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GEODESIC GRAPHOIDAL COVERING NUMBER OF GRAPHS<br>T. Gayathri*1 and S. Meena ${ }^{2}$<br>${ }^{1}$ Department of Mathematics, Sri Manakula Vinayagar Engineering College, Puducherry-605 107, India.<br>${ }^{2}$ Department of Mathematics, Government Arts and Science College, Chidambaram-608 102, India.

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#### Abstract

A geodesic graphoidal cover of a graph G is a collection $\psi$ of shortest paths in $G$ such that every path in $\psi$ has at least two vertices, every vertex of $G$ is an internal vertex of at most one path in $\psi$ and every edge of $G$ is in exactly one path in $\psi$. The minimum cardinality of a geodesic graphoidal cover of $G$ is called the geodesic graphoidal covering number of $G$ and is denoted by $\eta_{g}$. In this paper we determine $\eta_{g}$ for bicyclic graphs.


Key words: Graphoidal covers, Acyclic graphoidal cover, Geodesic Graphoidal cover

## 1. INTRODUCTION

A graph is a pair $G=(V, E)$, where $V$ is the set of vertices and $E$ is the set of edges. Here we consider only nontrivial, finite, connected, undirected graph without loops or multiple edges. The order and size of $G$ are denoted by $p$ and $q$ respectively. For graph theoretic terminology we refer to Harary [4]. The concept of graphoidal cover was introduced by B.D Acharya and E. Sampathkumar [1] and the concept of acyclic graphoidal cover was introduced by Arumugam and Suresh Suseela [4].The reader may refer [5], [2] and [7] for the terms not defined here.

Let $p=\left(v_{1}, v_{2}, v_{3}, \ldots, v_{r}\right)$ be a path or a cycle in a graph $G=(V, E)$. Then vertices $\left(v_{2}, v_{3}, \ldots, v_{r-1}\right)$ are called internal vertices of $P$ and $v_{1}$ and $v_{r}$ are called external vertices of $P$. Two paths $P$ and $Q$ of a graph $G$ are said to be internally disjoint if no vertex of $G$ is an internal vertex of both P and $Q$.

Definition 1.1 [1]: A graphoidal cover of a graph $G$ is called a collection $\psi$ of (not necessarily open) paths in G satisfying the following conditions:
(i) Every path in $\psi$ has at least two vertices.
(ii) Every vertex of G is an internal vertex of at most one path in $\psi$.
(iii) Every edge of G is in exactly one path in $\psi$

The minimum cardinality of a graphoidal cover of G is called the graphoidal covering number of G and is denoted by $\eta(G)$.

Definition 1.2 [3]: A graphoidal cover $\psi$ of a graph G is called an acyclic graphoidal cover if every member of $\psi$ is an open path. The minimum cardinality of an acyclic graphoidal cover of $G$ is called the acyclic graphoidal covering number of G and is denoted by $\eta_{a}(G)$ or $\eta_{a}$.

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Definition 1.3 [4]: A geodesic graphoidal cover of a graph $G$ is a collection $\psi$ of shortest paths in $G$ such that every path in $\psi$ has at least two vertices, every vertex of G is an internal vertex of at most one path in $\psi$ and every edge of G is an exactly one path in $\psi$. The minimum cardinality of a geodesic graphoidal cover of G is called the geodesic graphoidal covering number of G and is denoted by $\eta_{g}$.

Definition 1.4 [1]: Let $\psi$ be a collection of internally disjoint paths in G. A vertex of G is said to be in the interior of $\psi$ if it is an internal vertex of some path in $\psi$. Any vertex which is not in the interior of $\psi$ is said to be an exterior vertex of $\psi$.

Theorem 1.5 [8]: For any graphoidal cover $\psi$ of $G$, let $t_{\psi}$ denote the number of exterior vertices of $\psi$. Let $t=\min t_{\psi}$ where the minimum is taken over all graphoidal covers of G . Then $\eta=q-p+t$

Corollary 1.6: For any graph $\mathrm{G}, \eta \geq q-p$. Morever the following are equivalent.
(i) $\eta=q-p$
(ii) There exists a graphoidal cover without exterior vertices.
(iii) There exists a set of internally disjoint and edge disjoint paths without exterior vertices.

In [4] given that $\eta \leq \eta_{a} \leq \eta_{g}$ and these inequalities can be strict and also for a tree $\eta=\eta_{a}=\eta_{g}=n-1$ and Theorem 1.5 and corollary 1.6 are true for geodesic graphoidal covers.
They observe that $\eta_{g}=q$ if and only if $G$ is Complete. Further for a cycle $C_{m}, \eta_{g}=\left\{\begin{array}{l}2 \text { if } \mathrm{m} \text { is even } \\ 3 \text { if } \mathrm{m} \text { is odd }\end{array}\right.$
Theorem 1.7 [4]: Let $G$ be a unicyclic graph with unique cycle $C$ which is even. Let $n$ denote the number of pendant vertices of $G$ and let $m$ denote the number of vertices on $C$ with degree greater than 2 . Then $\eta_{g}=\left\{\begin{array}{l}2 \text { if } m=0 \\ n\left(\begin{array}{l}\text { if } \mathrm{m} \geq 2 \text { and every }(v, w) \text {-section of C in which all vertices } \\ \text { except } v \text { and } w \text { have degree } 2 \text { is a shortest path }\end{array}\right. \\ n+1 \text { otherwise }\end{array}\right)$

Theorem 1.8 [4]: Let $G$ be a unicyclic graph with unique cycle C of odd length $2 \mathrm{k}+1, k \geq 1$. Let $n$ denote the number of pendant vertices of $G$ and let $m$ denote the number of vertices of degree greater than 2 on $C$ with. Then $\eta_{g}=\left\{\begin{array}{l}3 \quad \text { if } m=0 \\ n+2 \quad \text { if } m=1\end{array}\right]\binom{$ if $\mathrm{m} \geq 2$ and every $(v, w)$-section of C in which all vertices }{ except $v$ and $w$ have degree 2 is a shortest path }

Definition 1.9: For two graphs $G$ and $H$, their Cartesian product $G \times H$ has vertex set $V(G) \times V(H)$ in which $\left(g_{1}, h_{1}\right)$ is joined $\left(g_{2}, h_{2}\right)$ iff $g_{1}=g_{2}$ and $h_{1} h_{2} \varepsilon E(H)$ or $h_{1}=h_{2}$ and $g_{1} g_{2} \varepsilon E(G)$.

Definition 1.10: A triangular cactus is a connected graph all of whose blocks are triangles. A triangular snake is a triangular cactus whose block-cutpoint-graph is a path (a triangular snake is obtained from a path $v_{1}, v_{2}, \ldots, v_{n}$ by joining $v_{i}$ and $v_{i+1}$ to a new vertex $w_{i}$ for $i=1,2, \ldots, n-1$ ).

Definition 1.11: A double triangular snake consists of two triangular snakes that have a common path. That is a double triangular snake is obtained from a path $v_{1}, v_{2}, \ldots, v_{n}$ by joining $v_{i}$ and $v_{i+1}$ to a new vertex $w_{i}$ for $i=1,2, \ldots, n-1$ and to a new vertex $u_{i}$ for $i=1,2, \ldots, n-1$.

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Definition 1.12: A tripple triangular snake consists of three triangular snakes that have a common path. That is, a tripple triangular snake is obtained from a path $u_{1}, u_{2}, \ldots, u_{n}$ by joining $u_{i} \& u_{i+1}$ to new vertex $v_{i}$ for $\mathrm{i}=1,2, \ldots, \mathrm{n}-1$ and to a new vertex $W$ for $\mathrm{i}=1,2, \ldots, \mathrm{n}-1$ for and also to a new vertex $Z_{i}$ for $\mathrm{i}=1,2, \ldots, \mathrm{n}-1$

Definition 1.13: Mongolian tent as a graph obtained from $P_{m} \times P_{n}$, n odd, by adding one extra vertex above the grid and joining every other vertex of the top row of $P_{m} \times P_{n}$ to the new vertex.

Definition 1.14: The book $B_{m}$ is the graph $S_{m} \times P_{2}$ where $S_{m}$ is the star with $m+1$ vertices
Definition 1.15: A gear graph, denoted $G_{n}$ is a graph obtained by inserting an extra vertex between each pair of adjacent vertices on the perimeter of a wheel graph $W_{n}$. Thus, $G_{n}$ has $2 n+1$ vertices and $3 n$ edges. Gear graphs are examples of square graphs, and play a key role in the forbidden graph characterization of square graphs. Gear graphs are also known as cogwheels and bipartite wheels.

Definition 1.16: A helm graph, denoted Hn is a graph obtained by attaching a single edge and node to each node of the outer circuit of a wheel graph Wn .

Definition 1.17: A graph $G$ is called the flower graph with $n$ petals if it has $3 n+1$ vertices which form an $n-c y c l e$.
Definition 1.18: A shell Sn is the graph obtained by taking $\mathrm{n}-3$ concurrent chords in a cycle Cn on n vertices. The vertex at which all the chords are concurrent is called the apex vertex. The shell is also called fan Fn-1.
i.e.. $\mathrm{S}_{\mathrm{n}}=\mathrm{F}_{\mathrm{n}}-1=\mathrm{P}_{\mathrm{n}}-1+\mathrm{K}_{1}$.

Definition 1.19: The cartesian product of two paths is known as grid graph which is denoted by $P_{m} \times P_{n}$. In particular the graph $L_{n}=P_{n} \times P_{2}$ is known as ladder graph.

Definition 1.20: A web graph is the graph obtained by joining the pendant vertices of a helm to form a cycle and then adding a single pendant edge to each vertex of this outer cycle.

Definition 1.21: A double-wheel graph $D W_{N}$ of size $N$ can be composed of $2 C_{N}+K_{1}$, i.e. it consists of two cycles of size $N$, where the vertices of the two cycles are all connected to a common hub.

Definition 1.22 [9]: For $n \geq 3, m \geq 1, A_{n}^{m}$ the plane graph of a convex polytope which is obtained as a combination of antiprism.

Definition 1.23 [9]: The m-prism $D_{n}{ }^{m}, n \geq 3, m \geq 1$ is a trivalent graph of a convex polytope which can be defined as cartesian product of a path on $\mathrm{m}+1$ vertices with a cycle on n vertices $\left(p_{m+1} \times c_{n}\right)$ embedded in the plane.

## 2. MAIN RESULTS

Theorem 2.1: Let $G$ be Triangular cactus graph with $n$ number of triangles then $\eta_{g}=2 n-1=q-p+n$

## Proof:

Let $V(G)=\left\{v_{0}, v_{i 1}, v_{i 2}\right\} i=1$ to $n$
$P=2 n+1$ and $q=3 n$
The Geodesic graphoidal path covering of G is as follows
$P=\left\{v_{11}, v_{12}, v_{0}, v_{n 2}, v_{n 1}\right\}$
$P_{i}=\left\{v_{i 1}, v_{i 2}, v_{0}\right\} i=2,3, \ldots, n-1$
$Q_{i}=\left\{v_{i 1}, v_{0}\right\} \quad i=1,2, \ldots, n$
$\psi=P \cup\left\{P_{i}\right\} \cup\left\{Q_{i}\right\}$ is minimum geodesic graphoidal covering of G
$\Rightarrow \eta_{g}=1+n-2+n=2 n-1$

Theorem 2.2: Let $G$ be Triangular snake graph with $n-1$ number of triangles then $\eta_{g}=2 n-3=q-p+n-1$

## Proof:

Let $V(G)=\left\{v_{1}, v_{2}, v_{3}, \ldots, v_{n}, \mathrm{w}_{1}, \mathrm{w}_{2}, \ldots, \mathrm{w}_{n-1}\right\}$

Here $w_{i}$ adjacent to $v_{i}$ and $v_{i+1}$
The Geodesic graphoidal path covering of G is as follows
$P_{i}=\left\{v_{i+1} w_{i}\right\}, i=1,2, \ldots, n-2$
$P_{n-1}=\left\{w_{1}, v_{1}, v_{2}, v_{3}, \ldots, v_{n}, w_{n-1}\right\}$
$R_{i}=\left\{v_{i} w_{i}\right\}, \quad i=1,2,3, \ldots, n-1$
$\psi=P_{1} \cup \ldots \cup P_{n-1} \cup R_{2} \cup \ldots \cup R_{n-1}$ is minimum geodesic graphoidal covering of G
$\Rightarrow \eta_{g}=n-1+n-2=2 n-3=q-p+n-1$

Also for triangular graph $\eta_{a}=q-p+n-1$

Theorem 2.3: Let $G$ be Double Triangular snake graph with $2 n-2$ number of triangles then $\eta_{g}=4 n-5=q-p+2 n-2$

## Proof:

Let $V(G)=\left\{v_{1}, v_{2}, v_{3}, \ldots, v_{n}, u_{1}, u_{2}, \ldots, u_{n-1}, \mathrm{w}_{1}, \mathrm{w}_{2}, \ldots, \mathrm{w}_{n-1}\right\}$

Here $u_{i}$ adjacent to $v_{i}$ and $v_{i+1}$ in upward direction and $w_{i}$ adjacent to $v_{i}$ and $v_{i+1}$ in downward direction.

$$
p=3 n-2, q=5(n-1)
$$

The Geodesic graphoidal path covering of G is as follows
$P_{i}=\left\{v_{i+1} u_{i}\right\}, \quad i=1,2,3, \ldots, n-2$
$P_{n}=\left\{u_{n-1} v_{n} w_{n-1}\right\}$
$Q_{i}=\left\{v_{i} u_{i}\right\}, i=2,3, \ldots, n-1$
$Q_{n}=\left\{v_{1}, v_{2}, v_{3}, \ldots, v_{n}\right\}$
$R_{i}=\left\{v_{i} w_{i}\right\}, i=2,3, \ldots, n-1$
$R_{n}=\left\{u_{1} v_{1} w_{1}\right\}$
$S_{i}=\left\{v_{i+1} w_{i}\right\}, \quad i=1,2,3, \ldots, n-2$
$\psi=\left\{P_{i}\right\} \cup\left\{Q_{i}\right\} \cup\left\{R_{i}\right\} \cup\left\{S_{i}\right\}$ is minimum geodesic graphoidal covering of G
$\Rightarrow \eta_{g}=4(n-2)+3=4 n-5=q-p+2 n-2$

## Note:

Let G be triple Triangular snake graph with $3 n-3$ number of triangles then $\eta_{g}=q-p+3(n-1)$

Theorem 2.4: For $p_{m} \times p_{n}$, the geodesic graphoidal covering number is $\eta_{g}=q-p+2$.
Proof: Let $V(G)=\left\{v_{i 1}, v_{i 2}, \ldots, v_{i n}\right\} i=1,2, \ldots, m$
Here $p=m n$ and $q=m(n-1)+n(m-1)$

The geodesic graphoidal cover of $p_{m} \times p_{n}$ is as follows:
$P_{i}=\left\{v_{i+1,1}, v_{i 1}, v_{i 2}, \ldots, v_{i n}\right\} i=1,2, \ldots, m-1$
$P_{n}=\left\{v_{1 n}, v_{2 n}, v_{3 n}, \ldots, v_{m n}, v_{m, n-1}, v_{m, n-2}, \ldots, v_{m 2}, v_{m 1}\right\}$
$\mathrm{S}=$ The remaining edges not covered by $P_{1}, P_{2}, P_{3}, \ldots, P_{n-1}, P_{n}$
$\psi=P_{1} \cup P_{2} \cup \ldots \cup P_{n} \cup S$ is minimum geodesic graphoidal covering of G
From above we see that all the paths are shortest paths and all the vertices of $p_{m} \times p_{n}$ are internal vertices in at least one path expect except $v_{1 n}$ and $v_{m 1}$
Therefore $\eta_{g}=q-p+2$

## Note:

(i) For $p_{m} \times p_{n}, \eta_{a}=\eta_{g}=q-p+2$
(ii) Let $G$ be a Ladder graph then $\eta_{g}=q-p+2$. Since Ladder is a particular case of $p_{m} \times p_{n}$

Theorem 2.5: Let G be a gear graph with $2 \mathrm{n}+1$ vertices and $3 n$ edges then $\eta_{g}=q-p+n$.
Proof: Let $V(G)=\left\{v_{0}, v_{1}, v_{2}, v_{3}, \ldots, v_{n}, \mathrm{w}_{1}, \mathrm{w}_{2}, \ldots, \mathrm{w}_{n}\right\}$
where $v_{0}$ is the centre vertex of wheel and $w_{i}$ adjacent to $v_{i}$ and $v_{i+1}$ and $w_{n}$ is adjacent to $v_{1}$ and $v_{\mathrm{n}}$ and $P=2 n+1$ and $q=3 n$

The Geodesic graphoidal path covering of $G$ is as follows
$P_{i}=\left\{v_{i}, w_{i}, v_{i+1}\right\} \quad i=1,2, \ldots, n-1$
$P_{n}=\left\{v_{n}, w_{n}, v_{1}\right\}$
$Q_{i}=\left\{v_{0}, v_{i}\right\} \quad i=2,3, \ldots, n-1$
$Q_{n}=\left\{v_{0}, v_{1}, v_{n}\right\}$
$\psi=\left\{P_{i}\right\} \cup\left\{Q_{i}\right\}$ is minimum geodesic graphoidal covering of G
$\Rightarrow \eta_{g}=2 n-1=q-p+n$

Theorem 2.6: Let $G$ be a Helm graph $2 \mathrm{n}+1$ vertices and 3 n edges then $\eta_{g}=q-p+n$.

Proof: Let $V(G)=\left\{v_{0}, v_{1}, v_{2}, v_{3}, \ldots, v_{n}, w_{1}, w_{2}, \ldots, w_{n}\right\}$
where $v_{0}$ is the centre vertex of wheel and is $v_{i}$ adjacent to wi and $v_{0}$ and $P=2 n+1$ and $q=3 n$
The Geodesic graphoidal path covering of G is as follows
$P_{i}=\left\{w_{i}, v_{i}, v_{i+1}\right\}, i=1,2, \ldots, n-1$
$P_{n}=\left\{w_{n}, v_{n}, v_{1}\right\}$
$Q_{i}=\left\{v_{0}, v_{i}\right\}, i=2,4,5, \ldots, n$
$Q_{n}=\left\{v_{3}, v_{0}, v_{1}\right\}$
$\psi=\left\{P_{i}\right\} \cup\left\{Q_{i}\right\}$ is minimum geodesic graphoidal covering of G
$\Rightarrow \eta_{g}=2 n-1=q-p+n$
For the Helm graph the graphoidal cover is also the same.
(i.e.) $\eta_{a}=q-p+n$

Theorem 2.7: Let G be $P_{m}\left(Q S_{n}\right)$ graph then $\eta_{g}=n(m+1)-1$

## Proof:

Let $V(G)=\left\{v_{1}, v_{2}, v_{3}, \ldots, v_{m}, l_{i 1}, l_{i 2}, \ldots, l_{i n}, r_{i 1}, r_{i 2}, \ldots, r_{i n}, w_{i 1}, w_{i 2}, \ldots, w_{i n}\right\} i=1,2, \ldots, n$
The Geodesic graphoidal path covering of G is as follows
$P_{i}=\left\{w_{i n}, l_{i n}, w_{i(n-1)}, \ldots, w_{i 2}, l_{i 2}, w_{i 1}, l_{i 1}, v_{i}, v_{i+1}, l_{i+1,1}, w_{i+1,1}, \ldots, w_{i+1(n-1)}, l_{i+1 n}, w_{i+1 n}\right\} i=1,2, \ldots, n-1$
$Q_{i}=\left\{v_{i}, r_{i 1}, w_{i 1}\right\}, \quad i=1,2, \ldots, n$
$R_{i}=\left\{w_{i 1}, r_{i 2}, w_{i 2}\right\}, \quad i=1,2, \ldots, n$
$S_{i}=\left\{w_{i(m-1)}, r_{i m}, w_{i m}\right\}, \quad i=1,2, \ldots, n$
$\psi=P \cup\left\{P_{i}\right\} \cup\left\{Q_{i}\right\} \cup\left\{R_{i}\right\} \cup \ldots \cup\left\{S_{i}\right\}$ is minimum geodesic graphoidal covering of G
$\Rightarrow \eta_{g}=n-1+m n=n(m+1)-1$
Theorem 2.8: Let G be $C_{m}\left(Q S_{n}\right)$ graph then $\eta_{g}=n+m n$

## Proof:

Let $V(G)=\left\{v_{1}, v_{2}, v_{3}, \ldots, v_{m}, l_{i 1}, l_{i 2}, \ldots, l_{i n}, r_{i 1}, r_{i 2}, \ldots, r_{i n}, w_{i 1}, w_{i 2}, \ldots, w_{i n}\right\} i=1,2, \ldots, n$
The Geodesic graphoidal path covering of G is as follows
$P_{i}=\left\{w_{i n}, l_{i n}, w_{i(n-1)}, \ldots, w_{i 2}, l_{i 2}, w_{i 1}, l_{i 1}, v_{i}, v_{i+1}, l_{i+1,1}, w_{i+1,1}, \ldots, w_{i+1(n-1)}, l_{i+1 n}, w_{i+1 n}\right\} i=1,2, \ldots, n-1$
$P_{n}=\left(v_{n}, v_{1}\right)$
$Q_{i}=\left\{v_{i}, r_{i 1}, w_{i 1}\right\}, \quad i=1,2, \ldots, n$
$R_{i}=\left\{w_{i 1}, r_{i 2}, w_{i 2}\right\}, \quad i=1,2, \ldots, n$
$S_{i}=\left\{w_{i(m-1)}, r_{i m}, w_{i m}\right\}, \quad i=1,2, \ldots, n$
$\psi=\left\{P_{i}\right\} \cup\left\{Q_{i}\right\} \cup\left\{R_{i}\right\} \cup \ldots \cup\left\{S_{i}\right\}$ is minimum geodesic graphoidal covering of G
$\Rightarrow \eta_{g}=n+m n$
Theorem 2.9: Let G be a web graph with $3 \mathrm{n}+1$ vertices and 5 n edges then $\eta_{g}=q-p+n$.
Proof: Let $V(G)=\left\{v_{0}, v_{i}, v_{i 1}, v_{i 2}\right\} \quad i=1,2, \ldots, n$
Here $v_{i 2}$ is adjacent to $v_{0}$ and $v_{i 1}$ and $v_{i 1}$ is adjacent to $v_{i}$ and $v_{i 2}$
The Geodesic graphoidal path covering of G is as follows
$P_{i}=\left\{v_{i 1}, v_{i+1,1} v_{i+1}\right\} \quad i=1,2, \ldots, n-1$
$P_{n}=\left\{v_{n 1}, v_{11}, v_{1}\right\}$
$Q_{i}=\left\{v_{i 1}, v_{i 2} v_{i+1,2}\right\} \quad i=1,2, \ldots, n-1$
$Q_{n}=\left\{v_{n 1}, v_{n 2}, v_{12}\right\}$
$R_{i}=\left\{v_{0} v_{i 2}\right\} i=2,4,5, \ldots, n$
$R_{n}=\left\{v_{12}, v_{0}, v_{32}\right\}$
$\psi=\left\{P_{i}\right\} \cup\left\{Q_{i}\right\} \cup\left\{R_{i}\right\}$ is minimum geodesic graphoidal covering of G
$\Rightarrow \eta_{g}=3 n-1=q-p+n$
For the Web graph the graphoidal cover is also the same.
(i.e.) $\eta_{a}=q-p+n$

Theorem 2.10: Let G be a shell graph with $\mathrm{n}+1$ vertices then $\eta_{g}=q-p+\frac{n}{2}=\frac{3 n-6}{2}$.
Proof: Let $V(G)=\left\{v_{1}, v_{2}, v_{3}, \ldots, v_{n}\right\}$
$p=n, q=2 n+1$
The Geodesic graphoidal path covering of G is as follows
$P_{i}=\left\{v_{1} v_{i}\right\} i=3,4, \ldots, n-1$
$Q=\left\{v_{2} v_{1} v_{n}\right\}$
$R_{i}=\left\{v_{i} v_{i+1} v_{i+2}\right\} i=2,4,6, \ldots, n-2$
$\psi=\left\{P_{i}\right\} \cup\{Q\} \cup\left\{R_{i}\right\}$ is minimum geodesic graphoidal covering of G
$\Rightarrow \eta_{g}=n-3+1+\frac{n}{2}-1=q-p+\frac{n}{2}=\frac{3 n-6}{2}$

Theorem 2.11: Let G be a book graph with $2 \mathrm{n}+2$ vertices then $\eta_{g}=q-p+n$.

Proof: Let $V(G)=\left\{v_{1}, v_{2}, b_{i 1}, b_{i 2}\right\} i=1,2, \ldots, n$
The Geodesic graphoidal path covering of G is as follows
$P_{1}=\left(v_{1}, v_{2}\right)$
$P_{2}=\left\{b_{11}, b_{12}, v_{2}, b_{22}\right\}$
$P_{n+1}=\left\{b_{11}, v_{1}, b_{21}, b_{22}\right\}$
$P_{i}=\left\{b_{i 1}, b_{i 2}, v_{1}\right\} \quad i=3,4 \ldots, n$
[ if this path not exists for some $i$ then for that particular $i, P_{i}=\left\{b_{i 2}, b_{i 1}, v_{1}\right\}$ ]
$Q_{i}=\left\{v_{2}, b_{i 1}\right\}$ o $\left\{v_{2}, b_{i 2}\right\} i=3,4 \ldots, n$
$\psi=\left\{P_{i}\right\} \cup\left\{Q_{i}\right\}$ is minimum geodesic graphoidal covering of G
$\Rightarrow \eta_{g}=3+n-2+n-2=2 n-1=q-p+n$

For book graph the graphoidal coveing number is also same as geodesic graphoidal covering number (i.e.) $\eta_{a}=q-p+n$

Theorem 2.12: Let $G$ be Mongolian tent graph then $\eta_{g}\left(M_{m, n}\right)=q-p+2$
Proof: Let $G=M_{m, n}$
Let $V(G)=\left\{v_{0}, v_{i 1}, v_{i 2}, \ldots, v_{i n}\right\} \quad i=1,2, \ldots, m$

The Geodesic graphoidal path covering of G is as follows
$P_{i}=\left\{v_{i+1,1}, v_{i 1}, v_{i 2}, \ldots, v_{i n}\right\} i=1,2, \ldots, m-1$
$P_{n}=\left\{v_{1 n}, v_{2 n}, v_{3 n}, \ldots, v_{m n}, v_{m, n-1}, v_{m, n-2}, \ldots, v_{m 2}, v_{m 1}\right\}$
$P_{n+1}=\left\{v_{11}, v_{0}, v_{1 n}\right\}$
$\mathrm{S}=$ The remaining edges
From above paths all the vertices are exterior points except $v_{1 n}$ and $v_{m 1}$
$\psi=P_{1} \cup P_{2} \cup \ldots \cup P_{n} \cup P_{n+1} \cup S$ is minimum geodesic graphoidal covering of G
$\Rightarrow \eta_{g}=q-p+2$
For book graph the graphoidal coveing number is also same as geodesic graphoidal covering number
Theorem 2.13: Let $G$ be double wheel graph with $2 \mathrm{n}+1$ vertices $\eta_{g}(G)=\left\{\begin{array}{l}3 n+2 \text { if } \mathrm{n} \text { is odd } \\ 3 n \text { if } \mathrm{n} \text { is even }\end{array}\right.$.
Proof: Let $V(G)=\left\{v_{0}, c_{11}, c_{12}, \ldots, c_{1 n}, c_{21}, c_{22}, \ldots, c_{2 n}\right\}$
$P_{1}=\left(c_{21}, \mathrm{v}_{0}, c_{11}\right)$
$P_{i j}=\left\{c_{i j}, c_{i j+1}, c_{i j+2}, i=1,2 \& j=1,3,5, \ldots, n-2\right.$
$Q_{i j}=\left\{\begin{array}{l}c_{i, n-2}, c_{i, 1}, i=1,2 \& n \text { is odd } \\ c_{i, n-1}, c_{i, \mathrm{n}}, \mathrm{c}_{\mathrm{i}, \mathrm{j}}, i=1,2 \& \mathrm{n} \text { is even }\end{array}\right.$
$\Rightarrow \eta_{g}=2 n-2+n-1+1=3 n-2$ if $n$ is odd
$\eta_{g}=2 n-2+n+1+1=3 n$ if $n$ is even

Theorem 2.14: $\eta_{g}\left(A_{5}^{3}\right)=q-p+2$.
Proof: Let $V(G)=\left\{x, c_{11}, c_{12}, \ldots, c_{1 n}, c_{21}, c_{22}, \ldots, c_{2 n}, c_{31}, c_{32}, \ldots, c_{3 n}, y\right\}$
The Geodesic graphoidal path covering of G is as follows
$P_{i}=\left\{x, \mathrm{c}_{1 i}, \mathrm{c}_{2 i}, \mathrm{c}_{3 i}, \mathrm{y}\right\} \quad i=1,2, \ldots, n$
Clearly all the vertices are internal vertices except x and y .
$\psi=\{P\}_{i} \cup$ Remaining Edges is minimum geodesic graphoidal covering of G
$\therefore \eta_{g}=q-p+2$

Theorem 2.15: $\eta_{g}\left(D_{7}^{3}\right)=q-p+2$.

Proof: Let $V(G)=\left\{\mathrm{y}, v_{i 1}, v_{i 2}, \ldots, v_{i n}\right\} \quad i=1,2, \ldots, m$
The geodesic graphoidal cover of $D_{7}^{3}$ is as follows:
$P_{i}=\left\{v_{i+1,1}, v_{i 1}, v_{i 2}, \ldots, v_{i n}\right\} i=1,2, \ldots, m-1$
$P_{n}=\left\{v_{1 n}, v_{2 n}, v_{3 n}, \ldots, v_{m n}, v_{m, n-1}, v_{m, n-2}, \ldots, v_{m 2}, v_{m 1}\right\}$
$Q=\left(v_{11}, y, v_{1 n}\right)$
$\mathrm{S}=$ The remaining edges

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$\psi=\{P\}_{i} \cup Q \cup S$ is minimum geodesic graphoidal covering of G and all the vertices are internal vertices except $v_{11}$ and $v_{1 n} y$.
$\therefore \eta_{g}=q-p+2$

## REFERENCES

1. B.D. Acharya and E. Sampathkumar, Graphoidal covers and graphoidal covering number of a graph, Indian J.Pure Appl.Math. 18 (10) (1987), 882-890.
2. S.Arumugam, B.D.Acharya and E.Sampathkumar,Graphoidal covers of a graph:a creative review,in Proc. National Workshop on Graph theoryand its applications, Manonmaniam Sundaranar University, Tirunelveli, Tata McGraw-Hill, New Delhi,(1997), 1-28.
3. S.Arumugam and J.Suresh Suseela, Acyclic graphoidal covers and path partitions in a graph, Discrete Math.190(1998), 67-77
4. In S. Arumugam and J. Suresh Suseela, Geodesic Graphoidal covering number of a graph J. Indian. Math. Soci., 72(2005), 99-106.
5. F.Harary, Graph Theory, Addison-Wesley, Reading, MA, 1969.
6. Hao Li, Perfect path double covers in every simple graph, Journal of Graph Theory, 14 (6) (1990), 645-650.
7. Joseph A. Gallian, Department of Mathematics and Statistics, University of Minnesota Duluth, Minnesota 55812, U.S.A, A Dynamic Survey of Graph Labeling,.
8. C.Packkiam and S.Arumugam, The graphoidal covering number of unicyclic graphs, Indian J.Pure appl.Math. 23 (2) (1992), 141-143.
9. Conjectures and open problems on Face Antimagic Evaluations of Graphs. M. Baca, E.T. Baskoro,Y.M. Cholily,s. Jendrol,Y.Lin,M.Miller, J. Ryan, R.Simanjuntak, Slam in, and K.A.Sugeng, 2000, Mathematics Subject Classification 05c78.

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