International Journal of Mathematical Archive-5(8), 2014, 62-70
Available online through www.ijma.info ISSN 2229 – 5046

# GEODESIC GRAPHOIDAL COVERING NUMBER OF GRAPHS

**T. Gayathri*[1] and S. Meena[2]**

*[1]Department of Mathematics,*
*Sri Manakula Vinayagar Engineering College, Puducherry-605 107, India.*

*[2]Department of Mathematics,*
*Government Arts and Science College, Chidambaram-608 102, India.*

*(Received On: 13-08-14; Revised & Accepted On: 31-08-14)*

*ABSTRACT*

*A geodesic graphoidal cover of a graph G is a collection $\psi$ of shortest paths in G such that every path in $\psi$ has at least two vertices, every vertex of G is an internal vertex of at most one path in $\psi$ and every edge of G is in exactly one path in $\psi$. The minimum cardinality of a geodesic graphoidal cover of G is called the geodesic graphoidal covering number of G and is denoted by $\eta_g$. In this paper we determine $\eta_g$ for bicyclic graphs.*

**Key words:** *Graphoidal covers, Acyclic graphoidal cover, Geodesic Graphoidal cover*

## 1. INTRODUCTION

A graph is a pair $G = (V, E)$, where $V$ is the set of vertices and $E$ is the set of edges. Here we consider only nontrivial, finite, connected, undirected graph without loops or multiple edges. The order and size of *G* are denoted by *p* and *q* respectively. For graph theoretic terminology we refer to Harary [4]. The concept of graphoidal cover was introduced by B.D Acharya and E. Sampathkumar [1] and the concept of acyclic graphoidal cover was introduced by Arumugam and Suresh Suseela [4].The reader may refer [5], [2] and [7] for the terms not defined here.

Let $p = (v_1, v_2, v_3, \ldots, v_r)$ be a path or a cycle in a graph $G = (V, E)$. Then vertices $(v_2, v_3, \ldots, v_{r-1})$ are called internal vertices of *P* and $v_1$ and $v_r$ are called external vertices of *P*. Two paths *P* and *Q* of a graph G are said to be internally disjoint if no vertex of *G* is an internal vertex of both P and *Q*.

**Definition 1.1 [1]:** A graphoidal cover of a graph G is called a collection $\psi$ of (not necessarily open) paths in G satisfying the following conditions:
(i) Every path in $\psi$ has at least two vertices.
(ii) Every vertex of G is an internal vertex of at most one path in $\psi$.
(iii) Every edge of G is in exactly one path in $\psi$.

The minimum cardinality of a graphoidal cover of G is called the graphoidal covering number of G and is denoted by $\eta(G)$.

**Definition 1.2 [3]:** A graphoidal cover $\psi$ of a graph G is called an acyclic graphoidal cover if every member of $\psi$ is an open path. The minimum cardinality of an acyclic graphoidal cover of G is called the acyclic graphoidal covering number of G and is denoted by $\eta_a(G)$ or $\eta_a$.

*Corresponding author: T. Gayathri*[1], [1]Department of Mathematics,*
*Sri Manakula Vinayagar Engineering College, Puducherry-605 107, India.*

**Definition 1.3 [4]:** A geodesic graphoidal cover of a graph G is a collection $\psi$ of shortest paths in G such that every path in $\psi$ has at least two vertices, every vertex of G is an internal vertex of at most one path in $\psi$ and every edge of G is an exactly one path in $\psi$. The minimum cardinality of a geodesic graphoidal cover of G is called the geodesic graphoidal covering number of G and is denoted by $\eta_g$.

**Definition 1.4** [**1**]**:** Let $\psi$ be a collection of internally disjoint paths in G. A vertex of G is said to be in the interior of $\psi$ if it is an internal vertex of some path in $\psi$. Any vertex which is not in the interior of $\psi$ is said to be an exterior vertex of $\psi$.

**Theorem 1.5 [8]:** For any graphoidal cover $\psi$ of G, let $t_\psi$ denote the number of exterior vertices of $\psi$. Let $t = \min t_\psi$ where the minimum is taken over all graphoidal covers of G. Then $\eta = q - p + t$

**Corollary 1.6:** For any graph G, $\eta \geq q - p$. Morever the following are equivalent.
(i) $\eta = q - p$
(ii) There exists a graphoidal cover without exterior vertices.
(iii) There exists a set of internally disjoint and edge disjoint paths without exterior vertices.

In [4] given that $\eta \leq \eta_a \leq \eta_g$ and these inequalities can be strict and also for a tree $\eta = \eta_a = \eta_g = n-1$ and Theorem 1.5 and corollary 1.6 are true for geodesic graphoidal covers.

They observe that $\eta_g = q$ if and only if G is Complete. Further for a cycle $C_m$, $\eta_g = \begin{cases} 2 \text{ if m is even} \\ 3 \text{ if m is odd} \end{cases}$

**Theorem 1.7 [4]:** Let G be a unicyclic graph with unique cycle C which is even. Let *n* denote the number of pendant vertices of G and let *m* denote the number of vertices on C with degree greater than 2. Then

$$\eta_g = \begin{cases} 2 \text{ if } m = 0 \\ n \begin{pmatrix} \text{if m} \geq 2 \text{ and every } (v,w)\text{-section of C in which all vertices} \\ \text{except } v \text{ and } w \text{ have degree 2 is a shortest path} \end{pmatrix} \\ n+1 \text{ otherwise} \end{cases}$$

**Theorem 1.8 [4]:** Let G be a unicyclic graph with unique cycle C of odd length 2k+1, $k \geq 1$. Let *n* denote the number of pendant vertices of G and let *m* denote the number of vertices of degree greater than 2 on C with. Then

$$\eta_g = \begin{cases} 3 \quad \text{if } m = 0 \\ n+2 \quad \text{if } m = 1 \\ n \begin{pmatrix} \text{if m} \geq 2 \text{ and every } (v,w)\text{-section of C in which all vertices} \\ \text{except } v \text{ and } w \text{ have degree 2 is a shortest path} \end{pmatrix} \\ n+1 \text{ otherwise} \end{cases}$$

**Definition 1.9:** For two graphs G and H, their Cartesian product $G \times H$ has vertex set $V(G) \times V(H)$ in which $(g_1, h_1)$ is joined $(g_2, h_2)$ iff $g_1 = g_2$ and $h_1 h_2 \varepsilon E(H)$ or $h_1 = h_2$ and $g_1 g_2 \varepsilon E(G)$.

**Definition 1.10:** A triangular cactus is a connected graph all of whose blocks are triangles. A triangular snake is a triangular cactus whose block-cutpoint-graph is a path (a triangular snake is obtained from a path $v_1, v_2, ..., v_n$ by joining $v_i$ and $v_{i+1}$ to a new vertex $w_i$ for $i = 1, 2, ..., n-1$).

**Definition 1.11:** A double triangular snake consists of two triangular snakes that have a common path. That is a double triangular snake is obtained from a path $v_1, v_2, ..., v_n$ by joining $v_i$ and $v_{i+1}$ to a new vertex $w_i$ for $i = 1, 2, ..., n-1$ and to a new vertex $u_i$ for $i = 1, 2, ..., n-1$.

**Definition 1.12:** A tripple triangular snake consists of three triangular snakes that have a common path. That is, a tripple triangular snake is obtained from a path $u_1, u_2, ..., u_n$ by joining $u_i$ & $u_{i+1}$ to new vertex $v_i$ for i = 1,2,…,n-1 and to a new vertex $w$ for i = 1,2,…,n-1 for and also to a new vertex $z_i$ for i = 1,2,…,n-1

**Definition 1.13:** Mongolian tent as a graph obtained from $P_m \times P_n$, n odd, by adding one extra vertex above the grid and joining every other vertex of the top row of $P_m \times P_n$ to the new vertex.

**Definition 1.14:** The book $B_m$ is the graph $S_m \times P_2$ where $S_m$ is the star with m + 1 vertices

**Definition 1.15:** A gear graph, denoted $G_n$ is a graph obtained by inserting an extra vertex between each pair of adjacent vertices on the perimeter of a wheel graph $W_n$. Thus, $G_n$ has 2n+1 vertices and 3n edges. Gear graphs are examples of square graphs, and play a key role in the forbidden graph characterization of square graphs. Gear graphs are also known as cogwheels and bipartite wheels.

**Definition 1.16:** A helm graph, denoted Hn is a graph obtained by attaching a single edge and node to each node of the outer circuit of a wheel graph Wn.

**Definition 1.17:** A graph G is called the flower graph with n petals if it has 3n+1 vertices which form an n- cycle.

**Definition 1.18:** A shell Sn is the graph obtained by taking n-3 concurrent chords in a cycle Cn on n vertices. The vertex at which all the chords are concurrent is called the apex vertex. The shell is also called fan Fn-1.
i.e.. $\mathrm{S_n = F_n\text{-}1 = P_n\text{-}1 + K_1}$.

**Definition 1.19:** The cartesian product of two paths is known as grid graph which is denoted by $P_m \times P_n$. In particular the graph $L_n = P_n \times P_2$ is known as ladder graph.

**Definition 1.20:** A web graph is the graph obtained by joining the pendant vertices of a helm to form a cycle and then adding a single pendant edge to each vertex of this outer cycle.

**Definition 1.21:** A double-wheel graph $DW_N$ of size N can be composed of $\mathrm{2C_N + K_1}$, i.e. it consists of two cycles of size N, where the vertices of the two cycles are all connected to a common hub.

**Definition 1.22** [9]: For $n \geq 3, m \geq 1, A_n^m$ the plane graph of a convex polytope which is obtained as a combination of antiprism.

**Definition 1.23** [9]: The m-prism $D_n^{\ m}, n \geq 3, m \geq 1$ is a trivalent graph of a convex polytope which can be defined as cartesian product of a path on m+1 vertices with a cycle on n vertices $\left(p_{m+1} \times c_n\right)$ embedded in the plane.

## 2. MAIN RESULTS

**Theorem 2.1:** Let G be Triangular cactus graph with *n* number of triangles then $\eta_g = 2n - 1 = q - p + n$

**Proof:**
Let $V(G) = \{v_0, v_{i1}, v_{i2}\}\ i = 1 \text{ to } n$
$P = 2n+1$ and $q = 3n$
The Geodesic graphoidal path covering of G is as follows

$P = \{v_{11}, v_{12}, v_0, v_{n2}, v_{n1}\}$

$P_i = \{v_{i1}, v_{i2}, v_0\}\ i = 2, 3, ..., n-1$

$Q_i = \{v_{i1}, v_0\}\ i = 1, 2, ..., n$

$\psi = P \cup \{P_i\} \cup \{Q_i\}$ is minimum geodesic graphoidal covering of G

$\Rightarrow \eta_g = 1 + n - 2 + n = 2n - 1$

**Theorem 2.2:** Let G be Triangular snake graph with $n$-1 number of triangles then $\eta_g = 2n-3 = q-p+n-1$

**Proof:**
Let $V(G) = \{v_1, v_2, v_3, ..., v_n, w_1, w_2, ..., w_{n-1}\}$

Here $w_i$ adjacent to $v_i$ and $v_{i+1}$

The Geodesic graphoidal path covering of G is as follows
$P_i = \{v_{i+1} w_i\},\ i = 1, 2, ..., n-2$
$P_{n-1} = \{w_1, v_1, v_2, v_3, ..., v_n, w_{n-1}\}$
$R_i = \{v_i w_i\},\ \ i = 1, 2, 3, ..., n-1$
$\psi = P_1 \cup \ldots \cup P_{n-1} \cup R_2 \cup \ldots \cup R_{n-1}$ is minimum geodesic graphoidal covering of G
$\Rightarrow \eta_g = n-1+n-2 = 2n-3 = q-p+n-1$

Also for triangular graph $\eta_a = q-p+n-1$

**Theorem 2.3:** Let G be Double Triangular snake graph with 2$n$-2 number of triangles then
$\eta_g = 4n-5 = q-p+2n-2$

**Proof:**
Let $V(G) = \{v_1, v_2, v_3, ..., v_n, u_1, u_2, ..., u_{n-1}, w_1, w_2, ..., w_{n-1}\}$

Here $u_i$ adjacent to $v_i$ and $v_{i+1}$ in upward direction and $w_i$ adjacent to $v_i$ and $v_{i+1}$ in downward direction.
$p = 3n-2,\ q = 5(n-1)$

The Geodesic graphoidal path covering of G is as follows
$P_i = \{v_{i+1} u_i\},\ \ i = 1, 2, 3, ..., n-2$
$P_n = \{u_{n-1} v_n w_{n-1}\}$
$Q_i = \{v_i u_i\},\ i = 2, 3, ..., n-1$
$Q_n = \{v_1, v_2, v_3, ..., v_n\}$
$R_i = \{v_i w_i\},\ i = 2, 3, ..., n-1$
$R_n = \{u_1 v_1 w_1\}$
$S_i = \{v_{i+1} w_i\},\ \ i = 1, 2, 3, ..., n-2$
$\psi = \{P_i\} \cup \{Q_i\} \cup \{R_i\} \cup \{S_i\}$ is minimum geodesic graphoidal covering of G
$\Rightarrow \eta_g = 4(n-2)+3 = 4n-5 = q-p+2n-2$

**Note:**
Let G be triple Triangular snake graph with 3$n$-$3$ number of triangles then $\eta_g = q-p+3(n-1)$

**Theorem 2.4:** For $p_m \times p_n$ **,** the geodesic graphoidal covering number is $\eta_g = q-p+2$.

**Proof:** Let $V(G) = \{v_{i1}, v_{i2}, \ldots, v_{in}\}$ $i = 1, 2, ..., m$

Here $p$ = $mn$ and $q$ = $m(n$-$1)$+$n(m$-$1)$

 

The geodesic graphoidal cover of $p_m \times p_n$ is as follows:

$P_i = \{v_{i+1,1}, v_{i1}, v_{i2}, \ldots, v_{in}\}$ $i = 1, 2, ..., m-1$

$P_n = \{v_{1n}, v_{2n}, v_{3n}, ..., v_{mn}, v_{m,n-1}, v_{m,n-2}, \ldots, v_{m2}, v_{m1}\}$

S = The remaining edges not covered by $P_1, P_2, P_3, \ldots, P_{n-1}, P_n$

$\psi = P_1 \cup P_2 \cup \ldots \cup P_n \cup S$ is minimum geodesic graphoidal covering of G

From above we see that all the paths are shortest paths and all the vertices of $p_m \times p_n$ are internal vertices in at least one path expect except $v_{1n}$ and $v_{m1}$

Therefore $\eta_g = q - p + 2$

**Note:**

(i) For $p_m \times p_n$, $\eta_a = \eta_g = q - p + 2$

(ii) Let G be a Ladder graph then $\eta_g = q - p + 2$ .Since Ladder is a particular case of $p_m \times p_n$

**Theorem 2.5:** Let G be a gear graph with 2n+1 vertices and 3n edges then $\eta_g = q - p + n$.

**Proof:** Let $V(G) = \{v_0, v_1, v_2, v_3, ..., v_n, \mathrm{w}_1, \mathrm{w}_2, ..., \mathrm{w}_n\}$

where $v_0$ is the centre vertex of wheel and $w_i$ adjacent to $v_i$ and $v_{i+1}$ and $w_n$ is adjacent to $v_1$ and $v_n$ and $P = 2n+1$ and $q = 3n$

The Geodesic graphoidal path covering of G is as follows

$P_i = \{v_i, w_i, v_{i+1}\}$ $i = 1, 2, ..., n-1$

$P_n = \{v_n, w_n, v_1\}$

$Q_i = \{v_0, v_i\}$ $i = 2, 3, ..., n-1$

$Q_n = \{v_0, v_1, v_n\}$

$\psi = \{P_i\} \cup \{Q_i\}$ is minimum geodesic graphoidal covering of G

$\Rightarrow \eta_g = 2n - 1 = q - p + n$

**Theorem 2.6:** Let G be a Helm graph 2n+1 vertices and 3n edges then $\eta_g = q - p + n$.

**Proof:** Let $V(G) = \{v_0, v_1, v_2, v_3, ..., v_n, w_1, w_2, ..., w_n\}$

where $v_0$ is the centre vertex of wheel and is $v_i$ adjacent to w$i$ and $v_0$ and $P = 2n+1$ and $q = 3n$

The Geodesic graphoidal path covering of G is as follows

$P_i = \{w_i, v_i, v_{i+1}\}$ , $i = 1, 2, ..., n-1$

$P_n = \{w_n, v_n, v_1\}$

$Q_i = \{v_0, v_i\}$ , $i = 2, 4, 5, ..., n$

$Q_n = \{v_3, v_0, v_1\}$

$\psi = \{P_i\} \cup \{Q_i\}$ is minimum geodesic graphoidal covering of G

$\Rightarrow \eta_g = 2n - 1 = q - p + n$

For the Helm graph the graphoidal cover is also the same.

$(i.e.)\eta_a = q - p + n$

 

**Theorem 2.7:** Let G be $P_m\left(QS_n\right)$ graph then $\eta_g = n(m+1)-1$

**Proof:**
Let $V\left(G\right)=\left\{v_1, v_2, v_3, ..., v_m, l_{i1}, l_{i2}, ..., l_{in}, r_{i1}, r_{i2}, ..., r_{in}, w_{i1}, w_{i2}, ..., w_{in}\right\}$ $i = 1, 2, ..., n$

The Geodesic graphoidal path covering of G is as follows

$P_i = \left\{w_{in}, l_{in}, w_{i(n-1)}, ...., w_{i2}, l_{i2}, w_{i1}, l_{i1}, v_i, v_{i+1}, l_{i+1,1}, w_{i+1,1}, ..., w_{i+1(n-1)}, l_{i+1n}, w_{i+1n}\right\}$ $i = 1, 2, ..., n-1$

$Q_i = \left\{v_i, r_{i1}, w_{i1}\right\}, \quad i = 1, 2, ..., n$

$R_i = \left\{w_{i1}, r_{i2}, w_{i2}\right\}, \quad i = 1, 2, ..., n$

$\cdots$

$S_i = \left\{w_{i(m-1)}, r_{im}, w_{im}\right\}, \quad i = 1, 2, ..., n$

$\psi = P \cup \left\{P_i\right\} \cup \left\{Q_i\right\} \cup \left\{R_i\right\} \cup \ldots \cup \left\{S_i\right\}$ is minimum geodesic graphoidal covering of G

$\Rightarrow \eta_g = n-1+mn = n(m+1)-1$

**Theorem 2.8:** Let G be $C_m\left(QS_n\right)$ graph then $\eta_g = n + mn$

**Proof:**
Let $V\left(G\right)=\left\{v_1, v_2, v_3, ..., v_m, l_{i1}, l_{i2}, ..., l_{in}, r_{i1}, r_{i2}, ..., r_{in}, w_{i1}, w_{i2}, ..., w_{in}\right\}$ $i = 1, 2, ..., n$

The Geodesic graphoidal path covering of G is as follows

$P_i = \left\{w_{in}, l_{in}, w_{i(n-1)}, ...., w_{i2}, l_{i2}, w_{i1}, l_{i1}, v_i, v_{i+1}, l_{i+1,1}, w_{i+1,1}, ..., w_{i+1(n-1)}, l_{i+1n}, w_{i+1n}\right\}$ $i = 1, 2, ..., n-1$

$P_n = \left(v_n, v_1\right)$

$Q_i = \left\{v_i, r_{i1}, w_{i1}\right\}, \quad i = 1, 2, ..., n$

$R_i = \left\{w_{i1}, r_{i2}, w_{i2}\right\}, \quad i = 1, 2, ..., n$

$S_i = \left\{w_{i(m-1)}, r_{im}, w_{im}\right\}, \quad i = 1, 2, ..., n$

$\psi = \left\{P_i\right\} \cup \left\{Q_i\right\} \cup \left\{R_i\right\} \cup \ldots \cup \left\{S_i\right\}$ is minimum geodesic graphoidal covering of G

$\Rightarrow \eta_g = n + mn$

**Theorem 2.9:** Let G be a web graph with 3n+1 vertices and 5n edges then $\eta_g = q - p + n$.

**Proof:** Let $V\left(G\right)=\left\{v_0, v_i, v_{i1}, v_{i2}\right\}$ $i = 1, 2, ..., n$

Here $v_{i2}$ is adjacent to $v_0$ and $v_{i1}$ and $v_{i1}$ is adjacent to $v_i$ and $v_{i2}$

The Geodesic graphoidal path covering of G is as follows

$P_i = \left\{v_{i1}, v_{i+1,1} v_{i+1}\right\}$ $i = 1, 2, ..., n-1$

$P_n = \left\{v_{n1}, v_{11,} v_1\right\}$

$Q_i = \left\{v_{i1}, v_{i2} v_{i+1,2}\right\}$ $i = 1, 2, ..., n-1$

$Q_n = \left\{v_{n1}, v_{n2}, v_{12}\right\}$

$R_i = \left\{v_0 v_{i2}\right\}$ $i = 2, 4, 5, ..., n$

$R_n = \left\{v_{12}, v_0, v_{32}\right\}$

$\psi = \{P_i\} \cup \{Q_i\} \cup \{R_i\}$ is minimum geodesic graphoidal covering of G

$\Rightarrow \eta_g = 3n - 1 = q - p + n$

For the Web graph the graphoidal cover is also the same.

$(i.e.)\eta_a = q - p + n$

**Theorem 2.10:** Let G be a shell graph with n+1 vertices then $\eta_g = q - p + \frac{n}{2} = \frac{3n-6}{2}$.

**Proof:** Let $V(G) = \{v_1, v_2, v_3, ..., v_n\}$

$p = n,\ q = 2n + 1$

The Geodesic graphoidal path covering of G is as follows

$P_i = \{v_1 v_i\}\ \ i = 3, 4, ..., n-1$

$Q = \{v_2 v_1 v_n\}$

$R_i = \{v_i v_{i+1} v_{i+2}\}\ \ i = 2, 4, 6, ..., n-2$

$\psi = \{P_i\} \cup \{Q\} \cup \{R_i\}$ is minimum geodesic graphoidal covering of G

$\Rightarrow \eta_g = n - 3 + 1 + \frac{n}{2} - 1 = q - p + \frac{n}{2} = \frac{3n-6}{2}$

**Theorem 2.11:** Let G be a book graph with 2n+2 vertices then $\eta_g = q - p + n$.

**Proof:** Let $V(G) = \{v_1, v_2, b_{i1}, b_{i2}\}\ \ i = 1, 2, ..., n$

The Geodesic graphoidal path covering of G is as follows

$P_1 = (v_1, v_2)$

$P_2 = \{b_{11}, b_{12}, v_2, b_{22}\}$

$P_{n+1} = \{b_{11}, v_1, b_{21}, b_{22}\}$

$P_i = \{b_{i1}, b_{i2}, v_1\}\ \ i = 3, 4..., n$

[ if this path not exists for some *i* then for that particular *i* , $P_i = \{b_{i2}, b_{i1}, v_1\}$ ]

$Q_i = \{v_2, b_{i1}\}\ \ o\ \ \{v_2, b_{i2}\}\ i = 3, 4..., n$

$\psi = \{P_i\} \cup \{Q_i\}$ is minimum geodesic graphoidal covering of G

$\Rightarrow \eta_g = 3 + n - 2 + n - 2 = 2n - 1 = q - p + n$

For book graph the graphoidal coveing number is also same as geodesic graphoidal covering number

$(i.e.)\eta_a = q - p + n$

**Theorem 2.12:** Let G be Mongolian tent graph then $\eta_g(M_{m,n}) = q - p + 2$

**Proof:** Let $G = M_{m,n}$

Let $V(G) = \{v_0, v_{i1}, v_{i2}, \ldots, v_{in}\}\ \ i = 1, 2, ..., m$

The Geodesic graphoidal path covering of G is as follows

$P_i = \{v_{i+1,1}, v_{i1}, v_{i2}, \ldots, v_{in}\}\ \ i = 1, 2, ..., m-1$

$P_n = \{v_{1n}, v_{2n}, v_{3n}, ..., v_{mn}, v_{m,n-1}, v_{m,n-2}, \ldots, v_{m2}, v_{m1}\}$

$P_{n+1} = \{v_{11}, v_0, v_{1n}\}$

S= The remaining edges

From above paths all the vertices are exterior points except $v_{1n}$ and $v_{m1}$

$\psi = P_1 \cup P_2 \cup \ldots \cup P_n \cup P_{n+1} \cup S$ is minimum geodesic graphoidal covering of G

$\Rightarrow \eta_g = q - p + 2$

For book graph the graphoidal coveing number is also same as geodesic graphoidal covering number

**Theorem 2.13:** Let G be double wheel graph with 2n+1 vertices $\eta_g(G) = \begin{cases} 3n+2 & \text{if n is odd} \\ 3n & \text{if n is even} \end{cases}$.

**Proof:** Let $V(G) = \{v_0, c_{11}, c_{12}, \ldots, c_{1n}, c_{21}, c_{22}, \ldots, c_{2n}\}$

$P_1 = (c_{21}, \text{v}_0, c_{11})$

$P_{ij} = \{c_{ij}, c_{ij+1}, c_{ij+2}, i = 1, 2\ \&\ j = 1, 3, 5, ..., n-2$

$Q_{ij} = \begin{cases} c_{i,n-2}, c_{i,1},\ i = 1, 2\ \&\ \text{n is odd} \\ c_{i,n-1}, c_{i,\text{n}}, \text{c}_{\text{i, j}},\ i = 1, 2\ \&\ \text{n is even} \end{cases}$

$\Rightarrow \eta_g = 2n - 2 + n - 1 + 1 = 3n - 2$ if n is odd

$\eta_g = 2n - 2 + n + 1 + 1 = 3n$ if n is even

**Theorem 2.14:** $\eta_g(A_5^3) = q - p + 2$.

**Proof:** Let $V(G) = \{x, c_{11}, c_{12}, \ldots, c_{1n}, c_{21}, c_{22}, \ldots, c_{2n}, c_{31}, c_{32}, \ldots, c_{3n}, y\}$

The Geodesic graphoidal path covering of G is as follows

$P_i = \{x, \text{c}_{1i}, \text{c}_{2i}, \text{c}_{3i}, \text{y}\}\ \ i = 1, 2, ..., n$

Clearly all the vertices are internal vertices except x and y.

$\psi = \{P\}_i \cup$ Remaining Edges is minimum geodesic graphoidal covering of G

$\therefore \eta_g = q - p + 2$

**Theorem 2.15:** $\eta_g(D_7^3) = q - p + 2$.

**Proof:** Let $V(G) = \{\text{y}, v_{i1}, v_{i2}, \ldots, v_{in}\}\ \ i = 1, 2, ..., m$

The geodesic graphoidal cover of $D_7^3$ is as follows:

$P_i = \{v_{i+1,1}, v_{i1}, v_{i2}, \ldots, v_{in}\}\ \ i = 1, 2, ..., m-1$

$P_n = \{v_{1n}, v_{2n}, v_{3n}, ..., v_{mn}, v_{m,n-1}, v_{m,n-2}, \ldots, v_{m2}, v_{m1}\}$

$Q = (v_{11}, y, v_{1n})$

S = The remaining edges

$\psi = \{P\}_i \cup Q \cup S$ is minimum geodesic graphoidal covering of G and all the vertices are internal vertices except $v_{11}$ and $v_{1n}y$.

$\therefore \eta_g = q - p + 2$

**REFERENCES**


1. B.D. Acharya and E. Sampathkumar, Graphoidal covers and graphoidal covering number of a graph, Indian J.Pure Appl.Math.18 (10) (1987), 882-890.
2. S.Arumugam, B.D.Acharya and E.Sampathkumar,Graphoidal covers of a graph:a creative review,in Proc. National Workshop on Graph theoryand its applications, Manonmaniam Sundaranar University, Tirunelveli, Tata McGraw-Hill, New Delhi,(1997), 1-28.
3. S.Arumugam and J.Suresh Suseela, Acyclic graphoidal covers and path partitions in a graph, Discrete Math.190(1998), 67-77
4. In S. Arumugam and J. Suresh Suseela, Geodesic Graphoidal covering number of a graph J. Indian. Math. Soci., 72(2005), 99-106.
5. F.Harary, Graph Theory, Addison-Wesley, Reading, MA, 1969.
6. Hao Li, Perfect path double covers in every simple graph, Journal of Graph Theory, 14 (6) (1990), 645–650.
7. Joseph A. Gallian, Department of Mathematics and Statistics, University of Minnesota Duluth, Minnesota 55812, U.S.A, A Dynamic Survey of Graph Labeling,.
8. C.Packkiam and S.Arumugam, The graphoidal covering number of unicyclic graphs, Indian J.Pure appl.Math.23 (2) (1992), 141-143.
9. Conjectures and open problems on Face Antimagic Evaluations of Graphs. M. Baca, E.T. Baskoro,Y.M. Cholily,s. Jendrol,Y.Lin,M.Miller, J. Ryan, R.Simanjuntak, Slam in, and K.A.Sugeng, 2000, Mathematics Subject Classification 05c78.


**Source of support: Nil, Conflict of interest: None Declared**