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DERIVATION OF MODIFIED MHD-STOCHASTIC REYNOLDS EQUATION WITH CONDUCTING COUPLESTRESS FLUID ON THE SQUEEZE FILM LUBRICATION OF POROUS ROUGH ELLIPTICAL PLATES

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ABSTRACT

The surface roughness effects on the squeeze film characteristics of porous elliptical plates with combined effect of conducting couplestress fluid (CCSF) and transverse magnetic field is analysed. Based upon the Christensen's Stochastic model, the MHD-Stochastic Reynolds-type equation is derived to take account of the presence of CCSF and MHD in the presence of roughness. The closed form expressions are obtained for the MHD squeeze film pressure, load carrying capacity and squeeze film time. The results are presented for different values of operating parameters. It is observed, that the combined effect of surface roughness and transverse magnetic field on the squeeze film lubrication between porous elliptical plates with CCSF is to increase the load carrying capacity significantly and to delay squeeze film time as compared to corresponding Non-conducting Newtonian fluid (NCNF).

***Keywords**: Surface roughness, Elliptical plates, Conducting couplestress fluid, Magnetic field.*

1. INTRODUCTION

Owing to the development of modern machine elements, different types of lubricants are selected to meet the specific requirements for bearings operating under various severe conditions. To avoid unexpected viscosity variation with temperature, the use of liquid metals as lubricants has received extensive interest. Compared with the conventional non-conducting lubricating oils, liquid metals posses a higher thermal conductivity and a higher electrical conductivity. The property of high thermal conductivity reveals that the heat from the source of generation is readily conducted away. In addition, the property of high electrical conductivity implies that hydrodynamic flow behaviour can be adjusted by the application of an external magnetic field. Since the motion of an electrically conducting liquid across a magnetic field will evoke an electrical-field intensity. This electric-field intensity results in a current density interacting with the magnetic field to produce a Lorentz body force acting on the lubricant film, this may provide a component opposite to the direction of motion. As a consequence, the hydrodynamic characteristics of thin film bearings with electrically conducting lubricants can be improved by the application of external magnetic fields. By the application of external magnetic fields, the hydrodynamic lubrication performance is then improved. Many authors have investigated the MHD performance of bearings lubricated with an electrically conducting fluid in the presence of external magnetic field. Experimental and theoretical research is observed in the MHD hydrostatic bearings by Maki and Kuzma [1]. Further theoretical studies are found in the MHD squeeze-film bearings by Kuzma[2], Shukla[3], Lin *et al.*[4] and Hsu *et al.* [5], Bujurjke *et al.*[6]. These authors have shown that, the use of an electrically conducting lubricant in the presence of transverse magnetic field result in a better performance of squeeze film bearings.

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The study of the effect of surface roughness on lubrication of various bearing surfaces has attracted the attention of many researchers in recent years due to the reason that, in industrial practice, all bearing surfaces are rough to some extent. In general, the roughness asperities are of the same order as the mean separation in the lubricated contacts. Under these conditions, the influence of surface roughness on the performance characteristics of the bearings should be considered. In the literature, two types of an averaged value method have been proposed to model the large number of roughness asperities on the bearing surfaces. An average flow model of a randomly generated rough surface with known statistical properties for the bearing surfaces was proposed by Patir and Cheng [7, 8]. A Stochastic model for the study of hydrodynamic lubrication of rough surface was developed by Christensen [9]. Many investigators used this method to study the effect of surface roughness on the performance of various bearing systems such as the squeeze films by Prakash and Toner [10] Lin *et al.*[11], Naduvinamani *et al.* [12,13,14].

The Stokes [15] couplestress fluid theory adequately describes the flow behaviour of fluids containing a substructure such as lubricants with polymer additives. Several investigators have used the Stokes couplestress fluid theory to analyze the performance characteristics of various types of bearings with surface roughness [16-20].

The aim of this paper is to study the combined effect of surface roughness and magnetic effect on the squeeze film lubrication of porous elliptical plates in the presence of CCSF. The study of the squeeze film lubrication between two rough surfaces is quiet important to clarify the contact situation between two mating surfaces during squeezing effect in conjunction with the surface topography. It is shown that the combined effect of surface roughness and magnetic field in the presence of CCSF enhances the pressure, load carrying capacity and squeeze film time and the results are compared with corresponding NCNF and non-roughness case.

2. MATHEMATICAL FORMULATION AND SOLUTION OF THE PROBLEM

The configuration of the bearing is shown in Figure (1).Consider squeezing flow between two elliptical plates when the lower plate with porous facing of thickness δ is assumed to be fixed while the upper plate moves normally to itself approaching the lower surface. The uniform magnetic field B_0 is applied perpendicular to the two plates with the account of velocity slip at the porous facing due to Sparrow [21].The flow in the porous medium obeys the modified form of Darcy's law while equations of hydromagnetic lubrication theory hold.

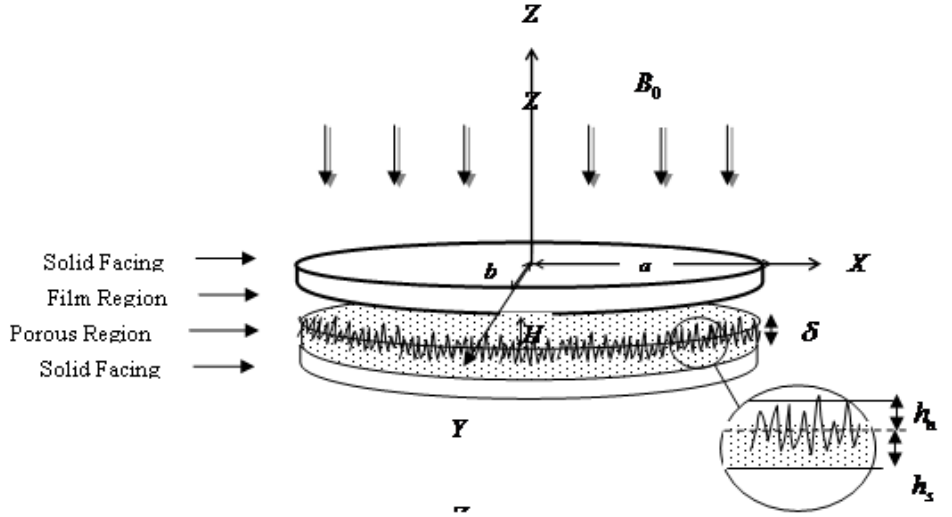

Figure 1. The physical representation of Squeeze film between porous rough Elliptical plates with transverse magnetic field

The basic equations of motion for the steady laminar conducting couplestress fluid flow in the film region with applied magnetic field are given by

$$\mu\frac{\partial^2 u}{\partial z^2}-\eta\frac{\partial^4 u}{\partial z^4}-\sigma B_0^2 u=\frac{\partial p}{\partial x} \tag{1}$$

$$\mu\frac{\partial^2 v}{\partial z^2}-\eta\frac{\partial^4 v}{\partial z^4}-\sigma B_0^2 v=\frac{\partial p}{\partial y} \tag{2}$$

$$\frac{\partial p}{\partial z}=0 \tag{3}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{4}$$

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For the porous region

$$u^* = -\frac{k}{\mu\left(1-\beta+\frac{kM_0^2}{M^{'}h_0^2}\right)}\frac{\partial p^*}{\partial x} \tag{5}$$

$$v^* = -\frac{k}{\mu\left(1-\beta+\frac{kM_0^2}{M^{'}h_0^2}\right)}\frac{\partial p^*}{\partial y} \tag{6}$$

$$w^* = -\frac{k}{\mu(1-\beta)}\frac{\partial p^*}{\partial z} \tag{7}$$

$$\frac{\partial u^*}{\partial x}+\frac{\partial v^*}{\partial y}+\frac{\partial w^*}{\partial z}=0 \tag{8}$$

The relevant boundary conditions for the velocity components are

(i) At the lower surface $z=0$

$u=v=0$ (no slip) (9a)

$w=0$ (9b)

$\frac{\partial^2 u}{\partial z^2}=\frac{\partial^2 v}{\partial z^2}=0$ (vanishing of couple stresses) (9c)

(ii) At the upper surface $z=h$

$u=v=0$ (no slip) (10a)

$\frac{\partial^2 u}{\partial z^2}=\frac{\partial^2 v}{\partial z^2}=0$ (vanishing of couplestresses) (10b)

$w=-w^*$ (10c)

The solution of equations (1) and (2) subject to equations (9a) and (10a) are

$$u=-\frac{h_0^2}{\mu M_0^2}\frac{\partial p}{\partial x}\left\{\frac{1}{(A^2-B^2)}\left(\frac{B^2 Cosh\frac{A(2z-h)}{2l}}{Cosh\frac{Ah}{2l}}-\frac{A^2 Cosh\frac{B(2z-h)}{2l}}{Cosh\frac{Bh}{2l}}\right)+1\right\} \tag{11}$$

$$v=-\frac{h_0^2}{\mu M_0^2}\frac{\partial p}{\partial y}\left\{\frac{1}{(A^2-B^2)}\left(\frac{B^2 Cosh\frac{A(2z-h)}{2l}}{Cosh\frac{Ah}{2l}}-\frac{A^2 Cosh\frac{B(2z-h)}{2l}}{Cosh\frac{Bh}{2l}}\right)+1\right\} \tag{12}$$

where $l=(\eta/\mu)^{1/2}$ is the couplestress parameter.

$$A=\left(\frac{1+\left(1-4l^2\sigma B_0^2/\mu\right)^{1/2}}{2}\right)^{1/2} \qquad B=\left(\frac{1-\left(1-4l^2\sigma B_0^2/\mu\right)^{1/2}}{2}\right)^{1/2}$$

Substituting equation (11) and (12) into (4) and integrating across the film thickness h .we obtain

$$\frac{\partial}{\partial x}\left\{\frac{h_0^2}{\mu M_0^2}\frac{\partial p}{\partial x}f(h,l,M_0)\right\}+\frac{\partial}{\partial y}\left\{\frac{h_0^2}{\mu M_0^2}\frac{\partial p}{\partial y}f(h,l,M_0)\right\}=w_h-w_0 \tag{13}$$

Since the upper plate is non-porous $w_h=0$

The velocity components in the y direction is continuous at the interface between the lower porous plate and the film so that

$$w_0 = -\left(\frac{dh}{dt} + \frac{k}{\mu(1-\beta)} \left[\frac{\partial p^*}{\partial z} \right]_{z=0} \right) \tag{14}$$

Substituting (14) in (13) we get

$$\frac{\partial}{\partial x}\left\{ \frac{h_0^2}{\mu M_0^2} \frac{\partial p}{\partial x} f(h,l,M_0) \right\} + \frac{\partial}{\partial y}\left\{ \frac{h_0^2}{\mu M_0^2} \frac{\partial p}{\partial y} f(h,l,M_0) \right\} = \left(\frac{dh}{dt} + \frac{k}{\mu(1-\beta)} \left[\frac{\partial p^*}{\partial z} \right]_{z=0} \right) \tag{15}$$

where

$$f(h,l,M_0) = \frac{2l}{A^2 - B^2} \left(\frac{B^2}{A} \tanh \frac{Ah}{2l} - \frac{A^2}{B} \tanh \frac{Bh}{2l} \right) + h \tag{16}$$

From equations (5) to (8) the fluid pressure in the porous region satisfies the equation

$$\frac{\partial^2 p^*}{\partial x^2} + \frac{\partial^2 p^*}{\partial y^2} + \frac{C_1^2}{1-\beta} \frac{\partial^2 p^*}{\partial z^2} = 0 \tag{17}$$

where

$$C_1^2 = \left(1 - \beta + \frac{\psi M_0^2 h_0}{\delta M'} \right)^{1/2} \tag{18}$$

and $\psi = \dfrac{k\delta}{h_0^3}$ (19)

Since the thickness of the porous layer δ is very small we can use the Morgan – Cameron approximation and eqn. (17) yields

$$\left. \frac{\partial p^*}{\partial z} \right|_{z=0} = -\frac{\delta(1-\beta)}{C_1^2} \left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} \right) \tag{20}$$

where $\dfrac{\partial p^*}{\partial z} = 0$ at $z = -\delta$ has been used.

Substitution of equation (20) in (15) we get

$$\frac{\partial}{\partial x}\left\{ \frac{h_0^2}{\mu M_0^2} \frac{\partial p}{\partial x} f(h,l,M_0) + \frac{k\delta}{\mu C_1^2} \right\} + \frac{\partial}{\partial y}\left\{ \frac{h_0^2}{\mu M_0^2} \frac{\partial p}{\partial y} f(h,l,M_0) + \frac{k\delta}{\mu C_1^2} \right\} = dh/dt \tag{21}$$

To introduce the surface roughness model on elliptical plates, the film thickness is considered to be made up of two parts

$$h = h_n(t) + h_s(x, y, \xi)$$

where $h_n(t)$ represents nominal smooth part of the film thickness, h_s denote the random part resulting from the surface roughness asperities measured from the nominal level and ξ is an index describing the definite roughness arrangements.

3. STOCHASTIC REYNOLDS EQUATION

In general two types of roughness structures are of interest namely (i) the longitudinal one-dimensional roughness in which the roughness pattern is in the form of long narrow ridges and valleys running in the x-direction, and (ii) the transverse one-dimensional roughness in which, the roughness striations are assumed to be running in the y-direction in the form of long narrow ridges and valleys. In the present paper the analysis is restricted to only one- dimensional longitudinal roughness, because, the longitudinal roughness and the transverse roughness can be made identical, by just a rotation of co-ordinate axes.

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For the longitudinal one-dimensional roughness, the film thickness assumes the form

$$h = h_n(t) + h_s(y, \xi)$$

Taking the expected values on both the sides of the equation (21) and assuming in accordance with the Christensen stochastic approach of the rough surfaces, the pressure gradient in the direction of roughness and flux perpendicular to it are the stochastic variables with zero or negligible variance. The variables with zero variance are the pressure gradient in x and the velocity along y directions.

The Reynolds equation for longitudinal roughness takes the form.

$$\frac{\partial}{\partial x}\left\{\frac{\partial E(p)}{\partial x} R_1\right\} + \frac{\partial}{\partial y}\left\{\frac{\partial E(p)}{\partial y} R_2\right\} = \frac{d\ h}{dt}$$

$$\frac{\partial^2 E(p)}{\partial x^2} + R\frac{\partial^2 E(p)}{\partial y^2} = \frac{d\ h}{dt} \tag{22}$$

where

$$R = \frac{R_2}{R_1} \tag{22a}$$

$$R_1 = \frac{h_0^2}{\mu M_0^2} E(f(h, l, M_0)) + \frac{k\delta}{\mu C_1^2} \tag{22b}$$

$$R_2 = \frac{h_0^2}{\mu M_0^2} \frac{1}{E\left(1/f(h, l, M_0)\right)} + \frac{k\delta}{\mu C_1^2} \tag{22c}$$

$E(*)$ denotes the expectancy operator defined by

where

$$E(*) = \int_{-\infty}^{\infty} (*)\, f(h_s) dh_s \tag{23}$$

where $f(h_s)$ is the probability distribution function of the random variable h_s .In most of the engineering application, the rough surfaces are of Gaussian type and hence, the following polynomial function is chosen to approximate the Gaussian distribution;

Following Christensen [9], we assume that

$$f(h_s) = \begin{cases} \frac{35}{32c^7}(c^2 - h_s^2)^3. & -c < h_s < c \\ 0 & elsewhere \end{cases} \tag{24}$$

where $\bar{\sigma} = \frac{c}{3}$ is the standard deviation.

The Stochastic Reynolds equation is

$$\frac{\partial^2 E(p)}{\partial x^2} + R\frac{\partial^2 E(p)}{\partial y^2} = \frac{dh/dt}{\left[\frac{h_0^2}{\mu M_0^2} E\left(f(h, l, M_0)\right) + \frac{\psi h_0^3}{\mu C_1^2}\right]} \tag{25}$$

and the boundary conditions are $p(x_1, y_1)=0$

where $\frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} = 1$ (26)

Solution of (25) with condition (26) we get the pressure equation as

$$E(p) = -\frac{dh/dt}{\left[\frac{h_0^2}{\mu M_0^2} E\left(f(h, l, M_0)\right) + \frac{\psi h_0^3}{\mu C_1^2}\right]} k(x_1, y_1) \tag{27}$$

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where $k(x_1, y_1)$ is the solution of the Poisson equation and is given by

$$k(x_1, z_1) = \frac{a^2 b^2}{2(Ra^2 + b^2)}\left(1 - \frac{x_1^2}{a^2} - \frac{y_1^2}{b^2}\right) \tag{28}$$

Substituting equation (28) into equation (27) the pressure distribution in the film region is

$$E(p) = -\frac{dh/dt}{\left[\frac{h_0^2}{\mu M_0^2} E\left(f(h, l, M_0)\right) + \frac{\psi h_0^3}{\mu C_1^2}\right]}\left\{\frac{a^2 b^2}{2(Ra^2 + b^2)}\left(1 - \frac{x_1^2}{a^2} - \frac{y_1^2}{b^2}\right)\right\} \tag{29}$$

The dimensionless form of the film pressure is given by

$$P = -\frac{E(p) h_0^3}{\mu (dh/dt)\pi ab} = \frac{1}{2\pi}\left(\frac{(a/b)}{R^*(a/b)^2 + 1}\right)\left(1 - \frac{x_1^2}{a^2} - \frac{y_1^2}{b^2}\right)\left[\frac{1}{M_0^2} E\left(F(h^*, l^*, M_0)\right) + \frac{\psi}{C_1^2}\right]^{-1} \tag{30}$$

where $l^* = \frac{2l}{h_0},\ \psi = \frac{k\delta}{h_0^3},\ h^* = \frac{h}{h_0}, x^* = \frac{x}{a}, y^* = \frac{y}{b}, \delta^* = \frac{\delta}{h_0}$

$$F(h^*, l^*, M_0) = \frac{l^*}{A^{*2} - B^{*2}}\left(\frac{B^{*2}}{A^*}\tanh\frac{A^* h^*}{l^*} - \frac{A^{*2}}{B^*}\tanh\frac{B^* h^*}{l^*}\right) + h^*$$

$$A^* = \sqrt{\frac{1 + \sqrt{1 - l^{*2} M_0^2}}{2}}, \quad B^* = \sqrt{\frac{1 - \sqrt{1 - l^{*2} M_0^2}}{2}}$$

$$R^* = \frac{R_2^*}{R_1^*},\ R_2^* = \frac{h_0^2}{\mu}\left(\frac{1}{M_0^2 E\left(1/\left(F(h^*, l^*, M_0)\right)\right)} + \frac{\psi}{C_1^2}\right),\ R_1^* = \frac{h_0^2}{\mu}\left(\frac{E(f(h^*, l^*, M_0))}{M_0^2} + \frac{\psi}{C_1^2}\right)$$

$$E(F(h^*, l^*, M_0)) = \frac{35}{32C^7}\int_{-C}^{C} F(h^*, l^*, M_0)\,(C^2 - h_s^{*2})^3 d\,h_s^*;$$

$$E\left(\frac{1}{F(h^*, l^*, M_0)}\right) = \frac{35}{32C^7}\int_{-C}^{C}\frac{(C^2 - h_s^{*2})^3}{F(h^*, l^*, M_0)} dh_s^*$$

The load carrying capacity of the squeeze film is obtained by integrating the pressure field over the area of the plate

$$E(W) = \int_{-a}^{a}\int_{-\frac{b}{a}\sqrt{a^2 - x_1^2}}^{\frac{b}{a}\sqrt{a^2 - x_1^2}} E(p)\, dy_1 dx_1$$

The non-dimensional mean instantaneous load carrying capacity W^* of the squeeze film is given by

$$W^* = \frac{1}{4\pi}\left(\frac{a/b}{R^*\left(a/b\right)^2 + 1}\right)\frac{1}{\left[\frac{1}{M_0^2} E\left(F(h^*, l^*, M_0)\right) + \frac{\psi}{C_1^2}\right]} \tag{31}$$

The time of approach in dimensionless form is obtained as

$$T^* = \int_1^{h^*} W^*\, dh^*$$

$$T^* = \int_1^{t_1/t_0}\frac{E(W) h_0^2 dt}{\mu\pi^2 a^2 b^2} = -\left(\frac{a/b}{4\pi}\right)\int_1^{h_1^*}\frac{dh^*}{\left(R^*\left(a/b\right)^2 + 1\right)\left[\frac{1}{M_0^2} E\left(F(h^*, l^*, M_0)\right) + \frac{\psi}{C_1^2}\right]} \tag{32}$$

As $C \to 0,\ M_0 \to 0,$ the equations for $P,\ W^*$ and T^* reduces to corresponding smooth and NCNF case.

4. RESULTS AND DISCUSSION

In the present paper, the influence of surface roughness on the MHD Reynolds equation is analysed with respect to the various dimensionless parameters namely couplestress parameter l^*, Hartmann number M_0, permeability parameter ψ and roughness parameter C. The parameter $l = \sqrt{\eta / \mu}$ arises due to the presence of polar additives in the lubricant. The dimension of (η / μ) is of length square and this length may be regarded as the chain length of the polar additives in the lubricant. Hence the parameter l^* provides the mechanism of interaction of fluid with the bearing geometry. For the purpose of numerical calculation and to compute the results for P, W^* and T^*, the parametric values of $\delta^* = 0.02,\ \beta = 0.2$ and $M' = 0.6$ are chosen.

4.1. Squeeze film Pressure

Figure 2 represents the variation of non-dimensional mean squeeze film pressure P with the dimensionless co-ordinate x^* for different values of l^*. It is observed that there is significant increase in squeeze film pressure P with increasing values of l^*.The effects are more pronounced for rough elliptical plates than smooth plates. Figure 3 shows the variation of non-dimensional mean squeeze film pressure P with the dimensionless co-ordinate x^* for different values of M_0. It is observed that the effect of applied magnetic field is to enhance the squeeze film pressure for rough plates as compared to a smooth plate case. Figure 4 shows the variation of non-dimensional mean squeeze film pressure P with the dimensionless co-ordinate x^* for different values of C. It is observed that there is significant increase in squeeze film pressure P with increasing values of C. Together, the conducting couplestress elliptical plates provide an enhancement in P as compared with the non-conducting Newtonian squeeze film plates.

4.2. Load carrying capacity

Figure 5 represents the variation of non-dimensional load carrying capacity W^* with h^* for different values of l^*. It is observed that with the increasing values of l^* there is significant increase in the load carrying capacity W^*. The effects are more prominent for CCSF as compared to conventional NCNF. The variation of non-dimensional means load carrying capacity W^* with h^* for different C is shown in Fig. 6. It is observed that W^* is prominent for increasing values of C. Figure 7 shows the variation of non-dimensional mean load carrying capacity W^* with h^* for different values of M_0. it is seen that increasing effect of M_0 is to enhance the mean load carrying capacity for rough elliptical plates as compared to smooth plate case. Figure 8 shows the variation of dimensionless mean load carrying capacity W^* with ψ for different values l^*. It is observe that W^* decreases for increasing values of ψ.

4.3. Squeeze film time

Variation on non-dimensional squeeze film time T^* with h_1^* for different values of l^* is shown in Figure 9. It is observed that with the increasing values of l^* there is significant increase in squeeze film time T^*. Figure 10 shows the variation on non-dimensional squeeze film time T^* with h_1^* for different values of C. It is observed that T^* increases with increasing values of C. The effects are more pronounced for CCSF as compared to NCNF. The variation of response time T^* as function h_1^* is given in Figure 11 for various values of M_0. It is observed that the effect of magnetic parameter is to enhance the squeezing effect for rough elliptical plates as compared to smooth plates.

5. CONCLUSIONS

On the basis of Christensen's stochastic model, this paper predicts the influence of the surface roughness on the squeeze-film characteristics between porous elliptical plates in the presence of CCSF and transverse magnetic field. The MHD-Stochastic Reynolds-type equation is derived to take account of the presence of CCSF and MHD in the presence of roughness.

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From the results obtained the following conclusions can be drawn.

- For the rough elliptical plates, load carrying capacity and squeezing film time increases with increasing the values of Harman number M_0 and couplestress parameter l^*. It is observed that effect of surface roughness C is to accentuate the squeeze film pressure and load carrying capacity as compared to smooth case.
- Further, it is observed that the squeeze film time is significant for increasing values of conduction couplestress parameter l^* and magnetic parameter M_0 with surface roughness effects as compared to NCNF.
- The present analysis provides useful information for design engineers in designing squeeze film bearing considering surface roughness with the combined effect of CCSF and transverse magnetic field with elliptical plates.

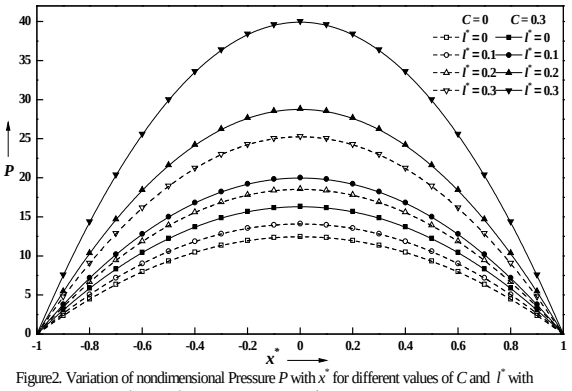

Figure2. Variation of nondimensional Pressure P with x^* for different values of C and l^* with $M_0 = 3$, $h^* = 0.4$, $y^* = 0$ $\lambda = 2$, $\psi = 0.001$, $M = 0.6$, $\delta^* = 0.01$, $\beta = 0.2$. .

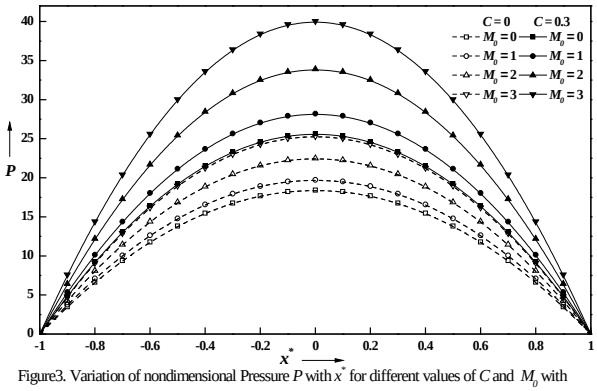

Figure3. Variation of nondimensional Pressure P with x^* for different values of C and M_0 with $l^* = 0.3$, $h^* = 0.4$, $y^* = 0$ $\lambda = 2$, $\psi = 0.001$, $m = 0.6$, $\delta^* = 0.01$, $\beta = 0.2$.

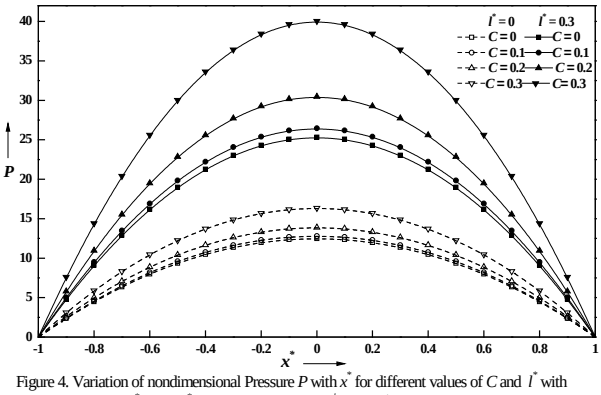

Figure 4. Variation of nondimensional Pressure P with x^* for different values of C and l^* with $M_0 = 3$, $h^* = 0.4$, $y^* = 0$ $\lambda = 2$, $\psi = 0.001$, $M = 0.6$, $\delta^* = 0.01$, $\beta = 0.2$. .

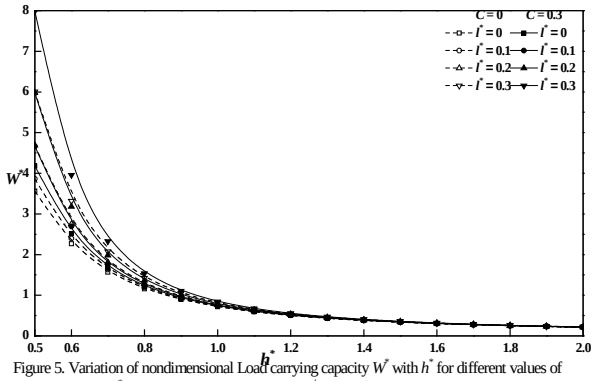

Figure 5. Variation of nondimensional Load carrying capacity W^* with h^* for different values of C and l^* with $M_0 = 3$, $\lambda = 2$, $\psi = 0.001$, $M = 0.6$, $\delta^* = 0.01$, $\beta = 0.2$. .

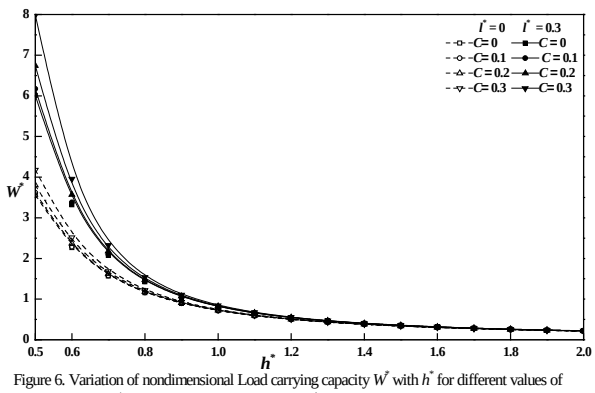

Figure 6. Variation of nondimensional Load carrying capacity W^* with h^* for different values of C and l^* with $M_0 = 3$, $\lambda = 2$, $\psi = 0.001$, $M = 0.6$, $\delta^* = 0.01$, $\beta = 0.2$. .

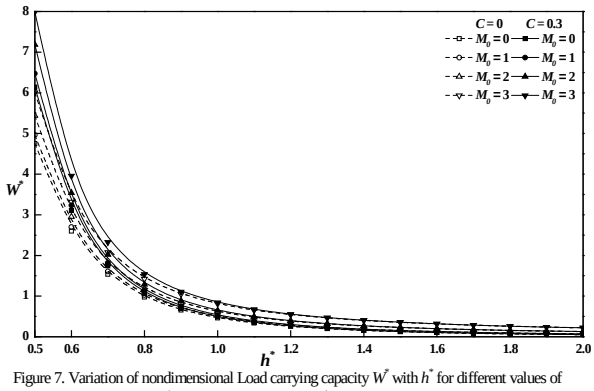

Figure 7. Variation of nondimensional Load carrying capacity W^* with h^* for different values of C and M_0 with $l^* = 0.3$, $\lambda = 2$, $\psi = 0.001$, $M = 0.6$, $\delta^* = 0.01$, $\beta = 0.2$. .

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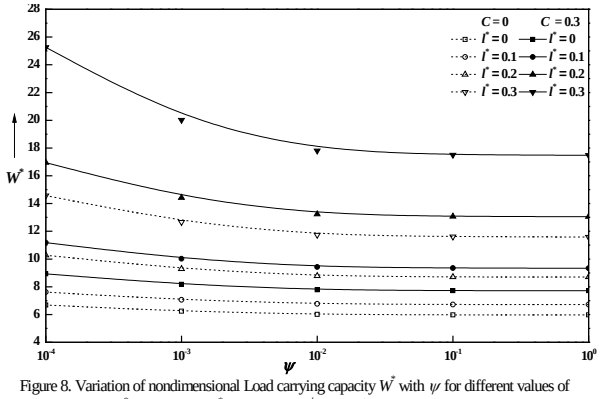

Figure 8. Variation of nondimensional Load carrying capacity W^* with ψ for different values of C and l^* with $M_0 = 3$, $h^* = 0.4$, $\lambda = 2$, $M' = 0.6$, $\delta^* = 0.01$, $\beta = 0.2$.

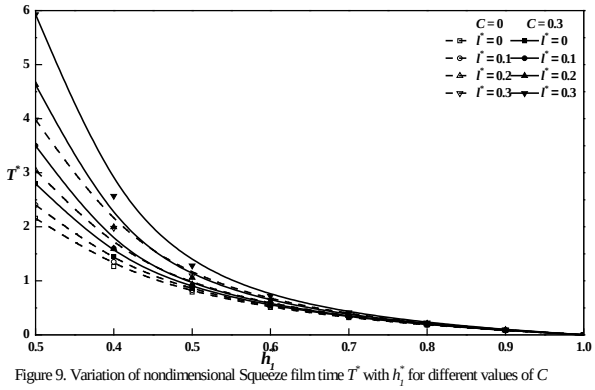

Figure 9. Variation of nondimensional Squeeze film time T^* with h_1^* for different values of C and l^* with $M_0 = 3$, $\lambda = 2$, $\psi = 0.001$, $M' = 0.6$, $\delta^* = 0.01$, $\beta = 0.2$. .

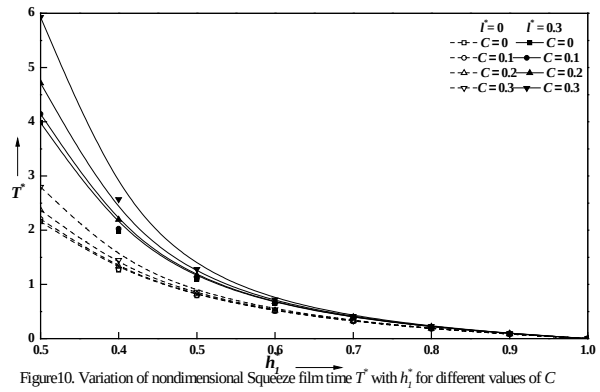

Figure10. Variation of nondimensional Squeeze film time T^* with h_1^* for different values of C and l^* with $M_0 = 3$, $\lambda = 2$, $\psi = 0.001$, $M' = 0.6$, $\delta^* = 0.01$, $\beta = 0.2$. .

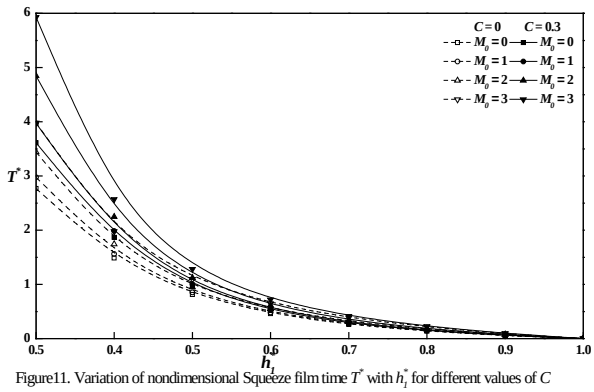

Figure11. Variation of nondimensional Squeeze film time T^* with h_1^* for different values of C and M_0 with $l^* = 0.3$, $\lambda = 2$, $\psi = 0.001$, $M' = 0.6$, $\delta^* = 0.01$, $\beta = 0.2$. .

NOMENCLATURE

a	length of semi major axis
b	length of semi minor axis
B_0	applied magnetic field in the $z-$direction
λ	a/b aspect ratio
δ	porous layer thickness
M_0	Hartmann number $\left(= B_0 h_0 (\frac{\sigma}{\mu})^{1/2}\right)$
M'	porousity
σ	Conductivity of fluid
h^*	Squeeze film thickness
h_1	film thickness after time Δt
h_1^*	dimensionless film thickness after time Δt
k	permeability of porous facing
l	couplestress parameter $(\eta/\mu)^{1/2}$
l^*	non-dimensional couple stress parameter $2l/h_0$
u, v, w	velocity components in film region
u^*, v^*, w^*	modified Darcy velocity components in x, y, z directions respectively
$E(p), p^*$	pressure in the film region and porous region respectively
P	non-dimensional pressure $\left\{-\frac{E(p)h_0^3}{\mu(dh/dt)\pi ab}\right\}$
t	mean time of approach

T^* non-dimensional time of approach $\left(\dfrac{8E(w)h_0^2}{\mu\pi R^4}\right)$

V squeezing velocity $\left(-dh/dt\right)$

W non-dimensional load carrying capacity $\left(-\dfrac{E(W)h_0^3}{\mu(dh/dt)\pi^2a^2b^2}\right)$

c maximum asperity deviation from the nominal film height

C dimensionless roughness parameter $\left(c/h_0\right)$

β ratio of microstructure size to pore size $\left(\left(\eta/\mu\right)/k\right)$

δ porous layer thickness

η material constant characterizing couple stress

μ viscosity coefficient

$\bar{\sigma}$ standard deviation $c/3$

σ^* parameter $\left(\sqrt{k}/\alpha\right)$

ψ permeability parameter $\left(=k\delta/h_0^3\right)$

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