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TRANSIENT CONVECTIVE HEAT AND MASS TRANSFER FLOW IN A HORIZONTAL WAVY CHANNEL WITH OSCILLATORY FLUX WITH CHEMICAL REACTION, THERMAL RADIATION AND RADIATION ABSORPTION

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ABSTRACT

We discuss the effect of chemical reaction on unsteady free connective heat and mass transfer flow through a porous medium in a horizontal wavy channel The unsteadiness in the flow is due to the oscillatory flux in the flow region. The coupled equations governing the flow, heat and mass transfer have been solved by using a perturbation technique with the slope δ as the perturbation parameter. The expression for the velocity, the temperature, the concentration, and the rate of heat and mass transfer are derived and are analysed for different variations of the governing parameters β,N, Sc, k, Q_1, N_1.

Keywords: *Oscillatory Flux, Wavy Channel, Heat and Mass transfer, Radiation, Chemical Reaction, Radiation Absorption.*

1. INTRODUCTION

Coupled heat and mass transfer phenomenon in porous media is gaining attention due to its interesting applications. The flow phenomenon is relatively complex rather than that of the pure thermal convection process. Underground spreading chemical wastes and other pollutants, grain storage, evaporation cooling and solidification are the few other application areas where the combined thermo-solutal natural convection in porous media are observed. Combined heat and mass transfer by free convection under boundary layer approximations has been studied by Bejan and Khair (1985), Lai and Kulacki (1991) and Murthy and Singh (1990). Coupled heat and mass transfer by mixed convection in Darcian fluid-saturated porous media has been analysed by Lai (1971). The free convection heat and mass transfer in a porous enclosure has been studied recently by Angirasa *et al.* (1997). The combined effects of thermal and mass diffusion in channel flows has been studied in recent times by a few authors, notably Nelson and Wood (1986, 1989), Lee *et al.* (1982) and others(1987).

In recent years, energy and material saving considerations have prompted an expansion of the efforts at producing efficient heat exchanger equipment through augmentation of heat transfer. It has been established (1985) that channels with diverging – converging geometries augment the transportation of heat transfer and momentum. As the fluid flows through a tortuous path viz., the dilated–constricted geometry, there will be more intimate contact between them. The flow takes place both axially (primary) and transversely (secondary) with the secondary velocity being towards the axis in the fluid bulk rather than confining within a thin layer as in straight channels. Hence it is advantageous to go for converging–diverging geometries for improving the design of heat transfer equipment. Vajravelu and Nayfeh (1981) have investigated the influence of the wall waviness on friction and pressure drop of the generated coquette flow. Vajravelu and Sastry (1978) have analysed the free convection heat transfer in a viscous, incompressible fluid confined between long vertical wavy walls in the presence of constant heat source. This problem has been extended to the case of wavy walls by McMichael and Deutsch (1984), Deshikachar *et al.* (1985), Rao *et al.* (2008) and Sree Ramachandra Murthy (1992). Hyan Goo Kwon *et al.* (2008) have analyzed that the Flow and heat/mass transfer in a wavy duct with various corrugation angles in two dimensional flow regimes. Mahdy *et al.* (2008) have studied the mixed convection heat and mass transfer on a vertical wavy plate embedded in a saturated porous media (PST/PSE). Comini *et al.* (2003) have analyzed the Convective heat and mass transfer in wavy finned-tube exchangers. Jer-Huan Jang *et al.* (1998) have analyzed that the Mixed convection heat and mass transfer along a vertical wavy surface.

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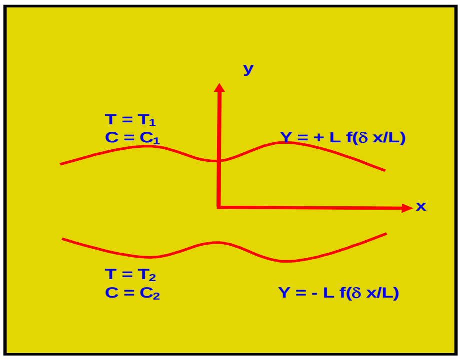
In many chemical engineering processes, there does occur the chemical reaction between a foreign mass and the fluid in which the plate is moving. These processes take place in numerous industrial applications viz., polymer production, manufacturing of ceramics or glassware and food processing. Das *et al.* (1994) have studied the effects of mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction. Muthukumaraswamy (2002) has studied the effects of reaction on a long surface with suction. Recently Gnaneswar (2008) has studied radiation and mass transfer on an unsteady two-dimensional laminar convective boundary layer flow of a viscous incompressible chemically reacting fluid along a semi-infinite vertical plate with suction by taking into account the effects of viscous dissipation.

The present trend in the field of chemical reaction analysis is to give a mathematical model for the system to predict the reactor performance. A large amount of research work has been reported in this field. In particular the study of heat and mass transfer with chemical reaction is of considerable importance in chemical and hydrometallurgical industries. Chemical reaction can be codified as either heterogeneous or homogeneous processes. This depends on whether they occur at an interface or as a single phase volume reaction. Frequently the transformations proceed in a moving fluid, a situation encountered in a number of technological fields. A common area of interest in the field of aerodynamics is the analysis of thermal boundary layer problems for two dimensional steady and incompressible laminar flow passing a wedge. Simulataneous heat and mass transfer from different geometrics embedded in a porous media has many engineering and geophysical application such as geothermal reservoirs, drying of porous solids thermal insulation, enhanced oil recovery, packed-bed catalytic reactors, cooling of nuclear reactors, and under ground energy transport. A very significant area of research in radiative heat transfer, at the present time is the numerical simulation of combined radiation and convection/conduction transport processes. The effort has arisen largely due to the need to optimize industrial system such as furnaces, ovens and boilers and the interest in our environment and in no conventional energy sources, such as the use of salt-gradient solar ponds for energy collection and storage. In particular, natural convection induced by the simultaneous action of buoyancy forces resulting from thermal and mass diffusion is of considerable interest in nature and in many industrial application such as geophysics, oceanography, drying process, solidification of binary alloy and chemical engineering. Kandaswamy *et al* (2006) have discussed the Effects of chemical reaction, heat and mass transfer on boundary layer flow over a porous wedge with heat radiation in the presence of suction or injection. Recently Madhusudan Reddy (2010) has analysed the effect of chemical reaction on double diffusive heat transfer flow of a viscous fluid in a wavy channel.

In this paper, we discuss the effect of chemical reaction on unsteady free connective heat and mass transfer flow through a porous medium in a horizontal wavy channel The unsteadiness in the flow is due to the oscillatory flux in the flow region. The coupled equations governing the flow, heat and mass transfer have been solved by using a perturbation technique with the slope δ as the perturbation parameter. The expression for the velocity, the temperature, the concentration, and the rate of heat and mass transfer are derived and are analysed for different variations of the governing parameters β,N, Sc, k, Q_1, N_1.

2. FORMULATION OF THE PROBLEM

We consider the effect of chemical reaction on the unsteady motion of viscous, incompressible fluid through a porous medium in a horizontal channel bounded by wavy walls . The thermal buoyancy in the flow field is created by an oscillatory flux in the fluid region. The walls are maintained at constant temperature and concentration. The Boussinesq approximation is used so that the density variation will be considered only in the buoyancy force. The viscous and Darcy dissipations are neglected in comparison with heat by conduction and convection in the energy equation. Also the Kinematic viscosity ν, the thermal conducting k are treated as constants. We choose a rectangular Cartesian system 0 (x, y) with x-axis in the horizontal direction and y-axis normal to the walls.



The walls of the channel are at $y = \pm L f(\frac{\delta x}{L})$.

The flow is maintained by an oscillatory volume flux for which a characteristic velocity is defined as

$$q(1 + k\, e^{i\omega t}) = \frac{1}{L}\int_{-Lf}^{Lf} u\, dy . \tag{1}$$

In view of the continuity equation we define the stream function ψ as

$$u = -\psi_y , \; v = \psi_x \tag{2}$$

Eliminating pressure p using the equations governing the flow in terms of ψ are

$$[(\nabla^2\psi)_t + \psi_x(\nabla^2\psi)_y - \psi_y(\nabla^2\psi)_x] = \nu\nabla^4\psi - \beta\, g(T-T_0)_y - \beta^*\, g(C-C_0)_y - \left(\frac{\mu}{k}\right)\nabla^2\psi \tag{3}$$

$$\rho_e C_p\left(\frac{\partial T}{\partial t} + \frac{\partial\psi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial T}{\partial y}\right) = \lambda\nabla^2 T - Q(T-T_o) + Q_1^{'}(C-C_e) + \frac{16\sigma^{\bullet}T_e^3}{3\beta_R}\frac{\partial^2 T}{\partial y^2} \tag{4}$$

$$\left(\frac{\partial C}{\partial t} + \frac{\partial\psi}{\partial y}\frac{\partial C}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial C}{\partial y}\right) = D\nabla^2 C - k_1(C-C_o) \tag{5}$$

where ρ_e is the density of the fluid in the equilibrium state, Te, Ce are the temperature and concentration in the equilibrium state,(u,v)are the velocity components along O(x, y) directions, p is the pressure, T, C are the temperature and Concentration in the flow region,ρis the density of the fluid,μ is the constant coefficient of viscosity, Cp is the specific heat at constant pressure, λ is the coefficient of thermal conductivity, qr is the radiative heat flux, β_0 is the coefficient of thermal expansion, Q is the strength of the constant internal heat source, β^* is the volumetric expansion with mass fraction coefficient D_1, is the molecular diffusivity and k_1 is the chemical reaction coefficient.

Introducing the non-dimensional variables in (2) & (3) as

$$x' = x/L,\; y' = y/L, t' = t\omega, \Psi' = \Psi/\nu, \theta = \frac{T-T_2}{T_1-T_2}, C' = \frac{C-C_2}{C_1-c_2} \tag{6}$$

the governing equations in the non-dimensional form (after dropping the dashes) are

$$R\left(\gamma^2(\nabla^2\psi)_t + \frac{\partial(\psi,\nabla^2\psi)}{\partial(x,y)}\right) = \nabla^4\psi + \left(\frac{G}{R}\right)(\theta_y + NC_y) - D^{-1}\nabla^2\psi \tag{7}$$

$$P_1\left(\gamma^2\frac{\partial\theta}{\partial t} + \frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y}\right) = \nabla^2\theta - \alpha_1\theta + Q_1N_2C \tag{8}$$

$$Sc\left(\gamma^2\frac{\partial C}{\partial t} + \frac{\partial\psi}{\partial y}\frac{\partial C}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial C}{\partial y}\right) = \nabla^2 C - KC \tag{9}$$

where

$R = \frac{UL}{\nu}$ (Reynolds number), $G = \frac{\beta_0 g\Delta T_e L^3}{\nu^2}$ (Grashof number)

$\mathrm{P} = \frac{\mu c_p}{k_1}$ (Prandtl number), $M^2 = \frac{\sigma\mu_e^2 H_o^2 L^2}{\nu^2}$ (Hartmann Number)

$Sc = \frac{\nu}{D_1}$ (SchmidtNumber), $\alpha = \frac{QL^2}{\lambda}$ (Heat source parameter)

$\gamma_1 = \frac{K_1L^2}{D_1}$ (Chemical reaction parameter), $\gamma^2 = \frac{\omega L^2}{\nu}$ (Wormsely Number)

$Q_1 = \frac{Q_1^{'}(C_1-C_2)_R L^2{}_1}{(T_1-T_2)\lambda C_p}$ (Radiation Absorption parameter), $N = \frac{\beta^{\bullet}\Delta C}{\beta\Delta T}$ (Buoyancy ratio)

$N_1 = \frac{\beta_r\lambda}{4\sigma^{\bullet}t_e^3}$ (Radiation parameter), $N_2 = \frac{3N_1}{3N_1+4}$ $P_1 = PN_2, \alpha_1 = \alpha N_2$

$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$

The corresponding boundary conditions are

$$\psi(+1)-\psi(-1)=1$$

$$\frac{\partial\psi}{\partial x}=0,\ \ \frac{\partial\psi}{\partial y}=0 \quad at\,\eta=\pm1 \tag{10}$$

$$\theta(x,y)=1, c=1 \quad on \ \ \eta=-1$$

$$\theta(x,y)=0, C=0 \quad on \ \ \eta=1$$

$$\frac{\partial\theta}{\partial y}=0,\frac{\partial C}{\partial y}=0 \quad at \quad \eta=0 \tag{11}$$

The value of ψ on the boundary assumes the constant volumetric flow in consistent with the hyphothesis. Also the wall temperature varies in the axial direction in accordance with the prescribed arbitrary function t.

3. METHOD OF SOLUTION

The main aim of the analysis is to discuss the perturbations created over a combined free and forced convection flow due to traveling thermal wave imposed on the boundaries. The perturbation analysis is carried out by assuming that the aspect ratio δ to be small.

Introduce the transformation such that

$$\overline{x}=\delta x,\frac{\partial}{\partial x}=\delta\frac{\partial}{\partial\overline{x}},$$

Then

$$\frac{\partial}{\partial x}\approx O(\delta)\rightarrow\frac{\partial}{\partial\overline{x}}\approx O(1)$$

For small values of δ<<1, the flow develops slowly with axial gradient of order δ

And hence we take $\frac{\partial}{\partial\overline{x}}\approx O(1)$

Using the above transformation the equations (8-10) reduces to

$$\delta R\left(\gamma^2(\nabla_1^2\psi)_t+\frac{\partial(\psi,\nabla_1^2\psi)}{\partial(x,y)}\right)=\nabla_1^4\psi+\left(\frac{G}{R}\right)(\theta_y+NC_y)-D^{-1}\nabla^2\psi \tag{12}$$

$$\delta P_1\left(\gamma^2\frac{\partial\theta}{\partial t}+\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y}\right)=\left(\frac{\partial^2\theta}{\partial y^2}+\delta^2\frac{\partial^2\theta}{\partial x^2}\right)-\alpha_1\theta+Q_1N_2C \tag{13}$$

$$\delta Sc\left(\gamma^2\frac{\partial C}{\partial t}+\frac{\partial\psi}{\partial y}\frac{\partial C}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial C}{\partial y}\right)=\nabla_1^2C-KC \tag{14}$$

where $\nabla_1^2=\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$

Introducing the transformation $\eta=\frac{y}{f(\overline{x})}$ the equations (12-14) reduces to

$$\delta Rf\left(\gamma^2(F^2\psi)_t+\frac{\partial(\psi,F^2\psi)}{\partial(\overline{x},\eta)}\right)=F^4\psi+\left(\frac{Gf^3}{R}\right)(\theta_\eta+NC_\eta)-(D^{-1}f^2)F^2\psi \tag{15}$$

$$\delta P\left(\gamma^2\frac{\partial\theta}{\partial t}+f\left(\frac{\partial\psi}{\partial\eta}\frac{\partial\theta}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial\eta}\right)\right)=F^2\theta-(\alpha f^2)\theta+(Q\ f^2)C \tag{16}$$

$$\delta Sc\left(\gamma^2\frac{\partial C}{\partial t}+f\left(\frac{\partial\psi}{\partial\eta}\frac{\partial C}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial C}{\partial\eta}\right)\right)=F^2C-KC \tag{17}$$

where

$$F^2=\delta^2\frac{\partial^2}{\partial\overline{x}^2}+\frac{\partial^2}{\partial\eta^2},M_1^2=M^2$$

We adopt the perturbation scheme and write

$$\begin{aligned}\psi(x,\eta,t)&=\psi_0(x,\eta,t)+ke^{it}\overline{\psi}_0(x,\eta,t)+\delta(\psi_1(x,\eta,t)+ke^{it}.\overline{\psi}_1(x,\eta,t))+.............\\ \theta(x,\eta,t)&=\theta_0(x,\eta,t)+ke^{it}\overline{\theta}_0(x,\eta,t)+\delta(\theta_1(x,\eta,t)+ke^{it}.\overline{\theta}_1(x,\eta,t))+.............\\ C(x,\eta,t)&=C_0(x,\eta,t)+ke^{it}C\overline{\phi}_0(x,\eta,t)+\delta(C_1(x,\eta,t)+ke^{it}.\overline{C}_1(x,\eta,t))+.............\end{aligned} \tag{18}$$

On substituting (18) in (15) - (17) and separating the like powers of δ the equations and respective conditions to the zeroth order are

$$\psi_{0,\eta\eta\eta\eta}-(M_1^2f^2)\psi_{0,\eta\eta y}=-\left(\frac{Gf^3}{R}\right)(\theta_{0,\eta}+NC_{0,\eta}) \tag{19}$$

$$\theta_{o,\eta\eta}-(\alpha_1 f^2)\theta_0=-(Q_1N_2f^2)C_o \tag{20}$$

$$C_{o,\eta\eta}-(Kf^2)C_0=0 \tag{21}$$

with

$\psi_0(+1)$ - $\Psi_0(-1) = 1$, $\psi_{0,\eta} = 0$, $\psi_{0,x} = 0$ at $\eta = \pm 1$ (22)

$$\begin{aligned}\theta_o&=1,\ C_0=1\quad on\quad \eta=-1\\ \theta_o&=0,\ C_0=0\quad on\quad \eta=1\end{aligned} \tag{23}$$

$$\overline{\theta}_{0,\mu\eta}-(iP_1\gamma^2f^2)\overline{\theta}_0=-(Q_1N_2f^2)\overline{C}_o \tag{24}$$

$$\overline{C}_{0,\eta\eta}-(K\gamma^2f^2)\overline{C}_o=0 \tag{25}$$

$$\overline{\psi}_{0,\eta\eta\eta\eta}-((M_1^2+i\gamma^2)f^2)\overline{\psi}_{0,\eta\eta}=-\left(\frac{Gf^3}{R}\right)(\overline{\theta}_{0,\eta}+N\overline{C}_{0.\eta}) \tag{26}$$

$$\overline{\theta}_o(\pm1)=0\qquad \overline{C}_o(\pm1)=0$$

$$\overline{\psi}_o(+1)-\overline{\psi}_o(-1)=1\quad \overline{\psi}_{o,\eta}(\pm1)=0,\overline{\psi}_{o,x}(\pm1)=0 \tag{27}$$

The first order equations are

$$\psi_{1,\eta\eta\eta\eta}-(M_1^2f^2)\psi_{1,\eta\eta}=-\left(\frac{Gf^3}{R}\right)(\theta_{1,\eta}+NC_{1,\eta})+(Rf)(\psi_{0,\eta}\psi_{0,x\eta\eta}-\psi_{0,x}\psi_{0,\eta\eta\eta}) \tag{28}$$

$$\theta_{1,\eta\eta}-(\alpha_1f^2)\theta_1=(P_1Rf)(\psi_{0,x}\theta_{o,\eta}-\psi_{0,\eta}\theta_{ox})-Q_1N^2C_1 \tag{29}$$

$$C_{1,\eta\eta y} - (Kf^2)C_1 = (Scf)(\psi_{0,x}C_{o,\eta} - \psi_{0,\eta}C_{ox}) \tag{30}$$

$$\bar{\psi}_{1,\eta\eta\eta\eta} - ((M_1^2 + i\gamma^2) f^2)\bar{\psi}_{1,\eta\eta} = -\left(\frac{Gf^3}{R}\right)(\bar{\theta}_{1,\eta} + N\bar{C}_{1,\eta}) + (Rf)(\bar{\psi}_{0,\eta}\psi_{0,x\eta\eta}$$
$$+\psi_{0,\eta}\bar{\psi}_{0,x\eta\eta} - \psi_{0,x}\bar{\psi}_{0,\eta\eta\eta} - \bar{\psi}_{0,x}\bar{\psi}_{0,\eta\eta\eta}) \tag{31}$$

$$\bar{\theta}_{1,\eta\eta} - ((iP_1\gamma^2 + \alpha) f^2)\bar{\theta}_1 = (PRf)(\psi_{0,\eta}\bar{\theta}_{o,x} + \bar{\psi}_{0,\eta}\theta_{o\ x} - \bar{\psi}_{0,x}\theta_{o,\eta} - \psi_{0,x}\bar{\theta}_{o\eta}) - (Q_1N_2)\bar{C}_1 \tag{32}$$

$$\bar{C}_{1,\eta\eta} - ((K + i\gamma^2) f^2)\bar{C}_1 = (Scf)(\psi_{0,\eta}\bar{C}_{o,x} + \bar{\psi}_{0,\eta}C_{o\ x} - \bar{\psi}_{0,x}C_{o,\eta} - \psi_{0,x}\bar{C}_{o\eta}) \tag{33}$$

with

$\psi_{1(+1)} - \psi_{1(-1)} = 0$

$\psi_{1,\eta} = 0$, $\psi_{1,x} = 0$ at $\eta = \pm 1$ (34)

$\theta_1(\pm 1) = 0$, $C_1(\pm 1) = 0$

$$\bar{\theta}_1(\pm 1) = 0,\ \bar{C}_1(\pm 1) = 0$$

$$\bar{\psi}_1(+1) - \bar{\psi}_1(-1) = 1 \quad \bar{\psi}_{1,\eta}(\pm 1) = 0, \bar{\psi}_{1,x}(\pm 1) = 0 \tag{35}$$

4. SOLUTION OF THE PROBLEM

Solving the equations (13) - (24) subject to the relevant boundary conditions we obtain

$$C_0 = 0.5\left(\frac{Ch\beta_1\eta}{Ch\beta_1} - \frac{Sh\beta_1\eta}{Sh\beta_1}\right)$$

$$\theta_0 = 0.5\left(\frac{Ch\beta_2\eta}{Ch\beta_2} - \frac{Sh\beta_2\eta}{Sh\beta_2}\right) + a_3\left(Ch\beta_1\eta - Ch\beta_1\frac{Ch\beta_2\eta}{Ch\beta_2}\right) + a_4\left(Sh\beta_1\eta - Sh\beta_1\frac{Sh\beta_2\eta}{Sh\beta_2}\right)$$

$$\psi_0 = a_{29}Ch(M_1\eta) + a_{30}Sh(M_1\eta) + a_{31}\eta + a_{32} + \varphi_3(\eta)$$

$$\varphi_3(\eta) = -a_{23}Ch(\beta_2\eta) - a_{24}Sh(\beta_2\eta) - a_{25}Ch(\beta_1\eta) - a_{26}Sh(\beta_1\eta) + a_{27}\eta Sh(\beta_2\eta) + a_{28}\eta Ch(\beta_2\eta)$$

$$\bar{C} = 0$$

$$\bar{\theta}_0 = 0$$

$$\bar{\Psi}_0 = a_{41}Sh(\beta_5\eta) + a_{42}\eta + a_{43}$$

$$\bar{C}_1 = a_{30}(Ch(M_1\eta) - Ch(M_1)) - a_{24}\beta_2(Ch(\beta_2\eta) - Ch(\beta_2))$$
$$-a_{25}\beta_1\left(Sh(\beta_1\eta) - Sh(\beta_1)\frac{Sh(M_1\eta)}{Sh(M_1)}\right) - a_{26}\beta_1(Ch(\beta_1\eta) - Ch(\beta_1))$$
$$+a_{27}\left(Sh(\beta_2\eta) - Sh(\beta_2)\frac{Sh(M_1\eta)}{Sh(M_1)}\right) + a_{28}\beta_2(\eta Sh(\beta_2\eta) - Sh(\beta_2))$$
$$+a_{28}(Ch(\beta_2\eta) - Ch(\beta_2)) - a_{23}\beta_2\left((Sh(\beta_2\eta) - Sh(\beta_2)\frac{Sh(M_1\eta)}{Sh(M_1)}\right)$$

$$-a_{26}\beta_1\left((Sh(\beta_1\eta)-Sh(\beta_1)\frac{Sh(M_1\eta)}{Sh(M_1)}\right)-a_{27}\beta_2\left((Sh(\beta_2\eta)-Sh(\beta_2)\frac{Sh(M_1\eta)}{Sh(M_1)}\right)$$

$$C_1 = b_{32}Ch(\beta_1\eta)+b_{33}Sh(\beta_1\eta)+\varphi_4(\eta)$$

$$\varphi_4(\eta) = -(b_1+\eta b_2+b_4\eta^2)Ch(\beta_1\eta)+(b_3\eta^2)-b_5\eta-b_6Sh(2\beta_1\eta)-b_7Ch(2\beta_1\eta)$$

5. NUSSELT NUMBER and SHERWOOD NUMBER

The local rate of heat transfer coefficient (Nusselt number Nu) on the walls has been calculated using the formula

$$Nu = \frac{1}{\theta_m-\theta_w}\left(\frac{\partial\theta}{\partial y}\right)_{\eta=\pm1} \text{ where } \theta_m = 0.5\int_{-1}^{1}\theta\, d\eta$$

and the corresponding expressions are

$$(Nu)_{\eta=+1} = \frac{(d_9+\delta d_{11})}{(\theta_m)} \qquad (Nu)_{\eta=-1} = \frac{(d_8+\delta d_{10})}{(\theta_m-1)},$$

where $\theta_m = d_{14}+\delta d_{15}$

The local rate of mass transfer coefficient(Sherwood Number Sh) on the walls has been calculated using the formula

$$Sh = \frac{1}{C_m-C_w}\left(\frac{\partial C}{\partial y}\right)_{y=\pm1} \qquad \text{where} \qquad C_m = 0.5\int_{-1}^{1}C\, dy$$

and the corresponding expressions are

$$(Sh)_{\eta=+1} = \frac{(d_4+\delta d_6)}{(C_m)} \qquad (Sh)_{\eta=-1} = \frac{(d_5+\delta d_7)}{(C_m-1)},$$

where $C_m = d_{12}+\delta d_{13}$

where $d_1.d_2,\ldots\ldots\ldots\ldots,d_{14}$ are constants.

6. DISCUSSION OF THE RESULTS

In this analysis we investigate the effect of chemical reaction, thermal radiation and radiation absorption on unsteady convective heat and mass transfer flow of a viscous flow through a porous medium in a horizontal wavy channel with oscillatory flux. The non-linear coupled equations governing the flow heat and mass transfer have been solved by employing the perturbation technique with the slope δ wavy wall as a perturbation parameter.

The axial velocity u shows in figure 1-6 for different values of β, Sc, N, K, Q_1, N_1. We notice that lesser the molecular diffusivity larger $|u|$ in the flow region (Fig.1). From fig.2, the variation of u with buoyancy ratio N shows that, when the molecular buoyancy force dominates over the thermal buoyancy force $|u|$ enhances, when the buoyancy forces act in the same directions and for the forces acting in opposite directions it depreciates in the flow region. The effect of wall waviness of u is exhibited in fig.3. Higher the constriction of the channel walls smaller $|u|$ in the region. The variation of u with chemical reaction parameter K with $|u|$ enhances with increasing $K \le 1.5$ and $|u|$ depreciates with $K \le 2.5$ (fig.4). An increasing the radiation absorption parameter $|Q|$ leads to depreciates in $|u|$ (fig.5).It is found that higher than the radiative heat flux larger $|u|$ in the flow region(fig .6).

The secondary velocity v which arraigns due to the wavy boundaries is show in figs 11-20 for a different parametric value. Fig (7) represents with Sc, it is found that lesser the molecular diffusivity larger $|v|$ and for further lowering of diffusivity larger $|v|$ in the flow region. The variation of v with buoyancy ratio N is shown in fig (8). It is found that, when the molecular buoyancy force dominates over the thermal buoyancy force $|v|$ enhances in flow region irrespective of directions of the buoyancy forces. The effect of wall waviness on v is shown in fig (9).It is observed that higher the constriction of which on a walls larger $|v|$ is entire floe region. The variation of v with chemical reaction K shows that, magnitude of experience with increasing chemical reaction K (fig.10). Fig (11) represents v with radiation absorption parameter $|Q|$ and worm sly number γ , it is found that $|v|$ depreciates with increasing $|Q|$, also it increases with γ . Fig (12) represents v with radiation parameter N_1..It is found that higher the radiative flux larger $|v|$ in the flow region.

The non-dimension temperature θ is exhibited in figures 13-18 for different parameters. It is found that the non-dimensional temperature θ positive for all variations. From fig (13), it can be seen that the actual temperature depreciates with increasing Schmidt number Sc. When the molecular buoyancy force dominates over the thermal buoyancy force the actual temperature depreciates irrespective of the directions of buoyancy forces in fig (14). The actual temperature depreciates with increasing chemical reactions K (fig.16). From fig (17), it can be observed that the actual temperature enhances with increasing $|Q|$ Also it depreciates with $\gamma \le 0.9$, an enhances with higher $\gamma \ge 1.2$. Fig (18) shows that higher the radiative flux, smaller the actual temperature in the flow region.

The non-dimensional concentration (c) is shown in figs (18)-(24) for different parametric values. It is observed that the non-dimensional concentration c positive for all variations From fig (19) we notice that lesser the molecular diffusivity larger the concentration in the flow field. The variation of C with N shows that the actual concentration depreciates with increased $|N|$ (fig.20). From fig (21), it can be seen that higher the constriction of the channel walls smaller the concentration. The variation of c with K shows that the actual concentration enhances with increase $K \le 1.5$ & depreciates with higher $K \ge 2.5$. An increasing $|Q|$ or γ leads to an enhancement with actual concentration (fig.23). From fig (24) it is observed that the actual concentration depreciates with increasing the radiation parameter N_1..

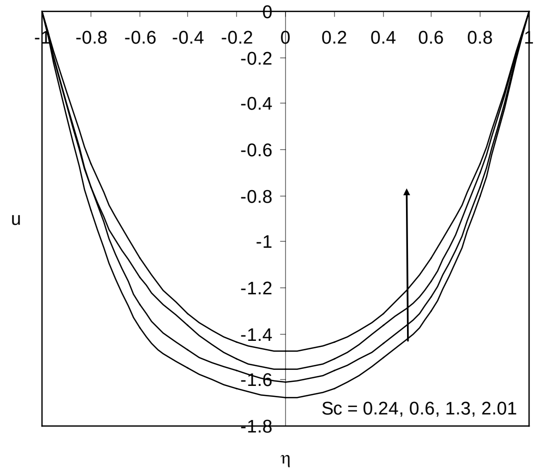


Fig. 1 : Variation of u with Sc
β=-03, k = 0.5, Q_1 = 1, N_1 = 1.5

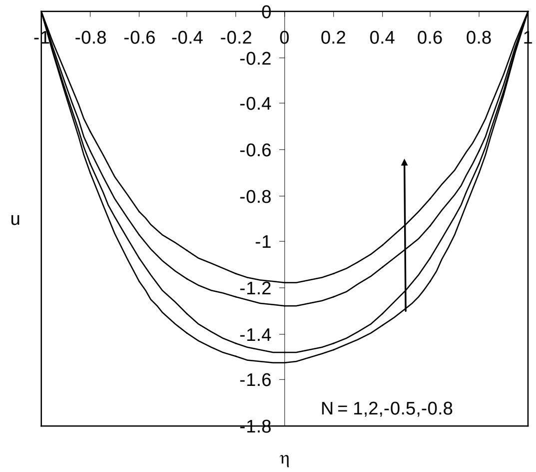


Fig. 2 : Variation of u with N
Sc=1.3, β=-03, k = 0.5, Q_1 = 1

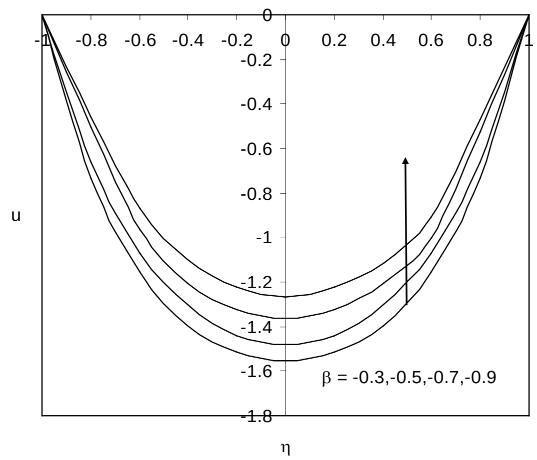


Fig. 3 : Variation of u with β
Sc=1.3, k = 0.5, Q_1 = 1, N_1 = 1.5

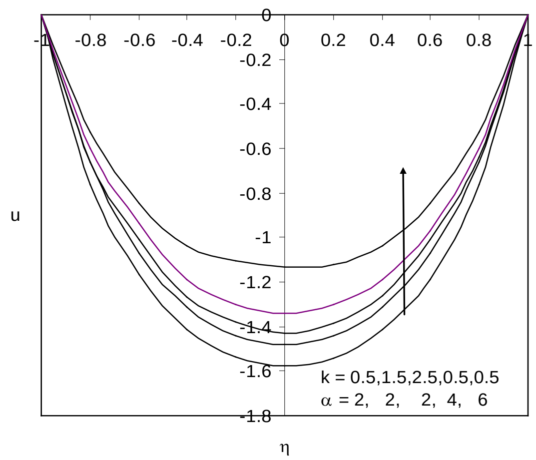


Fig. 4 : Variation of u with k & α
Sc=1.3, β=-03, Q_1 = 1, N_1 = 1.5

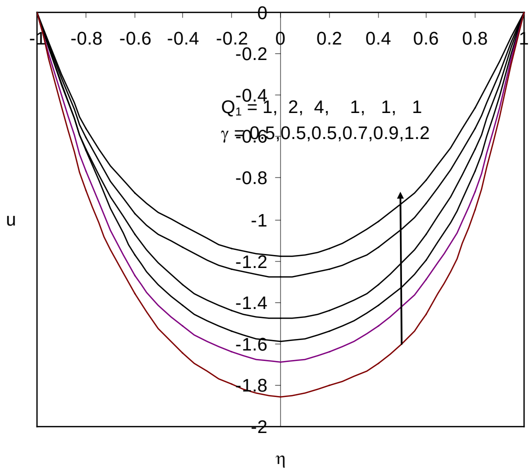


Fig. 5 : Variation of u with Q_1 & γ
Sc=1.3, β=-03, k = 0.5, N_1 = 1.5

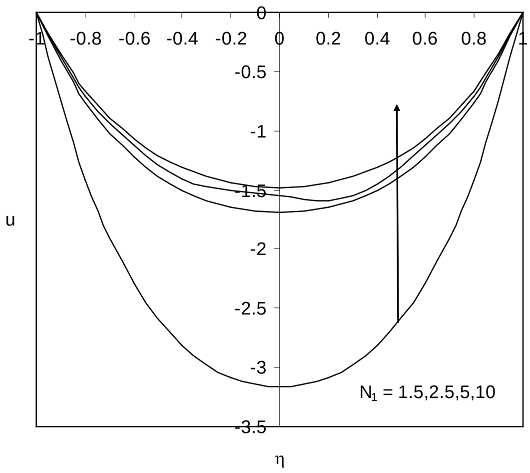


Fig. 6 : Variation of u with N_1
Sc=1.3, β=-03, k = 0.5, Q_1 = 1

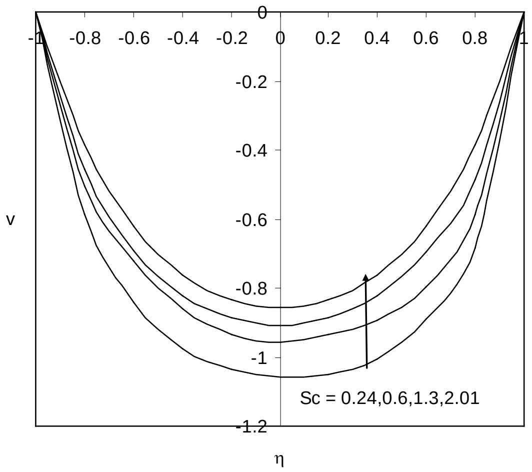


Fig. 7 : Variation of v with Sc
β=-03, k = 0.5, Q_1 = 1, N_1 = 1.5

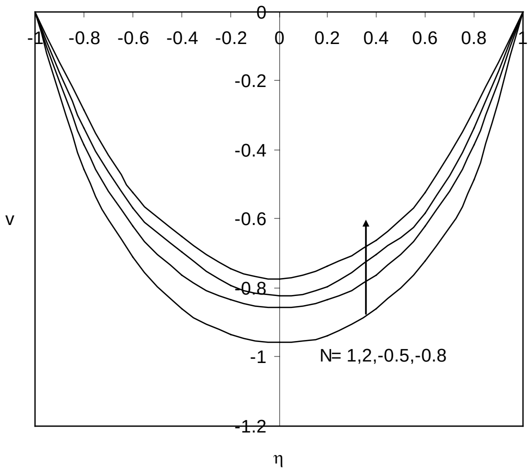


Fig. 8 : Variation of v with N
Sc=1.3, β=-03, k = 0.5, Q_1 = 1

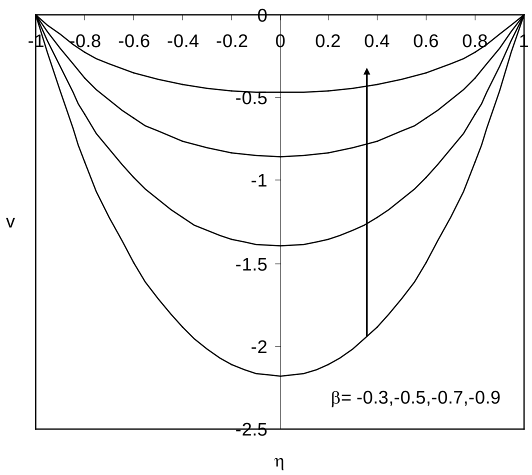


Fig. 9 : Variation of v with β
Sc=1.3, k = 0.5, Q_1 = 1, N_1 = 1.5

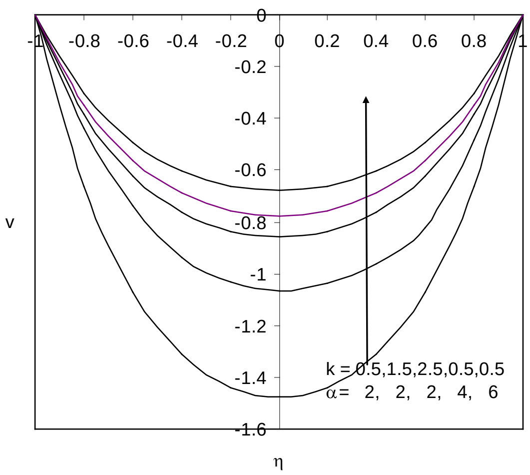


Fig. 10 : Variation of v with k & α
Sc=1.3, β=-03, Q_1 = 1, N_1 = 1.5

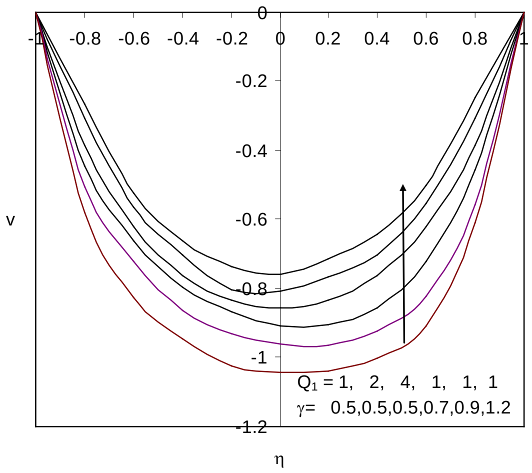


Fig. 11 : Variation of v with Q_1 & γ
Sc=1.3, β=-03, k = 0.5, N_1 = 1.5

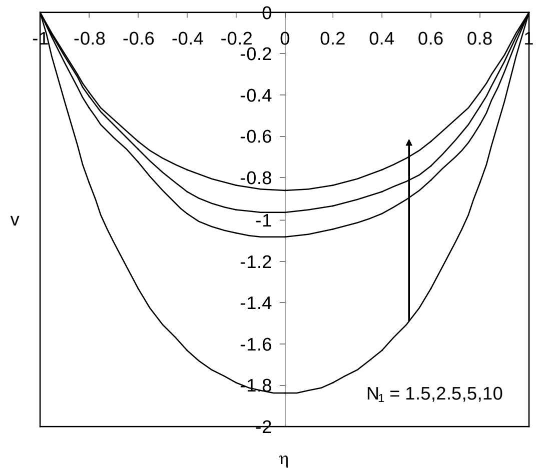


Fig. 12 : Variation of v with N_1
Sc=1.3, β=-03, k = 0.5, Q_1 = 1

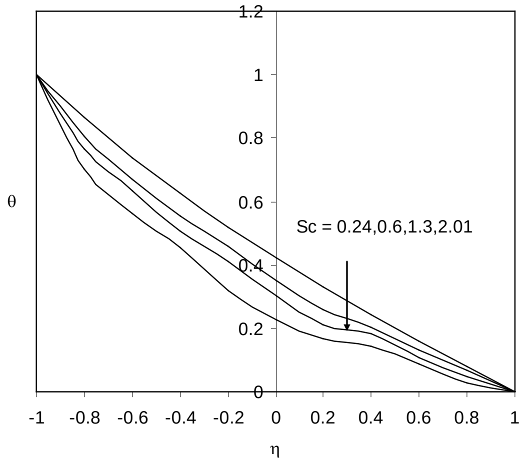


Fig. 13 : Variation of θ with Sc
β=-03, k = 0.5, Q_1 = 1, N_1 = 1.5

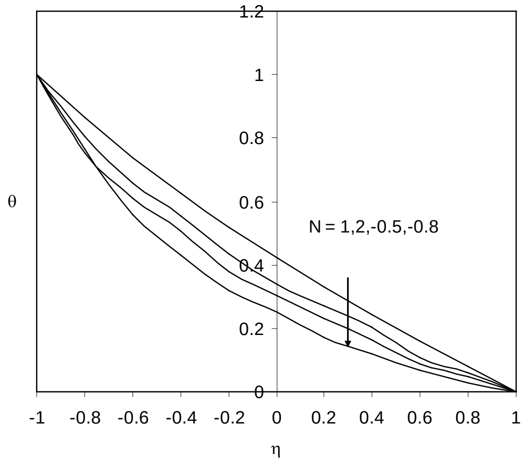


Fig. 14 : Variation of θ with N
Sc=1.3, β=-03, k = 0.5, Q_1 = 1

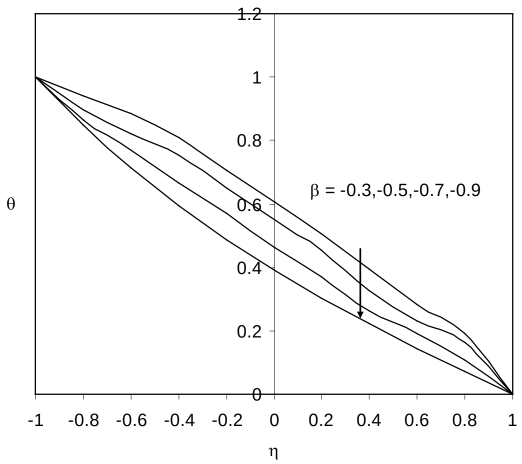


Fig. 15 : Variation of θ with β
Sc=1.3, k = 0.5, Q_1 = 1, N_1 = 1.5

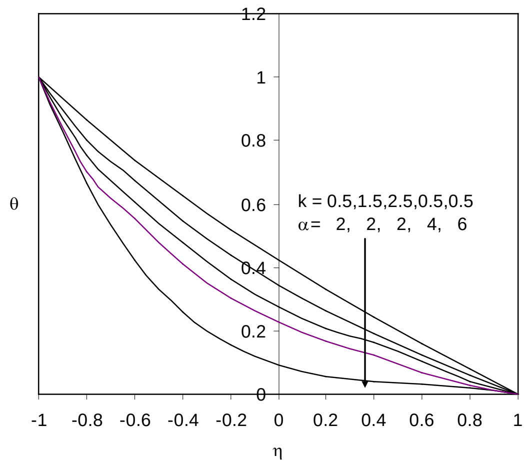


Fig. 16 : Variation of θ with k & α
Sc=1.3, β=-03, Q_1 = 1, N_1 = 1.5

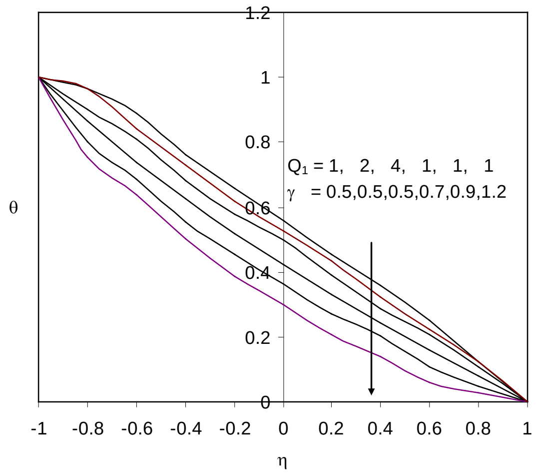


Fig. 17 : Variation of θ with Q_1 & γ
Sc=1.3, β=-03, k = 0.5, N_1 = 1.5

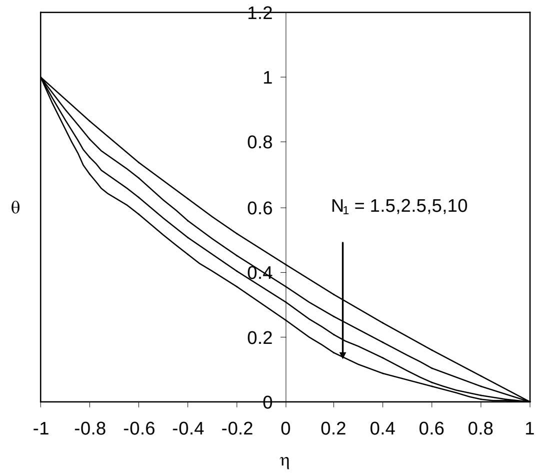


Fig. 18 : Variation of θ with N_1
Sc=1.3, β=-03, k = 0.5, Q_1 = 1

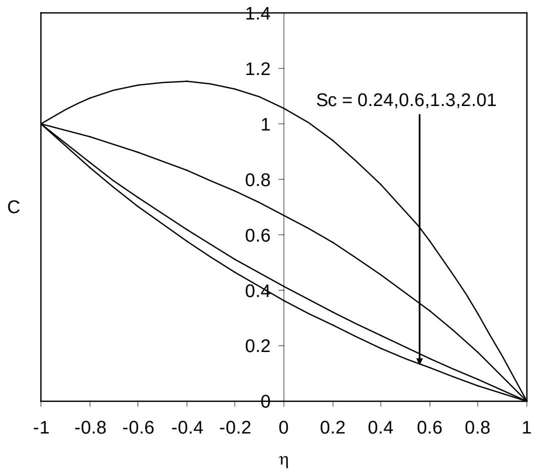


Fig. 19 : Variation of C with Sc
β=-03, k = 0.5, Q_1 = 1, N_1 = 1.5

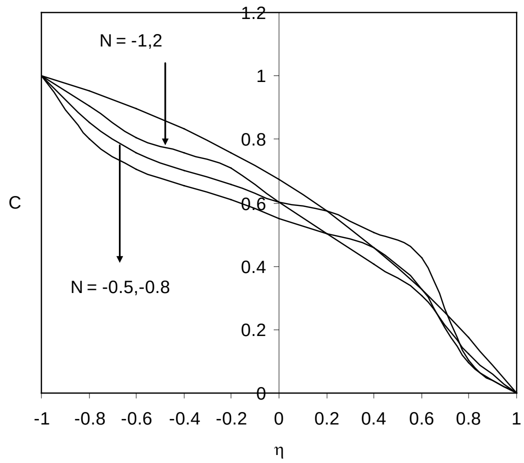


Fig. 20 : Variation of C with N
Sc=1.3, β=-03, k = 0.5, Q_1 = 1

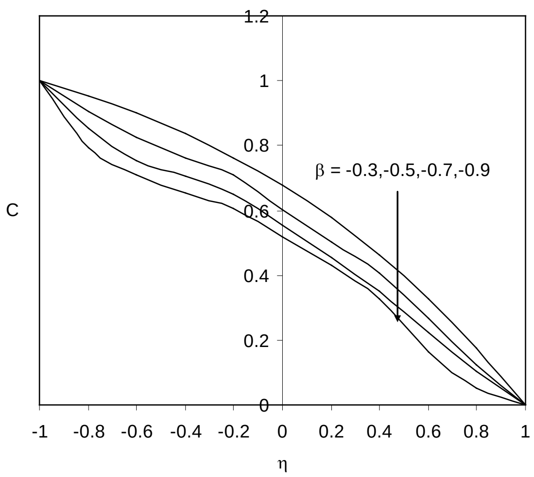


Fig. 21 : Variation of C with β
Sc=1.3, k = 0.5, Q_1 = 1, N_1 = 1.5

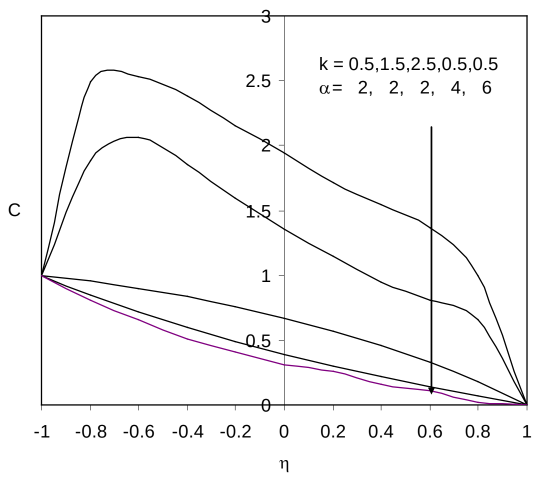


Fig. 22 : Variation of C with k & α
Sc=1.3, β=-03, Q_1 = 1, N_1 = 1.5

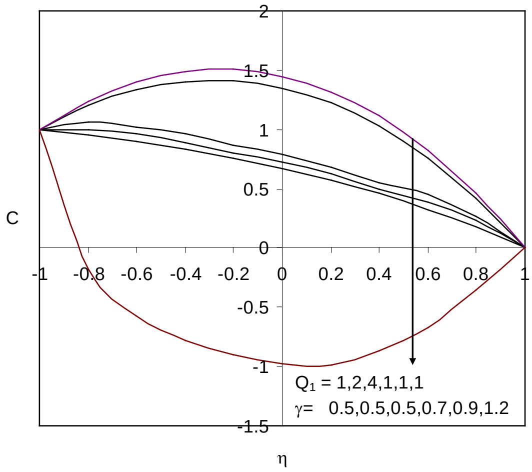


Fig. 23 : Variation of C with Q_1 & γ
Sc=1.3, β=-03, k = 0.5, Q_1 = 1, N_1 = 1.5

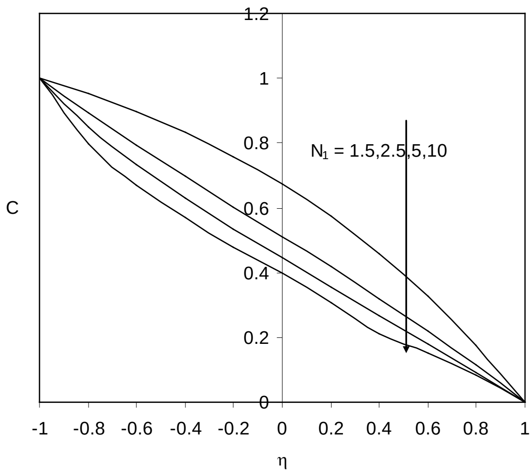


Fig. 24 : Variation of C with N_1
Sc=1.3, β=-03, k = 0.5, Q_1 = 1, N_1 = 1.5

The rate of heat transfer Nu at the walls θ=±1 is exhibited in tables 1&2 for different values of G, K, β, Q, N_1 .It is found that the rate of heat transfer depreciates η=1 and enhances η=-1 with increase G>0, while for G<0, $|Nu|$ enhances at η=1 and depreciates at η=-1. With increase in chemical reaction K, it can be shown that $|Nu|$ enhances η=1 with increases K. while η=-1, $|Nu|$ depreciates with $K \le 1.5$ and enhances with higher $K \ge 2.5$. The variation of Nu with β shows that higher the constriction of the channel walls $(|\beta| = 0.3)$ larger $|Nu|$ at η=1 and smaller η=-1 and for further higher $|\beta| = 0.5$ larger $|Nu|$ at growth the walls and for further higher $|\beta| = 0.9$, $|Nu|$ depreciates at η=±1 for G >0and for G<0, it depreciates at η=+1 and enhances η=-1. An increase the radiation absorption Q_1 results in an enhancement with Q_1 at η=1 and depreciates at η=-1. Also higher the radiative heat flux smaller $|Nu|$ at η=1 larger $|Nu|$ at η=-1 (tables 1 &2).

The rate of mass transfer Sh at η±1 is shown in tables 11-20 for different parametric values. It is found that the rate of mass transfer an enhances with increase G>0 at η=±1 and for G<0, $|Sh|$ enhances at η=1 and reduces at η=-1. The variation of Sh with Sc shows that lesser the molecular diffusivity larger $|Sh|$ at η=±1. When the molecular buoyancy force dominates over the thermal buoyancy force $|Sh|$ reduces at η=1 and enhances η=-1 Higher the constriction of the channel walls larger $|Sh|$ at η=1 and smaller η=-1. It can be seen that the rate of mass transfer enhances with increasing Q_1 An increasing the radiation parameter $N_1 \le 5$, reduces $|Sh|$ at η=±1, while for higher $N_1 \ge 10$, $|Sh|$ enhances at η=1 reduces at η=-1. (Table 3&4)

Table - 1
Nusselt Number (Nu) at η = -1

G	I	II	III	IV	V	VI
100	-3.0956	1.62613	-3.4075	-0.93212	-13.071	5.53849
300	-8.9231	2.25503	4.751	-1.60651	7.28034	1.72347
-100	-1.1486	0.58132	0.65312	-0.40413	-1.6761	-0.49421
-300	-0.7743	-0.4958	1.98386	0.02047	-0.2674	-1.94409
K	0.5	1.5	2.5	0.5	0.5	0.5
β	-0.5	-0.5	-0.5	-0.3	-0.7	-0.9

Table - 2

Nusselt Number (Nu) at η = -1

G	I	II	III	IV	V	VI	VII
100	-3.0955	-6.2153	1.11974	-2.58452	-2.8206	-2.9771	-3.1247
300	-8.9231	3.66535	0.24093	-4.26597	-4.3725	-4.5832	-4.9465
-100	-1.1486	-1.0836	-0.6738	-1.44311	-1.6853	-1.7921	-1.9403
-300	-0.1743	-0.3827	-0.3035	-0.61758	-0.8187	-0.8819	-0.9582
Q	1	2	4	1	1	1	1
N1	0.5	0.5	0.5	1.5	5	10	100

Table - 6

Nusselt Number (Nu) at η = +1

G	I	II	III	IV	V	VI
100	3.93981	5.79446	-216.15	2.11207	4.1624	3.06388
300	3.5501	8.41267	-104	2.38928	3.6002	1.98344
-100	5.31257	0.86171	-51.139	1.71256	-6.1852	-3.44084
-300	-6.5693	-11.886	-68.535	1.0869	-7.2755	0.17944
K	0.5	1.5	2.5	0.5	0.5	0.5
β	-0.5	-0.5	-0.5	-0.3	-0.7	-0.9

Table - 9

Nusselt Number (Nu) at η = +1

G	I	II	III	IV	V	VI	VII
100	3.93981	2.38819	1.31219	3.34012	3.23307	3.19816	3.09896
300	3.5501	1.79319	1.04232	4.37806	3.68275	3.46906	3.267
-100	5.31257	4.6577	0.33386	4.27721	3.11052	2.86421	2.65282
-300	-6.5693	6.86688	0.58859	4.15261	2.67777	2.44226	2.25151
Q	1	2	4	1	1	1	1
N1	0.5	0.5	0.5	1.5	5	10	100

Table - 11

Sherwood Number (Sh) at η = + 1

G	I	II	III	IV	V	VI
100	3.97909	-4.3119	-2.246	-0.55801	46.0525	324.526
300	12.4526	-11.751	-6.0896	-1.26623	142.977	1095.74
-100	-4.3581	4.58405	2.18497	0.14792	-48.09	-292.481
-300	-12.562	15.4119	7.34903	0.85157	-130.95	-797.326
K	0.5	1.5	2.5	0.5	0.5	0.5
β	-0.5	-0.5	-0.5	-0.3	-0.7	-0.9

Table - 14

Sherwood Number (Sh) at η = + 1

G	I	II	III	IV	V	VI	VII
100	3.97909	38.6477	109.048	-0.22322	-0.2223	-0.2224	-0.223
300	12.4526	118.014	336.174	-0.21642	-0.2124	-0.2126	-0.2129
-100	-4.3581	-38.55	-106.81	-0.2316	-0.2282	-0.2323	-0.2323
-300	-12.562	-113.67	-312.23	-0.23676	-0.2341	-0.2421	-0.2426
Q	1	2	4	1	1	1	1
N1	0.5	0.5	0.5	1.5	5	10	100

Table - 16

Sherwood Number (Sh) at η = - 1

G	I	II	III	IV	V	VI
100	-0.8468	-1.1126	-2.2942	-0.91765	-0.8236	-0.89825
300	-0.8776	-0.9171	-1.843	-0.921	-0.8405	-1.37275
-100	-0.817	-1.4201	-3.0025	-0.91432	-0.7213	-0.61198
-300	-0.7883	-1.9738	-4.275	-0.91102	-0.631	-0.42048
K	0.5	1.5	2.5	0.5	0.5	0.5
β	-0.5	-0.5	-0.5	-0.3	-0.7	-0.9

Table - 19

Sherwood Number (Sh) at η = - 1

G	I	II	III	IV	V	VI	VII
100	-0.8468	-0.8674	-0.9095	-0.80686	-0.8034	-0.8028	-0.7986
300	-0.8776	-0.8947	-0.931	-0.80509	-0.7987	-0.7977	-0.7969
-100	-0.817	-0.8418	-0.8904	-0.80861	-0.8081	-0.8079	-0.8078
-300	-0.7883	-0.8176	-0.8732	-0.81036	-0.8127	-0.813	-0.8126
Q	1	2	4	1	1	1	1
N1	0.5	0.5	0.5	1.5	5	10	100

7. CONCLUSIONS

- $|u|$ enhances with increasing $K \le 1.5$ and $|u|$ depreciates with $K \le 2.5$ (fig.4). An increasing the radiation absorption parameter Q_1 leads to depreciates in $|u|$ (fig.5).It is found that higher than the radiative heat flux larger $|u|$ in the flow region.
- The effect of wall waviness on v is shown in fig (9). It is observed that higher the constriction of which on a walls larger $|v|$ is entire floe region. The variation of v with chemical reaction K shows that, magnitude of experience with increasing chemical reaction K.
- The actual temperature depreciates with increasing chemical reactions K.
- Higher the constriction of the channel walls smaller the concentration.
- An increase the radiation absorption Q_1 results in an enhancement with Q_1 at η=1 and depreciates at η=-1. Also higher the radiative heat flux smaller $|Nu|$ at η=1 larger $|Nu|$ at η=-1
- An increasing the radiation parameter $N_1 \le 5$, reduces $|5h|$ at η=±1, while for higher $N_1 \ge 10$, $|5h|$ enhances at η=1 reduces at η=-1.

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