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NONLINEAR CONTRACTIONS IN COMPLETE FUZZY METRIC SPACES

Siva Krishna[1], Venugoplam[2] and Vijaya Kumar*[3]

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ABSTRACT

Our main purpose in this paper is to introduce the notion of φ-contractive mapping in fuzzy metric spaces and on the existence and the approximation of fixed point of nonlinear contractions mappings in fuzzy metric spaces. results are analogous in metric spaces.

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1. INTRODUCTION

Our terminology and notation for fuzzy metric spaces conform of that George et al. [5, 6]. Recently Gregori *et al.* [4] have showed that the study of the intuitionistic fuzzy metric space (IFMS) (X, M, N,*, ◊) can be reduced to the study of the fuzzy metric space (FMS) (X,M,*). More exactly, the topology $T_{(MN)}$ of an IFMS (X, M, N, *, ◊) coincides with the topology $\tau_{(M)}$ generated by the FMS (X,M,*), which has as a base the family of open sets. So, our study is limited to FMSs.

Definition 1.1: A subset B of FMS(X, M, *) is called fuzzy bounded if for each t > 0 there exists $\lambda \in (0,1)$ such that $M(x, y, t) \geq \lambda$ for all x, y ∈ B.

Remark 1.2: Let(X, d) be a metric space. Denote $a*b= \min \{a, b\}$ for all a, b ∈ [0, 1] and let M_d be a fuzzy set on $X^2 \times (0,\infty)$ defined as follows:

$$M_d(x,y,t) = \frac{t}{t+d(x,y)}$$

It easy to check is that(X, M_d,*) is a fuzzy metric space, and

$$B(x,y,t) = \left\{ y \in : d(x,y) < \frac{rt}{1-r} \right\}$$

So (X, M_d,*) is a complete fuzzy metric space if and only if (X, d) is a complete metric space. Moreover, a nonempty subset A of (X, M_d, *) is fuzzy bounded if and only if A is bounded in (X, d)

Lemma 1.3: Every convergence sequence of a Fuzzy metric space is fuzzy bounded.

Definition 1.4: Let (X, M, *) be a FMS. Let $\varphi:[0,\infty) \to [0,\infty)$ is an upper semi-continuous from the right function such that $\varphi(0) = 0$ and $\varphi(t) < t$ for $t > 0$.

We will say the sequence $\{x_n\}$ in X is fuzzy φ-contractive if for each t > 0,

$$\frac{t}{M(x_{n+2},x_{n+1},t)} - \leq \phi\left(\frac{t}{M(x_{n+2},x_{n+1},t)} - t \right)$$

for all n ∈ N.

Corresponding author: Vijaya Kumar[3]*

A selfmap f on (X, M, *) is called fuzzy φ-contractive if for each t > 0,

$$\frac{t}{M\left(f(x), f(y), t\right)} - t \le \phi\left(\frac{t}{M\left(x, y, t\right)} - t)\right)$$

for all x, y ∈ X.

Remark 1.5: It is not hard to prove that every φ-contractive selfmap f on a metric (X, d) is fuzzy φ-contractive on (X, M_d,*) (* a t-norm such that (X, M_d,*) is a fuzzy metric space). As it very easy to check that a fuzzy contractive mapping of contractive constant k is a φ-contractive with r → φ(r) = rk

Lemma 1.6: Every fuzzy bounded φ-contractive sequence $\{x_n\}$ of a FMS (X, M,*) is a fuzzy Cauchy sequence.

2. MAIN RESULTS

Theorem 2.1: Let A, B, S and T be self mappings of a fuzzy metric space .Let (X, M,*) be a complete fuzzy metric space and A, B, S and T be a φ-contractive self-map on X. If there exists x_0 such that the sequence $\{(A, B, S, T), (x_0)\}$ is fuzzy bounded. Then A, B, S and T has a unique fixed point in X. Furthermore, the Picard iterates associate to each point of X converge to the fixed point

Proof: Let $x_0 \in X$, such that $x_n = \{(A, B, S, T), (x_0)\}$n ∈ IN is bounded. Since A, B, S and T is φ-contractive therefore $\{x_n\}$ is fuzzy bounded φ-contractive sequence. Hence from the Lemma 1.2 $\{x_n\}$ is a Cauchy sequence in a complete fuzzy metric space. Then there exists a point x ∈ X such that $x_n \to x$ as $n \to \infty$.

Now we shall show that Ax=x.

Since A, B, S, and T is φ-contractive then

$$\lim_{n\to+\infty} Ax_n = \lim_{n\to+\infty} Sx_n = \lim_{n\to+\infty} By_n = \lim_{n\to+\infty} Ty_n = z \tag{2.1}$$

As n→∞, then $\lim_{n\to\infty} M(A(x_n), A(x), t) = 1$ for each t>0. Therefore,

$\lim_{n\to\infty} x_n = A(x)$ that is A(x)=x. Hence f has a fixed point x ∈ X. Next,

Let z ∈ X then

$$\frac{1}{M\left(Au, By_n, t\right)} - 1 \le \left(\frac{1}{\min\{M\left(Su, Ty_n, t\right), M\left(Au, Su, t\right), M\left(By_n, Ty_n, t\right)\}} - 1\right) \tag{2.2}$$

As n→∞, then $\lim_{n\to\infty} M((A, B, S, T)(z), x, t) = 1$ for each t > 0.Therefore, $\lim_{n\to\infty} A^n(z) = x$. We claim that x is the unique fixed point of f. For this suppose that y (x ≠ y) is another fixed point of f in X. Then

$$\frac{1}{M\left(Au, z, t\right)} - 1 \le r\left(\frac{1}{\min\{M\left(Au, z, t\right)\}} - 1\right) \tag{2.3}$$

That is a contradiction yielding Au=Su, therefore, u is a coincidence point of the pair (A, S)

If T(x) is closed subset of X, then $\lim_{n\to\infty}$, Ty_n =z ∈ T(x). There fore, there exists a point w∈ X such that Tw =z.

Now, we assert that Bw = Tw, if not, then according to we have

$$\frac{1}{M\left(Ax_n, Bw, t\right)} - 1 \le r\left(\frac{1}{\min\{M\left(Sx_n, Tw, t\right), M\left(Ax_n, Sx_n, t\right)M\left(Bw, Tw, t\right)\}} - 1\right) \tag{2.4}$$

Which on making n→ + ∞, for every t > 0 , reduces to

$$\frac{1}{M\left(z, Bw, t\right)} - 1 \le r\left(\frac{1}{\min\{M\left(z, Bw, t\right)\}} - 1\right) \tag{2.5}$$

Which is a contraction as earlier . it follows that Bw = Tw which shows that w is a point of coincoidence of the pair (B,T) . Since the Pair (A,S) is weakly compatible and Au = Su , hence Az=ASu=SAu=Sz

$$\frac{1}{M(Az,Bw,t)}-1\le r\left(\frac{1}{\min\{M(Az,Bw,t)\}}-1\right) \tag{2.6}$$

Implying thereby that AZ =Bw=Z.

Finally, using the notion of weak compatibility of the pair (B,T) together with we get Bz=z=Tz. Hence z is a common fixed point of both the pairs (A, S) and (B, T)

Uniqueness of the common fixed point z say is an easy consequence of the condition.

Letting n→∞ we obtain M(x, y, t) = 1 for each t > 0, that is x = y, which is a contradiction. This completes the proof of the theorem.

Corollary 2.2: Suppose that A, B, S and T is φ-contractive self map on a complete metric space (X,d). Then A, B, S and T has a unique fixed point z and moreover, for any x belong to X. the sequence of iterates {aA, B, S, T(x)} converges to z

Corollary2.3: Suppose that A,B,S and T is φ-contractive self map on a complete metric space. Let (X, M,*) be complete fuzzy metric space in which contractive sequences are Cauchy, A, B, S, T: X→X be an fuzzy contractive mapping. Then A, B, S and T has a unique fixed point.

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