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NUMERICAL SOLUTION OF LAPLACE EQUATION USING FUZZY DATA BY NINE-POINTS FINITE DIFFERENCE

***Mhassin A. A.**
Faculty of Education for Pure Science, Al-anbar University, Iraq.

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ABSTRACT

The fuzzification of Laplace equation in two dimensions are discussed. The interval of fuzzy interval can be determined. Finite difference method applied of two different grids using five points and nine – points to solve Laplace equation numerically.

Keywords: *Fuzzy membership function (f.m.f.), interval of confidence, triangular fuzzy number (t.f.n.), α – cuts, five and nine points finite difference.*

1. INTRODUCTION

The concept of Fuzzy differential equation was first introduced by Chang Zadeh [10]. Dubois and Prade [5] has given extension principle. Raphel [8], used five points only. Here implementing nine-points for finite difference method to solve Laplace equation in two variables numerically, then fuzzified.

2- DEFINITIONS

A triangular Fuzzy number μ is defined by three real numbers with base as the interval $[a,c]$ and b as the vertex of triangle. The membership function are defined as follows [8]:

$$\mu(x)(=)\begin{cases} \dfrac{x-a}{b-a} & ; \ where \ \ a \le x \le b \\ \dfrac{x-c}{b-c} & ; \ where \ \ b \le x \le c \\ 0 & ; \ \ otherwise \end{cases}.$$

The α – cuts are defined by $\Delta_L(\alpha)(=)a+\alpha(b-a)$ and $\Delta_R(\alpha)(=)c+\alpha(b-c)$

2.1 Finite difference using to solve Laplace equation

The Laplace equation in two variables is defined by

$$u_{xx}(x,y)+u_{yy}(x,y)=0 \tag{1}$$

This equation is encountered in many application, fluid mechanics, study state , electrostatics, mass transfer, and for other areas of mechanics and physics. Replacing u_{xx} and u_{yy} by the central difference formula the value of $u(x_i,x_j)$ at any mesh point is the arithmetic mean of the values at four neighboring mesh to the left, right, above and below which is called standard five points formula (SFPF) Fig.2, and diagonal five points formula (DFPF) Fig.3, and standard nine-points formula (SNPF) Fig.4, use for finding the initial data respectively as follows

$$u_{i,j}=\frac{1}{4}\left[u_{i-1,j}+u_{i+1,j}+u_{i,j-1}+u_{i,j+1}\right] \tag{2}$$

$$u_{i,j}=\frac{1}{4}\left[u_{i+1,j+1}+u_{i+1,j-1}+u_{i-1,j+1}+u_{i-1,j-1}\right] \tag{3}$$

*Corresponding author: *Mhassin A. A.*
Faculty of Education for Pure Science, Al-anbar University, Iraq.

and $u_{i,j} = \frac{1}{20}\left[4\{u_{i-1,j} + u_{i+1,j} + u_{i,j-1} + u_{i,j+1}\} + u_{i+1,j+1} + u_{i+1,j-1} + u_{i-1,j+1} + u_{i-1,j-1}\right]$ (4)

3. APPLICATION OF FUZZY INTERVAL IN LAPLACE EQUATION

From c_1 to c_{16} represents the boundary conditions of the mesh square with fuzzy interval as in table 1.

$c_1(=)\lfloor l_{1,1}; l_{1,2}; l_{1,3}\rfloor$	$c_2(=)\lfloor l_{2,1}; l_{2,2}; l_{2,3}\rfloor$	$c_3(=)\lfloor l_{3,1}; l_{3,2}; l_{3,3}\rfloor$	$c_4(=)\lfloor l_{4,1}; l_{4,2}; l_{4,3}\rfloor$
$c_5(=)\lfloor l_{5,1}; l_{5,2}; l_{5,3}\rfloor$	$c_6(=)\lfloor l_{6,1}; l_{6,2}; l_{6,3}\rfloor$	$c_7(=)\lfloor l_{7,1}; l_{7,2}; l_{7,3}\rfloor$	$c_8(=)\lfloor l_{8,1}; l_{8,2}; l_{8,3}\rfloor$
$c_9(=)\lfloor l_{9,1}; l_{9,2}; l_{9,3}\rfloor$	$c_{10}(=)\lfloor l_{10,1}; l_{10,2}; l_{10,3}\rfloor$	$c_{11}(=)\lfloor l_{11,1}; l_{11,2}; l_{11,3}\rfloor$	$c_{12}(=)\lfloor l_{12,1}; l_{12,2}; l_{12,3}\rfloor$
$c_{13}(=)\lfloor l_{13,1}; l_{13,2}; l_{13,3}\rfloor$	$c_{14}(=)\lfloor l_{14,1}; l_{14,2}; l_{14,3}\rfloor$	$c_{15}(=)\lfloor l_{15,1}; l_{15,2}; l_{15,3}\rfloor$	$c_{16}(=)\lfloor l_{16,1}; l_{16,2}; l_{16,3}\rfloor$

Table -1

Fig.1

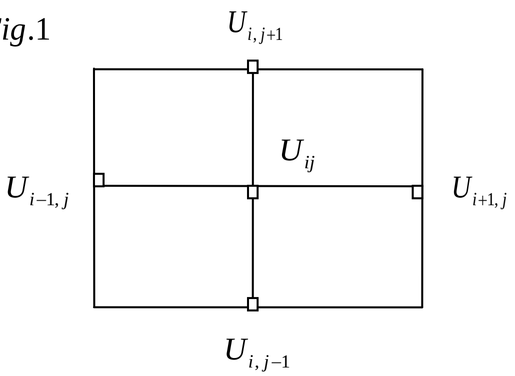


Fig.2

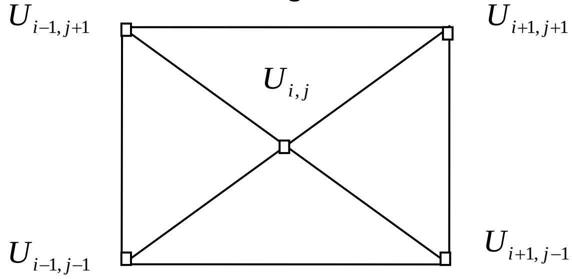


Fig.3

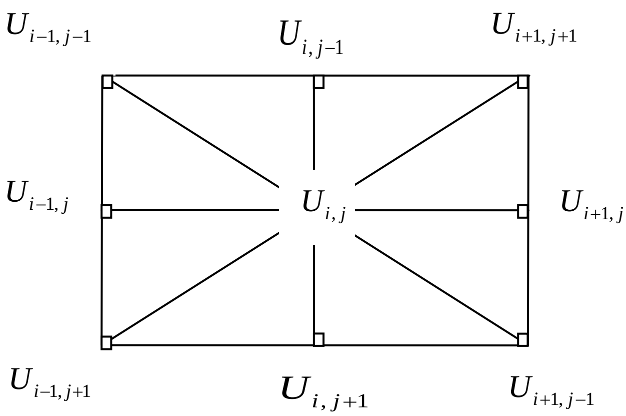
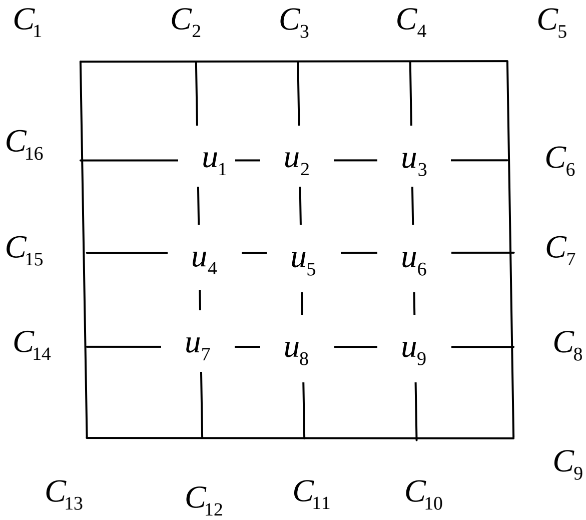


Fig.4



The interior points due to the square grid are u_1 to u_9.

Now to find the initial value of $u_5^{(0)}$ using standard nine-points formula (4) as

$u_5^{(0)}(=)\frac{1}{20}\left[4\{c_3(+)c_7(+)c_{11}(+)c_{15}\}(+)c_1(+)c_5(+)c_9(+)c_{13}\right]$ (5)

We can write equation (9) in some details as

$$\left[u_5^{(0)}\right]^{(\alpha)}(=)\begin{bmatrix}\dfrac{4\{l_{3,1}+l_{7,1}+l_{11,1}+l_{15,1}\}+l_{1,1}+l_{5,1}+l_{9,1}+l_{13,1}}{20}, \\ \dfrac{4\{l_{3,2}+l_{7,2}+l_{11,2}+l_{15,2}\}+l_{1,2}+l_{5,2}+l_{9,2}+l_{13,2}}{20}, \\ \dfrac{4\{l_{3,3}+l_{7,3}+l_{11,3}+l_{15,3}\}+l_{1,3}+l_{5,3}+l_{9,3}+l_{13,3}}{20}\end{bmatrix} \tag{6}$$

Fuzzy membership functions (f.m.f) are respective α -cuts of $c_1, c_3, c_5, c_7, c_9, c_{11}, c_{13}$ *and* c_{15} are respectively as

$$\mu_{C_i}(x)(=)\begin{Bmatrix}\dfrac{x-l_{i,1}}{l_{i,2}-l_{i,1}} & ; \ where \ \ l_{i,1}\le x\le l_{i,2} \\ \dfrac{x-l_{i,3}}{l_{i,2}-l_{i,3}} & ; \ where \ \ l_{i,2}\le x\le l_{i,3} \\ 0 \quad ; & otherwise\end{Bmatrix}$$

Hence the $\alpha-$cuts of c_i is given by

$[c_i]^{(\alpha)}(=)\left[l_{i,1}+\alpha(l_{i,2}-l_{i,1}), l_{i,3}+\alpha(l_{i,2}-l_{i,3})\right]$ Where $i=1,2,\cdots,9$. Then from equation (6) we have

$$\mu_{u_5^{(0)}}(x)(=)\frac{1}{20}\begin{bmatrix}4\{(l_{3,2}+l_{7,2}+l_{11,2}+l_{15,2})-(l_{3,1}+l_{7,1}+l_{11,1}+l_{15,1})\}\alpha+ \\ \{(l_{13,2}+l_{1,2}+l_{5,2}+l_{9,2})-(l_{13,1}+l_{1,1}+l_{5,1}+l_{9,1})\}\alpha+ \\ 4(l_{3,1}+l_{7,1}+l_{11,1}+l_{15,1})+(l_{1,1}+l_{5,1}+l_{9,1}+l_{13,1}) \quad , \\ \\ 4\{(l_{3,2}+l_{7,2}+l_{11,2}+l_{15,2})-(l_{3,3}+l_{7,3}+l_{11,3}+l_{15,3})\}\alpha+ \\ \{(l_{1,2}+l_{5,2}+l_{9,2}+l_{13,2})-(l_{1,3}+l_{5,3}+l_{9,3}+l_{13,3})\}\alpha+ \\ 4(l_{3,3}+l_{7,3}+l_{11,3}+l_{15,3})+(l_{1,3}+l_{5,3}+l_{9,3}+l_{13,3})\end{bmatrix}$$

or

$$\mu_{u_5^{(0)}}(x)(=)\begin{bmatrix}\dfrac{\{4H_2-4H_1+X_2-X_1\}\alpha+4H_1+X_1}{20}, \\ \\ \dfrac{\{4H_2-4H_3+X_2-X_3\}\alpha+4H_3+X_3}{20}\end{bmatrix}$$

Where

$H_2=l_{3,2}+l_{7,2}+l_{11,2}+l_{15,2}$, $H_1=l_{3,1}+l_{7,1}+l_{11,1}+l_{15,1}$, $H_3=l_{3,3}+l_{7,3}+l_{11,3}+l_{15,3}$
$X_1=l_{1,1}+l_{5,1}+l_{9,1}+l_{13,1}$, $X_2=l_{1,2}+l_{5,2}+l_{9,2}+l_{13,2}$ and $X_3=l_{1,3}+l_{5,3}+l_{9,3}+l_{13,3}$.

Let

$x_1=\dfrac{[4(H_2-H_1)+X_2-X_1]\alpha+4H_1+X_1}{20}$ and $x_2=\dfrac{[4(H_2-H_3)+X_2-X_3]\alpha+4H_3+X_3}{20}$.

Hence f.m.f. for $u_5^{(0)}$ is

$$\mu_{u_5^{(0)}}(x)(=)\begin{Bmatrix}\dfrac{20x-4H_1-X_1}{4H_2-4H_1+X_2-X_1}; & \dfrac{1}{20}[4H_1+X_1]\le x\le\dfrac{1}{20}[4H_2+X_2] \\ \dfrac{20x-4H_3-X_3}{4H_2-4H_3+X_2-X_3}; & \dfrac{1}{20}[4H_2+X_2]\le x\le\dfrac{1}{20}[4H_3+X_3] \\ 0 & ;otherwise\end{Bmatrix} \tag{7}$$

Where $\alpha\in[0,1]$.

But to find the initial values of u_1, u_3, u_9 and u_7 using five points diagonally (DFPF) by the equation (3), and to find The initial values of u_2, u_6, u_8 and u_4 by (SFPF) by the equation (2).

For example to find initial value of $u_1^{(0)}$ we doing the following steps

$$u_1^{(0)}(=)\frac{1}{4}\left[c_1(+)c_3(+)u_5^{(0)}(+)c_{15}\right] \text{ or } u_1^{(0)}(=)\left[\frac{l_{1,1}+l_{3,1}+u_{5,1}^{(0)}+l_{15,1}}{4},\frac{l_{1,2}+l_{3,2}+u_{5,2}^{(0)}+l_{15,2}}{4},\frac{l_{1,3}+l_{3,3}+u_{5,3}^{(0)}+l_{15,3}}{4}\right] \tag{8}$$

The interval of confidence and the correspondence Hence $\alpha-cuts$ as follows

$$\mu_{C_1}(x)(=)\left\{\begin{array}{l} \dfrac{x-l_{1,1}}{l_{1,2}-l_{1,1}} \;;\; where \quad l_{1,1}\le x\le l_{1,2} \\ \dfrac{x-l_{1,3}}{l_{1,2}-l_{1,3}} \;;\; where \quad l_{1,2}\le x\le l_{1,3} \\ 0 \quad ; \quad otherwise \end{array}\right\},$$

Hence $\alpha-cuts$ of c_1

$[c_1]^{(\alpha)}(=)\left[l_{1,1}+\alpha(l_{1,2}-l_{1,1}), l_{1,3}+\alpha(l_{1,2}-l_{1,3})\right]$, as well, $\alpha-cuts$ of c_3, c_{15}, and $u_5^{(0)}$ are $[c_3]^{(\alpha)}(=)\left[l_{3,1}+\alpha(l_{3,2}-l_{3,1}), l_{3,3}+\alpha(l_{3,2}-l_{3,3})\right]$, $[c_{15}]^{(\alpha)}(=)\left[l_{15,1}+\alpha(l_{15,2}-l_{15,1}), l_{15,3}+\alpha(l_{15,2}-l_{15,3})\right]$ and $\left[u_5^{(0)}\right]^{(\alpha)}(=)\left[u_{5,1}^{(0)}+\alpha(u_{5,2}^{(0)}-u_{5,1}^{(0)}), u_{5,3}^{(0)}+\alpha(u_{5,2}^{(0)}-u_{5,3}^{(0)})\right]$

Hence the interval of confidence of $u_1^{(0)}$ is

$$u_1^{(0)}(=)\left[\begin{array}{l} \dfrac{(l_{1,2}-l_{1,1})+(l_{3,2}-l_{3,1})+(l_{15,2}-l_{15,1})+(u_{5,2}^{(0)}-u_{5,1}^{(0)})}{4}\alpha+\dfrac{(l_{1,1}+l_{3,1}+u_{5,1}^{(0)}+l_{15,1})}{4}, \\ \dfrac{(l_{1,2}-l_{1,3})+(l_{3,2}-l_{3,3})+(l_{15,2}-l_{15,3})+(u_{5,2}^{(0)}-u_{5,3}^{(0)})}{4}\alpha+\dfrac{(l_{1,3}+l_{3,3}+u_{5,3}^{(0)}+l_{15,3})}{4} \end{array}\right]$$

Le t $x_1=\dfrac{(l_{1,2}-l_{1,1})+(l_{3,2}-l_{3,1})+(l_{15,2}-l_{15,1})+(u_{5,2}^{(0)}-u_{5,1}^{(0)})}{4}\alpha+\dfrac{(l_{1,1}+l_{3,1}+u_{5,1}^{(0)}+l_{15,1})}{4}$ and $x_2=\dfrac{(l_{1,2}-l_{1,3})+(l_{3,2}-l_{3,3})+(l_{15,2}-l_{15,3})+(u_{5,2}^{(0)}-u_{5,3}^{(0)})}{4}\alpha+\dfrac{(l_{1,3}+l_{3,3}+u_{5,3}^{(0)}+l_{15,3})}{4}$, solve for α

$$\alpha=\frac{4x_1-(l_{1,1}+l_{3,1}+u_{5,1}^{(0)}+l_{15,1})}{(l_{1,2}-l_{1,1})+(l_{3,2}-l_{3,1})+(l_{15,2}-l_{15,1})+(u_{5,2}^{(0)}-u_{5,1}^{(0)})} \quad \text{and} \quad \alpha=\frac{4x_2-(l_{1,3}+l_{3,3}+u_{5,3}^{(0)}+l_{15,3})}{(l_{1,2}-l_{1,3})+(l_{3,2}-l_{3,3})+(l_{15,2}-l_{15,3})+(u_{5,2}^{(0)}-u_{5,3}^{(0)})}.$$

Hence f.m.f. for $u_1^{(0)}$ is

$$\mu_{u_1^{(0)}}(x)(=)\left\{\begin{array}{l} \dfrac{4x-(l_{1,1}+l_{3,1}+u_{5,1}^{(0)}+l_{15,1})}{(l_{1,2}+l_{3,2}+l_{15,2}+u_{5,2}^{(0)})-(l_{1,1}+l_{3,1}+l_{15,1}+u_{5,1}^{(0)})} \\ where \quad \dfrac{1}{4}(l_{1,1}+l_{3,1}+l_{15,1}+u_{5,1}^{(0)})\le x\le \dfrac{1}{4}(l_{1,2}+l_{3,2}+l_{15,2}+u_{5,2}^{(0)}) \\ \dfrac{4x-(l_{1,3}+l_{3,3}+u_{5,3}^{(0)}+l_{15,3})}{(l_{1,3}+l_{3,3}+l_{15,3}+u_{5,3}^{(0)})-(l_{1,2}+l_{3,2}+l_{15,2}+u_{5,2}^{(0)})} \\ where \quad \dfrac{1}{4}(l_{1,2}+l_{3,2}+l_{15,2}+u_{5,2}^{(0)})\le x\le \dfrac{1}{4}(l_{1,3}+l_{3,3}+l_{15,3}+u_{5,3}^{(0)}) \\ 0 \quad ; \quad otherwise \end{array}\right\}$$

This process also for u_3, u_9 and u_7. In a similar way we evaluate u_2, u_6, u_8 and u_4, as for $u_2^{(0)}$ we find the $\alpha-cuts$ of c_3, $u_3^{(0)}$, $u_5^{(0)}$ and $u_1^{(0)}$ we get

$$u_2^{(0)}(=)\left[\begin{array}{l} \dfrac{(l_{3,2}-l_{3,1})+(u_{3,2}^{(0)}-u_{3,1}^{(0)})+(u_{5,2}^{(0)}-u_{5,1}^{(0)})+(u_{1,2}^{(0)}-u_{1,1}^{(0)})}{4}\alpha+\dfrac{(l_{3,1}+u_{3,1}^{(0)}+u_{5,1}^{(0)}+u_{1,1}^{(0)})}{4}, \\ \dfrac{(l_{3,2}-l_{3,3})+(u_{3,2}^{(0)}-u_{3,3}^{(0)})+(u_{5,2}^{(0)}-u_{5,3}^{(0)})+(u_{1,2}^{(0)}-u_{1,3}^{(0)})}{4}\alpha+\dfrac{(l_{3,3}+u_{3,3}^{(0)}+u_{5,3}^{(0)}+u_{1,3}^{(0)})}{4} \end{array}\right]$$

$$\mu_{u_2^{(0)}}(x)(=)\begin{cases} \dfrac{4x-(l_{3,1}+u_{3,1}^{(0)}+u_{5,1}^{(0)}+u_{1,1}^{(0)})}{(l_{3,2}+u_{3,2}^{(0)}+u_{1,2}^{(0)}+u_{5,2}^{(0)})-(l_{3,1}+u_{3,1}^{(0)}+u_{1,2}^{(0)}+u_{5,1}^{(0)})} \\ where\ \ \frac{1}{4}(l_{3,1}+u_{3,1}^{(0)}+u_{5,1}^{(0)}+u_{1,1}^{(0)})\le x\le\frac{1}{4}(l_{3,2}+u_{3,2}^{(0)}+u_{5,2}^{(0)}+u_{1,2}^{(0)}) \\ \dfrac{4x-(l_{3,3}+u_{3,3}^{(0)}+u_{5,3}^{(0)}+u_{1,3}^{(0)})}{(l_{1,3}+l_{3,3}+l_{15,3}+u_{5,3}^{(0)})-(l_{1,2}+l_{3,2}+l_{15,2}+u_{5,2}^{(0)})} \\ where\ \ \frac{1}{4}(l_{3,2}+u_{3,2}^{(0)}+u_{5,2}^{(0)}+u_{1,2}^{(0)})\le x\le\frac{1}{4}(l_{3,3}+u_{3,3}^{(0)}+u_{5,3}^{(0)}+u_{1,3}^{(0)}) \\ 0 \quad ; \quad otherwise \end{cases}$$

Next successive approximations with their f.m.f. as required be obtain from previous approximations and specified boundary conditions.

4. NUMERICAL EXAMPLE

Let us consider the Laplace equation [8],

$$u_{xx}(+)u_{yy}(=)0 \tag{9}$$

In the domain $0\le x\le 4, 0\le y\le 4$ with boundary conditions $u(0,y)(=)0$, $u(x,0)(=)\frac{x^2}{2}$, $u(x,4)(=)x^2$, and $u(4,y)(=)8(+)2y$. Leibmann's process will be applied to solve equation (9).

Solution: The boundary conditions given the numerical value of $c_1=0$, $c_2=1$, $c_3=4$, $c_5=16$, $c_6=14$, $c_7=12$, $c_8=10$, $c_9=8$, $c_{10}=4.5$, $c_{11}=2$, $c_{12}=0.5$, $c_{13}=0$, $c_{14}=0$, $c_{15}=0$, and $c_{16}=0$.

The initial values of $u_i=1,2,3,\ldots,9$, may be calculated the initial values with the help of standard nine-points, standard five points and diagonal five points formulas, then use nine-points to get the approximate solution.

$$u_5^{(0)}(=)[7.799,4.8,4.801] \tag{10}$$

To find f.m.f. and respective interval of confidence these eight c_i's as follows

$$\mu_{C_1}(x)(=)\begin{cases} \dfrac{x+0.001}{0+0.001} \ ; \ where \ \ -0.001\le x\le 0 \\ \dfrac{x-0.001}{0-0.001} \ ; \ where \ \ 0\le x\le 0.001 \\ 0 \quad ; \quad otherwise \end{cases}, [c_1]^{(\alpha)}(=)[0.001\alpha-0.001, -0.001\alpha+0.001]$$

$$\mu_{C_3}(x)(=)\begin{cases} \dfrac{x-3.999}{4-3.999} \ ; \ where \ \ 3.999\le x\le 4 \\ \dfrac{x-4.001}{4-4.001} \ ; \ where \ \ 4\le x\le 4.001 \\ 0 \quad ; \quad otherwise \end{cases}, [c_3]^{(\alpha)}(=)[0.001\alpha+3.999, -0.001\alpha+4.001]$$

$$\mu_{C_5}(x)(=)\begin{cases} \dfrac{x-15.999}{16-15.999} \ ; \ where \ \ 15.999\le x\le 16 \\ \dfrac{x-16.001}{16-16.001} \ ; \ where \ \ 16\le x\le 16.001 \\ 0 \quad ; \quad otherwise \end{cases}, [c_5]^{(\alpha)}(=)[0.001\alpha+15.999, -0.001\alpha+16.001]$$

$$\mu_{C_7}(x)(=)\begin{cases} \dfrac{x-11.999}{12-11.999} \ ; \ where \ \ 11.999\le x\le 12 \\ \dfrac{x-12.001}{12-12.001} \ ; \ where \ \ 12\le x\le 12.001 \\ 0 \quad ; \quad otherwise \end{cases}, [c_7]^{(\alpha)}(=)[0.001\alpha+11.999, -0.001\alpha+12.001]$$

$$\mu_{C_9}(x)(=)\begin{Bmatrix} \dfrac{x-7.999}{12-11.999} \ ; \ where \ \ 7.999 \le x \le 8 \\ \dfrac{x-8.001}{8-8.001} \ ; \ where \ \ 8 \le x \le 8.001 \\ 0 \quad ; \quad otherwise \end{Bmatrix}, \left[c_9\right]^{(\alpha)}(=)\left[0.001\alpha+7.799,-0.001\alpha+8.001\right]$$

$$\mu_{C_{11}}(x)(=)\begin{Bmatrix} \dfrac{x-1.999}{2-1.999} \ ; \ where \ \ 1.999 \le x \le 2 \\ \dfrac{x-2.001}{2-2.001} \ ; \ where \ \ 2 \le x \le 2.001 \\ 0 \quad ; \quad otherwise \end{Bmatrix}, \left[c_{11}\right]^{(\alpha)}(=)\left[0.001\alpha+1.999,-0.001\alpha+2.001\right]$$

$$\mu_{C_{13}}(x)(=)\begin{Bmatrix} \dfrac{x+0.001}{0+0.001} \ ; \ where \ \ -0.001 \le x \le 0 \\ \dfrac{x-0.001}{0-0.001} \ ; \ where \ \ 0 \le x \le 0.001 \\ 0 \quad ; \quad otherwise \end{Bmatrix}, \left[c_{13}\right]^{(\alpha)}(=)\left[0.001\alpha-0.001,-0.001\alpha+0.001\right]$$

$$\mu_{C_{15}}(x)(=)\begin{Bmatrix} \dfrac{x+0.001}{0+0.001} \ ; \ where \ \ -0.001 \le x \le 0 \\ \dfrac{x-0.001}{0-0.001} \ ; \ where \ \ 0 \le x \le 0.001 \\ 0 \quad ; \quad otherwise \end{Bmatrix} \text{and} \left[c_{15}\right]^{(\alpha)}(=)\left[0.001\alpha-0.001,-0.001\alpha+0.001\right].$$

Hence the interval of confidence for $u_5^{(0)}$, $\left[u_5^{(0)}\right]^{(\alpha)}(=)\left[0.001\alpha+4.799,-0.001\alpha+4.801\right]$, we are to retain two roots with $\alpha \in [0,1]$. Let $0.001\alpha + 4.799 = x_1$ and $-0.001\alpha + 4.799 = x_2$, then solving for α we get $\alpha = \dfrac{x_1 - 4.799}{0.001}$ and $\alpha = \dfrac{x_2 - 4.801}{-0.001}$, hence f.m.f. for $u_5^{(0)}$ is

$$\mu_{u_5^{(0)}}(x)(=)\begin{Bmatrix} \dfrac{x-4.799}{4.8-4.799} \ ; \ where \ \ 4.799 \le x \le 4.8 \\ \dfrac{x-4.801}{4.8-4.801} \ ; \ where \ \ 4.8 \le x \le 4.801 \\ 0 \quad ; \quad otherwise \end{Bmatrix}$$

In the same way we find f.m.f. for $u_1^{(0)}, u_3^{(0)}, u_9^{(0)}, u_7^{(0)}, u_2^{(0)}, u_6^{(0)}, u_8^{(0)}$ and $u_4^{(0)}$ are respectively.

$$\mu_{u_1^{(0)}}(x)(=)\begin{Bmatrix} \dfrac{x-2.199}{2.2-2.199} \ ; \ where \ \ 2.199 \le x \le 2.2 \\ \dfrac{x-2.201}{2.2-2.201} \ ; \ where \ \ 2.2 \le x \le 2.201 \\ 0 \quad ; \quad otherwise \end{Bmatrix}, \quad \mu_{u_3^{(0)}}(x)(=)\begin{Bmatrix} \dfrac{x-9.199}{9.200-9.199} \ ; \ where \ \ 9.199 \le x \le 9.200 \\ \dfrac{x-9.201}{9.200-9.201} \ ; \ where \ \ 9.200 \le x \le 9.201 \\ 0 \quad ; \quad otherwise \end{Bmatrix}$$

$$\mu_{u_9^{(0)}}(x)(=)\begin{Bmatrix} \dfrac{x-6.699}{6.700-6.699} \ ; \ where \ \ 6.699 \le x \le 6.700 \\ \dfrac{x-6.701}{6.700-6.701} \ ; \ where \ \ 6.700 \le x \le 6.701 \\ 0 \quad ; \quad otherwise \end{Bmatrix}, \quad \mu_{u_7^{(0)}}(x)(=)\begin{Bmatrix} \dfrac{x-1.699}{1.700-1.699} \ ; \ where \ \ 1.699 \le x \le 1.700 \\ \dfrac{x-1.701}{1.700-1.701} \ ; \ where \ \ 1.700 \le x \le 1.701 \\ 0 \quad ; \quad otherwise \end{Bmatrix}$$

$$\mu_{u_2^{(0)}}(x)(=)\begin{cases} \dfrac{x-5.049}{5.050-5.049} \;;\; where \;\; 5.049 \le x \le 5.050 \\ \dfrac{x-5.051}{5.050-5.051} \;;\; where \;\; 5.050 \le x \le 5.051 \\ 0 \;;\; otherwise \end{cases}, \quad \mu_{u_6^{(0)}}(x)(=)\begin{cases} \dfrac{x-8.174}{8.175-8.174} \;;\; where \;\; 8.174 \le x \le 8.175 \\ \dfrac{x-8.175775}{8.175-8.176} \;;\; where \;\; 8.175 \le x \le 8.176 \\ 0 \;;\; otherwise \end{cases}$$

$$\mu_{u_8^{(0)}}(x)(=)\begin{cases} \dfrac{x-3.799}{3.800-3.799} \;;\; where \;\; 3.799 \le x \le 3.800 \\ \dfrac{x-3.801}{3.800-3.801} \;;\; where \;\; 3.800 \le x \le 3.801 \\ 0 \;;\; otherwise \end{cases} \text{ and } \mu_{u_4^{(0)}}(x)(=)\begin{cases} \dfrac{x-2.174}{2.175-2.174} \;;\; where \;\; 2.174 \le x \le 2.175 \\ \dfrac{x-2.176}{2.175-2.176} \;;\; where \;\; 2.175 \le x \le 2.176 \\ 0 \;;\; otherwise \end{cases}$$

In the following there are f.m.f of the forth approximations using nine-points by the method of Lebmann's iteration process applied to equation (4) have been found as

$$\mu_{u_1^{(4)}}(x)(=)\begin{cases} \dfrac{x-2.06673458557634}{2.06683458557382-2.06673458557634} \;;where \;\; 2.06673458557634 \le x \le 2.06683458557382 \\ \dfrac{x-2.06693458557129}{2.06683458557382-2.06693458557129} \;;where \;\; 2.06683458557382 \le x \le 2.06693458557129 \\ 0 \;;\; otherwise \end{cases}$$

$$\mu_{u_2^{(4)}}(x)(=)\begin{cases} \dfrac{x-4.99644306641130}{4.99654306640878-4.99644306641130} \;;\; where \;\; 4.99644306641130 \le x \le 4.99654306640878 \\ \dfrac{x-4.99664306640625}{4.99654306640878-4.99664306640625} \;;\; where \;\; 4.99654306640878 \le x \le 4.99664306640625 \\ 0 \;;\; otherwise \end{cases}$$

$$\mu_{u_3^{(4)}}(x)(=)\begin{cases} \dfrac{x-9.06482723694353}{9.06492723694100-9.06482723694353} \;;\; where \;\; 9.06482723694353 \le x \le 9.06492723694100 \\ \dfrac{x-9.06502723693848}{9.06492723694100-9.06502723693848} \;;\; where \;\; 9.06492723694100 \le x \le 9.06502723693848 \\ 0 \;;\; otherwise \end{cases}$$

$$\mu_{u_4^{(4)}}(x)(=)\begin{cases} \dfrac{x-2.13183978539018}{2.13193978538766-2.13183978539018} \;;where \;\; 2.13183978539018 \le x \le 2.13193978538766 \\ \dfrac{x-2.13203978538513}{2.13193978538766-2.13203978538513} \;;where \;\; 2.13193978538766 \le x \le 2.13203978538513 \\ 0 \;;\; otherwise \end{cases}$$

$$\mu_{u_5^{(4)}}(x)(=)\begin{cases} \dfrac{x-4.77449682693987}{4.77459682693734-4.77449682693987} \;;\; where 4.77449682693987 \le x \le 4.77459682693734 \\ \dfrac{x-4.77469682693481}{4.77459682693734-4.77469682693481} \;;\; where \;\; 4.77459682693734 \le x \le 4.77469682693481 \\ 0 \;;\; otherwise \end{cases}$$

$$\mu_{u_6^{(4)}}(x)(=)\begin{cases} \dfrac{x-8.12990883331804}{8.1300088331551-8.12990883331804} \;;where \;\; 8.12990883331804 \le x \le 8.1300088331551 \\ \dfrac{x-8.1301088331299}{8.1300088331551-8.1301088331299} \;;where \;\; 8.1300088331551 \le x \le 8.1301088331299 \\ 0 \;;\; otherwise \end{cases}$$

$$\mu_{u_7^{(4)}}(x)(=)\begin{cases} \dfrac{x-1.61869028549700}{1.61879028549447-1.61869028549700} & ;where\ \ 1.61869028549700 \le x \le 1.61879028549447 \\ \dfrac{x-1.61889028549194}{1.61879028549447-1.61889028549194} & ;where\ \ 1.61879028549447 \le x \le 1.61889028549194 \\ 0 \quad ; \quad otherwise \end{cases}$$

$$\mu_{u_8^{(4)}}(x)(=)\begin{cases} \dfrac{x-3.76553324203996}{3.76563324203744-3.76553324203996} & ; \ where 3.76553324203996 \le x \le 3.76563324203744 \\ \dfrac{x-3.76573324203491}{3.76563324203744-3.76573324203491} & ; \ where\ \ 3.76563324203744 \le x \le 3.76573324203491 \\ 0 \quad ; \quad otherwise \end{cases}$$

$$\mu_{u_9^{(4)}}(x)(=)\begin{cases} \dfrac{x-6.61775806885271}{6.61785806885018-6.61775806885271} & ; \ where\ \ 6.61775806885271 \le x \le 6.61785806885018 \\ \dfrac{x-6.6180}{6.61785806885018-6.61795806884766} & ; \ where\ \ 6.61785806885018 \le x \le 6.61795806884766 \\ 0 \quad ; \quad otherwise \end{cases}$$

5. CONCLUSION

For the given initial values the fourth approximations to solve the above example numerically is very significant results in comparison with example solved in [8] using five points only. However may increased the accuracy as desired if we take more iterations. As well, using nine-points is more accurate than five points.

6. REFERENCES

[1] Baruah, Hemanta K. Set Superimposition and its applications to the theory of Fuzzy sets. Journal of Assam Science Society, Vol. 40. Nos. 1 & 2, 25-31 (1999).

[2] Baruah, Hemanta K. Construction of the membership function of Fuzzy number, ICIC express letters (2010a).

[3] Baruah, Hemanta K. The mathematics of Fuzziness Myths and realities, Lambert Academic Publishing, Saarbruken, Germany (2010b).

[4] Chang, S. L., Zadeh, L. A. On Fuzzy mapping and control, IEEE Trans. Systems man cyber net, 2 (1972) pp.30-34.

[5] Dubois, D., Prade, H. Towards Fuzzy differential calculus, Fuzzy Sets and system, Part 3, 8 (1982) pp. 225-233.

[6] Grewal, B. S. Numerical methods in engineering and science with program in C & C++, Khanna Publishers, New Delhi-110002,(2010) pp. 343-348.

[7] Kaufmann, A. and Gupta, M.M. Introduction to Fuzzy arithmetic. Theory and applications, Van Nostrand Reinhold Co. Inc., Wokingham, Berkshire (1984).

[8] Raphel Kumar Saikia, Numerical Solution of Laplace Equation using Fuzzy Data, International Journal of computer applications (0975-8887) Vol. 38, No. 8, January, (2012).

[9] Sastry, S.S. Introductory methods of numerical analysis, PHI Learning private limited, New Delhi 110001, (2009).

[10] Zadeh, L. A. Probability measure of Fuzzy events, Journal Mathematical Analysis and Applications, Vol. 23 No.2, August, (1968) pp. 421-427.

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