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FIXED POINT THEOREMS ON EXPANSION IN FUZZY METRIC SPACE

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ABSTRACT

In this paper introducing fixed point theorem on expansion in fuzzy metric space, we also introduce the concept of R- weak commutatively of fuzzy metric spaces. Some illustrative examples.

Key Words: *fixed point, fixed point theorem. Common fixed point, Fuzzy metric space.*

AMS Subject Classification: *47H10, 54H25.*

1. INTRODUCTION

Zadeh [1], introduction of fuzzy sets by the fuzzyness invaded almost all the branches of crisp mathematics. introduced the concept of fuzzy metric space, George and Veeramani [4] modified the concept of fuzzy metric space introduced by kramosil and michalek [5]. In this paper effort has been made to obtain some results on fixed points of expansion type mapping in fuzzy metric space.

2. PRELIMINARIES

Definition 2.1: [7] A binary operation $*:[0, 1] \times [0, 1] \mapsto [0, 1]$ is cal led a continuous t-norm if ([0,1],*) is an abelian topological monoid with the unit 1 such that $a*b \le c*d$ whenever
$a \le c$ and $b \le d$ for all a, b, c, $d \in [0, 1]$.

Examples of t-norms are $a * b = ab$ and $a * b = \min\{a, b\}$

Definition 2.2: [4] the 3-tuple (X, M, *) is called a fuzzy metric space (FM-space) if X is an arbitrary set * is a continuous t-norm and M is a fuzzy set in $X^2 \times [0, \infty]$ satisfying the following conditions for all x, y, $z \in X$ and t, s > 0.

2. (1) $M(x, y, 0) > 0$
2. (2) $M(x, y, t) = 1, \forall t > 0$ iff $x = y$
2. (3) $M(x, y, t) = M(y, x, t)$,
2. (4) $M(x, y, t) * M(y, z, s) \le M(x, z, t + s)$
2. (5) $M(x, y, .) : [0, \infty] \to [0, 1]$ is continuous.

Remark 2.3: since * is continuous, it follows from (2. (4)) that the limit of a sequence in FM-space is uniquely determined

Example 2.4: Let X = N define $a * b = \max\{0, a + b – 1\}$ and $a \lozenge b = a + b – ab$ for all a, $b \in [0, 1]$ and let M and N be fuzzy sets on as follows

$$M(x,y,t)=\begin{cases}\dfrac{x}{y} if\ x\le y\\ \hline \dfrac{y}{x} if\ y\le x\end{cases}$$

For all x, $y \in X$ and t > 0 then (X, M, *) is fuzzy metric space.

Lemma 2.1: in fuzzy metric space x, M(x, y, .) is non decreasing and non increasing for all x, $y \in X$

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Lemma 2.2: Let (X, M, *) be fuzzy metric space if there exist $k \in (0, 1)$ such that $M(x, y, kt) \geq M(x, y, t)$ for all $x, y \in X$ then x = y

Lemma 2.3: Let (X, M, *) be a fuzzy metric space if there exists a number $k \in (0, 1)$ such that

$$M(y_{n+2}, y_{n+1}, qt) \geq M(y_{n+1}, y_n, t) \tag{1}$$

And n = 1, 2… then $\{y_n\}$ is a Cauchy sequence in X

3. MAIN RESULTS

Theorem 3.1: Let (X, M,, *,) be a complete FM – space and f be a self map of X, onto itself there exist a constant k > 1

$$M(f_x, f_y, kt) \leq M(x, y, t) \tag{1}$$

For all $x, y \in X$ and t > 0. Then f has a unique fixed point in x.

Proof: Let $x_0 \in X$ as f is onto, there is an element $x_1 \in f^{-1}x_0$. In the same way $x_n \in f^{-1}x_{n-1}$. For all n = 2, 3, 4… thus we get a sequence $\{x_n\}$, if $x_m = x_{m-1}$ for some m then x_m is a fixed point of f now suppose

$x_n \neq x_{n-1}$ for all n = 1,2……… then it follows from (1) that
$M(x_n, x_{n+1}, kt) = M(f_{x_{n+1}}, f_{x_{n+2}}, kt) \leq M(x_{n+1}, x_{n+2}, t)$ and for all $\{x_n\}$ is a

Cauchy sequence in X since X is complete $\{x_n\}$ has limit $u \in X$ as f is onto there is an element $v \in X$ such that $v \in f^{-1}u$.

Now

$$M(x_n, u, kt) = M(f_{x_{n+1}}, f_v, kt) \leq M(x_{n+1}, v, t)$$

Which as $n \to \infty$ gives M(u, v, t) = 1 for all t > 0, therefore (FM 2.(.2) it follows that u = v yielding thereby fu = u and so u is the fixed point of f. let u and v be the two fixed points of f ie fu = u and fv = v then (1) yields

$$M(u, v, kt) = M(fu, fv, t) \leq M(u, v, t)$$

for all t > 0 hence in view of lemma 2.2 we obtain u = v which shows the uniqueness of u as a fixed point of f this completes the proof

Theorem 3.2: let (X, M, *) be a complete FM-space with $t * t \geq t$ and $(1 - t) \lozenge (1 - t) \leq (1 - t)$ for all $t \in [0, 1]$ and f be mapping from X onto itself there exist a number k > 1 such that

$$M(f_x, f_y, kt) \leq M(x, y, t) * M(x, f_x, t) * M(y, fy, t) \tag{1}$$

For all $x, y \in X$ and t > 0 then f has a unique fixed point in X

Proof: a sequence $\{x_n\}$ is developed similarly as in theorem 3.1, if $x_{m-1} = x_m$ for some m, f has a fixed point x_m, suppose $x_{n-1} \neq x_n$ for every positive integer n then from (1)

$$\begin{aligned} M(x_n, x_{n+1}, kt) &= M(f_{x_{n+1}}, f_{x_{n+2}}, kt) \\ &\leq M(x_{n+1}, x_{n+2}, t) * M(x_{n+1}, f_{x_{n+1}}, t) * M(x_{n+2}, f_{x_{n+3}}, t) \\ &= M(x_{n+1}, x_{n+2}, t) * M(x_{n+1}, x_n, t) * M(x_{n+2}, x_{n+1}, t) \end{aligned}$$

yielding there by

$$M(x_n, x_{n+1}, kt) \leq M(x_n, x_{n+1}, t) * M(x_{n+2}, x_{n+2}, t) \tag{2}$$

Now suppose

$$M(x_{n+1}, x_{n+2}, t) < M(x_n, x_{n+1}, t)$$

For all t > 0 then in view of lemma 2.3 $\{x_n\}$ is a Cauchy sequence in X which is complete therefore there exists some $u \in X$ such that $x_n \to u$ since f is onto there is an element now $u \in f^{-1}$ and

$$\begin{aligned} M(x_n, u, kt) &= M(f_{x_{n+1}}, f_v, kt) \\ &\leq M(x_{n+1}, v, t) * M(x_{n+1}, x_n, t) * M(v, u, t) \end{aligned}$$

Which as letting $n \to \infty$ gives M(u, v, t) = 1 for all t > 0.

Therefore by (FM 2.4.2) it is noting that u = v and so fu = u i.e. u is a fixed point of f the uniqueness of u can be shown easily from (1) hence the theorem proved

Theorem 3.3: let (X, M, *) be a complete FM- space with $t * t \geq t$ and $(1 - t) \lozenge (1 - t) \leq (1 - t)$ for all $t \in [0, 1]$ and f, g be two self maps of X onto itself if there exist a number k > 1 such that

$$M(f_x, g_y, kt) \leq M(x, y, t) * M(x, f_x, t) * M(y, g_y, t) \tag{1}$$

for all $x \in X$ and t > 0 then f has a unique common fixed point in X

Proof: choose an element $x_0 \in X$ as f is onto there is an element $x_1 \in f^{-1}x_0$ since g is onto there exist an element x_2 satisfying $x_2 \in g^{-1}x_1$ thus in general a sequence $\{x_n\}$ is defined as $x_{2n+1} \in f^{-1}x_{2n}$, $x2n+2 \in g^{-1}x_1$, for all n = 0, 1, 2, ……… now we have two cases as follows

Case - (1): when $x_m \neq x_{m+1}$ for all m = 0, 1, 2…… in this case it follows from (1) that

$$\begin{aligned} M\left(x_{2n}, x_{2n+1}, kt\right) &= M\left(f_{x_{3n+1}}, f_{x_{2n+2}}, kt\right) \\ &\leq M\left(x_{2n+1}, x_{2n+2}, t\right) * M\left(x_{2n+1}, x_{2n}, t\right) * M\left(x_{2n+1}, x_{2n+2}, t\right) \\ &\quad M\left(x_{2n+1}, x_{2n+2}, t\right) * M\left(x_{2n}, x_{2n+1}, t\right) \end{aligned} \tag{2}$$

Suppose

$$M\left(x_{2n+1}, x_{2n+2}, t\right) < M\left(x_{2n}, x_{2n+1}, t\right)$$

Then from (2) we obtain

$$M\left(x_{2n}, x_{2n+1}, kt\right) \leq M\left(x_{2n}, x_{2n+1}, t\right)$$

Which in view of lemma (2.2) implies $x_{2n} = x_{2n+1}$ which is a contradiction therefore

Let $M\left(x_{2n+1}, x_{2n+2}, t\right) \geq M\left(x_{2n}, x_{2n+1}, t\right)$

Then (2) yields

$$M\left(x_{2n}, x_{2n+1}, kt\right) \leq M\left(x_{2n}, x_{2n+2}, t\right)$$

For all t > 0 similarly it can be show that

$$M\left(x_{2n+1}, x_{2n+2}, kt\right) \leq M\left(x_{2n+2}, x_{2n+1}, t\right)$$

For all t > 0, thus in general we obtain

$$M\left(x_{n}, x_{n+1}, kt\right) \leq M\left(x_{n+1}, x_{n+2}, t\right)$$

For all t > 0 and n = 0, 1, 2…… hence in view of lemma (2.3) $\{x_n\}$ is a Cauchy sequence in X which is complete therefore $\{x_n\}$ has a limit point in x since $\{x_{2n}\}$ and $\{x_{2n+1}\}$ are subsequence of $\{x_n\}$, $x_{2n} \to u$ and $x_{2n} \to u$ as $n \to \infty$ as f and g are onto there exist v, w $\in$ X satisfying $v \in f^{-1}u$ and $w \in f^{-1}u$ now

$$\begin{aligned} M\left(x_{2n}, u, kt\right) &= M\left(f_{x_{2n+1}}, gw, kt\right) \\ &\leq M\left(x_{2n+1}, w, t\right) * M\left(x_{2n+1}, x_{2n}, t\right) * M\left(w, gw, t\right) \end{aligned}$$

Which as $n \to \infty$ gives M(u, w, t)=1, for all t>0 then by (FM 2.4.2) it follows that u=w in the similar pattern taking x = v and y = x_{2n+2} in (1) and therefore proceeding as above we obtain u = v therefore u = v = w which immediately implies fu = gu = u and so u is a common fixed point of f and g now let u and v be two common fixed point of f and g.

i.e. fu = gu and fv = gv = v then

$$\begin{aligned} M(u, v, kt) &= M(fu, fv, kt) \\ &\leq M(u, v, t) * M(u, fu, t) * M(v, gv, t) \\ &= M(u, v, t) * 1 * 1 = M(u, v, t) \end{aligned}$$

For all t > 0 further by application of lemma (2.2) we obtain u = v

Case - II: When $x_{m-1} = x_m$ for some m here m may be even or odd, positive integer without loss of generality suppose m is an integer say m = 2p then $x_{2p-1} = x_{2p}$ i.e. $gx_{2p} = fx_{2p-1}$ which implies $x_{2p} = x_{2p+1}$ (as we have $fx \neq fy$ if $x \neq y$) therefore we have $x_{2p-1} = x_{2p} = x_{2p+1} = \ldots\ldots$ which shows that $\{x_n\}$ is convergent sequence and so Cauchy sequence in X the rest of the proof is similar to as in case (I) and this complete the proof.

REFERENCES

[1] Zadeh L A, Fuzzy sets, inform, and control 8 (1965) 338.
[2] Kaleva O, Seikkala, S, On fuzzy metric spaces. Fuzzy sets and system (1962) 122.
[3] Deng Z K, Fuzzy pseudo- metric spaces, J. Math. Anal , Appl. 86 (1982) 74.
[4] George A & Veeramani P, On some results in fuzzy metric spaces, fuzzy sets and systems 64, (1994) 395.
[5] Kramosil I & Michalek J, Fuzzy metric and statistical metric spaces kyberntika Appl. 50, (1985)142.
[6] Grabiec M, Fixed points in fuzzy metric spaces, fuzzy sets and systems, 27 (1988) 385.
[7] Scweizer B & Sklar A, statistical metric spaces pacific J Math. 10,(1960) 313.
[8] Rosenholtz, local expansion derivatives and fixed points, Fund. Math 91, (1976).
[9] Rhodes B E, A comparision of various definitions of contractive mappings, trans, Amer. Math, Soc., 57(1976) 257.
[10] Banach S Theoric les operations linearires manograie, Mathmatyezne Warsaw, Poland, (1932).
[11] Wang S Z, Li B Y, Gao Z M & Iseki K, Some fixed point theorems on expansion mappings Math. Japanica, 29, (1984) 631.
[12] Cho Y J, Pathak H K, Kang S M & Jang J S, Common fixed points of compatible maps of type (b) of fuzzy metric spaces, fuzzy sets and systems 93, (1998) 99.
[13] Fang J X, On fixed point theorems in fuzzy metric spaces fuzzy sets and systems 46, (1992) 107.
[14] Edelstein M, On fixed and periodic points under contraction mappings , J. London Math . Soc. 37, (1962) 74.
[15] Mishra S N, Sharma N & Singh S I, common fixed point of maps on fuzzy metric spaces , Inter. J. Math. Sci. 17, (1994) 253.
[16] Park J H, Intuitionistic fuzzy metric spaces, Chaos solitons and fractals 22 (2004) 1039.
[17] Park J H, Kwun Y C, & Park J H, A fixed point theorem in the intuitionistic fuzzy metric spaces , far east J. Math. Sci. 16 (2005), 137-149.
[18] Saadati R & Park J H, On the intuitionistic fuzzy topological spaces, chaos, solitons and fractals 27 (2006), 331.
[19] Turkoglu D, Alaca C, Cho Y J & Yildiz C, common fixed point theorems in intuitionistic fuzzy metric spaces J. Appl, Math, and computing 22(2006), 41.
[20] Turkoglu D, Altun I & Cho Y J, Common fixed points of compatible mappings of type (I) and type (II) in fuzzy metric spaces, J. fuzzy math. 15(2007), 435.
[21] Atarassov K, Intuitionistic fuzzy set and system 20 (1986), 87. [22] Choudhary B S, A unique common fixed point theorem for sequence of self maps in menger spaces Bull. Korean math, Soc. 37 (2000). No. 3 [23] Jungck, commuting mappings and fixed points, Amer. Math. Monthly, 83 (1976), 261.

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