \le n-1$
$f^*(w_iu_{i+1}) = 5i$ $\quad 1 \le i \le n-1$
$f^*(u_iu_{i+1}) = 5i-2$ $\quad 1 \le i \le n-1$

Clearly, E (G) = {1, 2 ….q}

Hence, C_2 (P_n) is a mean graph.

For example, C_2 (P_3) is a mean graph as shown in figure **3**.10

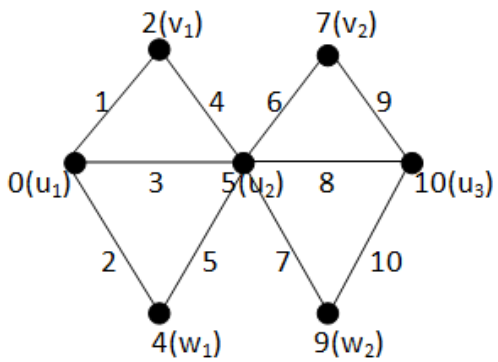

Figure 3.10

Theorem 3.11: C_3*C_n is a mean graph.

Proof: Let G = C_3*C_n

Let V (G) = {(u_1, u_2, u_3), (u_{1i}:1≤ I ≤ n-1)}.

Let E(G) = {(u_1u_2) ⊡ (u_2u_3) ⊡ (u_1u_3) ⊡ (u_1u_{11}) ⊡ ($u_1u_{1(n-1)}$) [($u_{1i}u_{i(i+1)}$):1≤i≤n-2)}.

Define f: V (G) > {0, 1, 2….q}

The vertex labellings are

f (v) = 0
f (w) = 2
f (u_1) = 3

Case (i):

When n is even

$f(u_1) = 3$
$f(u_2) = 5$
$f(u_{i+2}) = 4+2i$ $\quad 1 \le i \le (n/2)-1$
$f(u_{n+1-i}) = 2i+5$ $\quad 1 \le i \le (n/2)-1$

For example, C_3*C_4 is a mean graph as shown in figure 3.12.

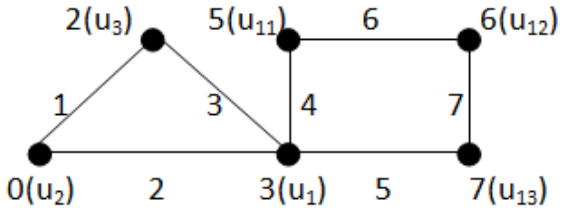

Figure 3.12 (a) (C_3*C_4)

Case (ii):

When n is odd

$f(u_1) = 3$

$f(u_2) = 5$

$f(u_{i+2}) = 4+2i \qquad 1 \le i \le (n-1)/2$

$f(u_{n+1-i}) = 2i+5 \qquad 1 \le i \le (n-3)/2$

For example, C_3*C_5 is a mean graph as shown in figure 3.12.

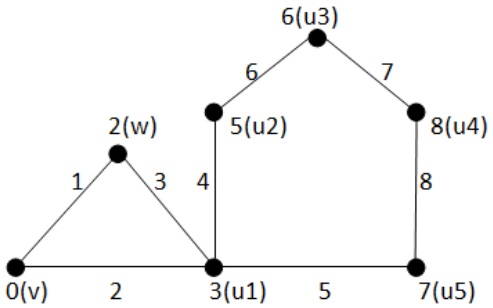

Figure 3.12 (b) (C_3*C_5)

Theorem 3.13: $P_n \times P_2$ is a mean graph.

Proof: Let $G = P_n \times P_2$

Let $V(G) = \{(u_{ij}):1 \le i \le n,\ 1 \le j \le 2\}$.

Let $E(G) = \{[(u_{i1}u_{i2}):1 \le i \le n] \cup [(u_{ij}u_{(i+1)j}):1 \le i \le n-1]\}$.

Define f: V (G) -> {0, 1, 2….q}

The vertex labellings are

$f(u_{i1}) = 3(i-1) \qquad 1 \le i \le n$

$f(u_{i2}) = 3i-2 \qquad 1 \le i \le n$

The induced edge labellings are

$f^*(u_{i1}u_{i2}) = 3i-2 \qquad 1 \le i \le n$

$f^*(u_{i1}u_{(i+1)1}) = 3i-1 \qquad 1 \le i \le n-1$

$f^*(u_{i2}u_{(i+1)2}) = 3i \qquad 1 \le i \le n-1$

Clearly, E (G) = {1, 2 ….q}

Hence, $P_n \times P_2$ is a mean graph.

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For example, $P_4 \times P_2$ is a mean graph as shown in figure 3.14.

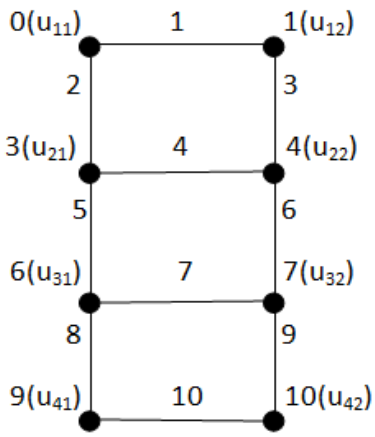

Figure 3.14: ($P_4 \times P_2$)

Theorem 3.15: Z-(P_n) is a mean graph.

Proof: Let G = Z-(P_n)

Let V (G) = $\{(u_i, v_i):1 \le i \le n\}$.

Let E (G) = $\{[(u_iu_{i+1}) \cup (v_iv_{i+1}) \cup [(u_iv_{(i+1)}):1 \le i \le n-1]\}$.

Define f: V (G) -> {0, 1, 2….q}

The vertex labellings are

$f(u_{n+1-1}) = (i-1) \qquad 1 \le i \le n$

$f(v_{n+1-i}) = 2n-3+i \qquad 1 \le i \le n$

The induced edge labellings are

$f^*(u_iv_{i+1}) = 2n-1-i \qquad 1 \le i \le n$

$f^*(u_iu_{(i+1)}) = n-i \qquad 1 \le i \le n-1$

$f^*(v_iv_{(i+1)}) = 3n-2-i \qquad 1 \le i \le n-1$

Clearly, E (G) = {1, 2 ….q}

Hence, Z-(P_n) is a mean graph.

For example, Z-(P_4) is a mean graph as shown in figure 3.16.

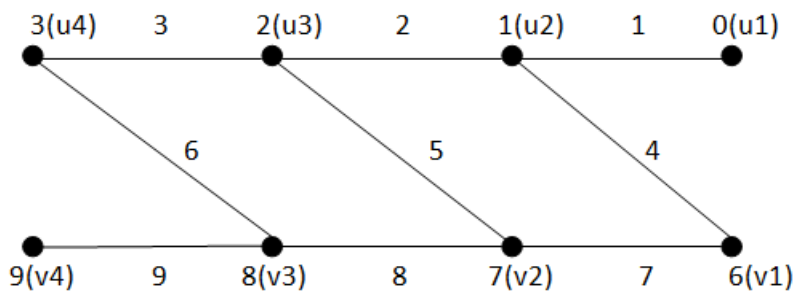

Figure: 3.16

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