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SOME COMMON FIXED POINT RESULTS FOR RATIONAL TYPE CONTRACTION MAPPING IN COMPLEX VALUED METRIC SPACE

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ABSTRACT

In this paper, we prove some Common fixed point results for rational type contraction mapping in complex valued metric space by using E.A. property.

1. INTRODUCTION AND PRELIMINARIES

The metric fixed point theory is very important and useful in mathematics because of its application in various areas such as variation and linear inequalities approximation theory physics and computer science. the Banach contraction principle[4]is very popular and effective tool in solving existing literature of fixed point theory contain a great no of generalizations of Banach contraction principle by using different form of contraction condition in various space but majority of such generalization are obtained by improving underlying contraction condition which also includes contraction condition described by rational expressions. Recently Azam *et al* [1] introduce the notation of complex valued metric space and established some fixed point results for pair of mapping for contraction condition satisfying a rational expression. Several authors studied many common fixed point results on complex valued metric space [3, 5-7].

In 2012, Rouzkard and imdad [2] extended and improved the common fixed point theorems which are more general than the result of Azam *et al*.[1]

The following definition of Azam *et al* [1] is needed in the sequel.

Let C be the set of complex numbers and let $z_1, z_2 \in C$. Define a partial order $\le$ on C as follows: $z_1 \le z_2$ if and only if $Re(z_1) \le Re(z_2)$, $Im(z_1) \le Im(z_2)$.

It follows that: $z_1 \le z_2$ if one of the following conditions is satisfied:

(1) $Re(z_1) = Re(z_2)$, $Im(z_1) < Im(z_2)$,
(2) $Re(z_1) < Re(z_2)$, $Im(z_1) = Im(z_2)$,
(3) $Re(z_1) < Re(z_2)$, $Im(z_1) < Im(z_2)$,
(4) $Re(z_1) = Re(z_2)$, $Im(z_1) = Im(z_2)$.

In particular, we will write $z_1 \lneq z_2$ if $z_1 \neq z_2$ and one of (1), (2) and (3) is satisfied and we will write $z_1 < z_2$ if only (3) is satisfied.

Definition 1: Let X be a non-empty set. Suppose that the mapping $d: X \times X \to C$ satisfies:

(a) $0 \le d(x, y)$ for all $x, y \in X$ and $d(x, y) = 0$ if and only if $x = y$;
(b) $d(x, y) = d(y, x)$ for all $x, y \in X$;
(c) $d(x, y) \le d(x, z) + d(z, y)$ for all $x, y, z \in X$.

Then d is called a complex valued metric on X and (X, d) is called a complex valued metric space.

Definition 2: A subset B is subset of X is called closed whenever each limit point of B belongs to B.

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Definition 3: Let$\{x_n\}$ be a sequence in X and x ЄX. If for every c Є C, with 0 < c there is n_0ЄN such that for all n>n_0, d(x_n, x)<c, then x is called the limit of $\{x_n\}$ and we Write lim n→∞x_n = x or x_n → x as n → ∞.

Definition 4: If every Cauchy sequence is convergent in (X, d), then (X, d) is called a complete complex valued metric space.

Lemma 1: Let (X, d) be a complex valued metric space and $\{x_n\}$ a sequence in X. Then $\{x_n\}$ converges to x if and only if |d (x_n , x)| → 0 as n → ∞.

Lemma 2: Let (X, d) be a complex valued metric space and $\{x_n\}$a sequence in X. Then $\{x_n\}$ is a Cauchy sequence if and only if |d (x_n, x_{n+m})| → 0 as n → ∞.

Definition 5: Let f and g be self-maps on a set X, if w = f x = g x for some x in X, then x is called a *coincidence point* of f and g, and w is called a *point of coincidence* of f and g.

Definition 6: Let f and g be two self-maps defined on a set X. Then f and g are said to be *weakly compatible* if they commute at their coincidence points.

Definition 7: A complex valued metric D and a metric d on X are said to be equivalent if they give rise to the same topology on X.

Definition 8: *Let T,S:X*→ X be two self mapping of a complex valued metric space (X, d).The pair (T,S) is said to satisfy (E.A.) property if there exists a sequence $\{x_n\}$ in X such that $lim_{n\to\infty}$ S x_n $= lim_{n\to\infty}$ Tx_n= t for some t ЄX.

Definition 9: The self mapping T and S from X to X are said to satisfy the common limit in the range of S property (CLRs) property if $lim_{n\to\infty}$ Sx_n = $lim_{n\to\infty}$ Tx_n = Sx for some x Є X.

2. MAIN RESULT

Theorem 1: Let (X, d) be a complex valued metrc space and A, B, S, T: X →X four self mapping satisfying the following conditions:

(i) A(X) ⊆ T(X) ,B(X) ⊆ S(X)

(ii) For all x, y Є X and 0 < α<1,

$$d(Ax, By) \le \alpha \frac{[d(Ax,Sx)d(Ax,Ty)+d(By,Ty)d(By,Sx)}{d(Ax,Ty)+d(By,Sx)} + \beta \frac{[\{d(Ax,Ty)\}^2+\{d(By,Sx)\}^2}{d(Ax,Ty)+d(By,Sx)}$$

(iii) The pair (A,S)and (B,T) is weakly compatible;

(iv) The pair (A,S)or (B,T) satisfies (E.A.)-property. If the range of mappings S(X) or T(X) is a closed subspace of X, then A, B S and T have a unique common fixed point in X.

Proof: suppose that the pair (B, T) satisfies (E.A.) property. Then there exists a sequence $\{x_{n\}}$ in X such that $lim_{n\to\infty}$ Bx_n = $lim_{n\to\infty}$ Tx_n = t for some t ЄX

Further since B(X) ⊆ S(X) there exists a sequence $\{y_n\}$ in X such that Bx_n = Sy_n hence $lim_{n\to\infty}$ S y_n $=$ t, we claim that $lim_{n\to\infty}$ Ay_n $=$ t, let $lim_{n\to\infty}$ A y_n $=$ r ≠ t then putting x = y_n, y = x_n in the condition (ii) we have,

$$d(Ay_n, Bx_n) \le \alpha \frac{[d(Ay_n,S\,y_n)d(A\,y_n,Tx_n)+d(Bx_n,Txn)d(Bx_n,S\,y_n)}{d(Ay_n,Tx_n)+d(Bx_n,Sy_n)} + \beta \frac{[\{d(Ay_n,Tx_n)\}^2+\{\{d(Bx_n,S\,y_n)\}^2}{d(Ay_n,Tx_n)+d(Bx_n,Sy_n)}$$

Let n → ∞ we have

$$d(r, t) \le \alpha \frac{[d(r,t)d(r,t)+d(t,t)d(t,t)}{d(r,t)+d(t,t)} + \beta \frac{[\{d(r,t)\}^2+\{d(t,t)\}^2}{d(r,t)+d(t,t)}$$

$$d(r, t) \le \alpha \frac{\{d(r,t)\}^2}{d(r,t)} + \beta \frac{\{d(r,t)\}^2}{d(r,t)}$$

d(r, t) ≤ (α+β) d(r, t)

(1-α-β) d(r, t) ≤ 0

Then| d(r, t)| ≤ 0 hence r=t and i.e. $lim_{n\to\infty}$ Ay_n = $lim_{n\to\infty}Bx_n$ = t

Now suppose that S(x) is a closed subspace of x then t=Su for some u ЄX.

Subsequently$lim_{n\to\infty}$ Ay_n = $lim_{n\to\infty}Bx_n$ =$lim_{n\to\infty}$ Sy_n= $lim_{n\to\infty}Tx_n$ = t =Su.

Now we prove Au =Su i.e. Au = t.

Put x =u and y =x_n in condition (ii)

$$d(Au, Bx_n) \le \alpha \frac{[d(Au,Su)d(Au,Tx_n)+d(Bx_n,Tx_n)d(Bx_n,Su)}{d(Au,Tx_n)+d(Bx_n,Su)} + \beta \frac{[\{d(Au,Tx_n)\}^2+\{\{d(Bx_n,Su)\}^2}{d(Au,Tx_n)+d(Bx_n,Su)} \text{ at } n\to\infty$$

$$d(Au, t) \le \alpha \frac{[d(Au,t)d(Au,t)+d(t,t)d(t,t)}{d(Au,t)+d(t,t)} + \beta \frac{[\{d(Au,t)\}^2+\{d(t,t)\}^2}{d(Au,t)+d(t,t)}$$

$$d(Au, t) \le \alpha \frac{\{d(Au,t)\}^2}{d(Au,t)} + \beta \frac{\{\{d(Au,t)\}^2}{d(Au,t)}$$

(1-α-β) d(Au, t) ≤ 0

Then| d(Au, t)| ≤ 0

So Au =t= Su. hence u is a coincidence point of (A,S)

Now the weak compatibility of pair (A, S) implies that ASu =SAu or At = St

On the other hand since A(X)⊆ T(X) then there exists v in X such that Au = Tv = t.

Now we show that v is coincidence point of (B, T) i.e. Bv =Tv =t.

So putting x=u, y= v in condition (ii)

$$d(Au, Bv) \le \alpha \frac{[d(Au,Su)d(Au,Tv)+d(Bv,Tv)d(Bv,Su)}{d(Au,Tv)+d(Bv,Su)} + \beta \frac{[\{d(Au,Tv)\}^2+\{d(Bv,Su)\}^2}{d(Au,Tv)+d(Bv,Su)} \text{ at } n\to\infty$$

$$d(t, Bv) \le \alpha \frac{[d(t,t)d(t,t)+d(Bv,Tv)d(Bv,t)}{d(t,Tv)+d(Bv,t)} + \beta \frac{[\{d(t,Tv)\}^2+\{d(Bv,t)\}^2}{d(t,Tv)+d(Bv,t)}$$

$$d(t, Bv) \le \alpha \frac{[d(Bv,Tv)d(Bv,t)}{d(Tv,Bv)} + \beta \frac{[\{d(t,t)\}^2+\{d(Bv,t)\}^2}{d(t,t)+d(Bv,t)}$$

$$d(t, Bv) \le \alpha\, d(Bv,t) + \beta \frac{\{d(Bv,t)\}^2}{d(Bv,t)}$$

(1-α-β) d(t,Bv) ≤ 0

Hence t =Bv

Then| d(t, Bv)| ≤ 0 , so Bv =Tv =t and v is the coincidence point of B and T.

Further the weak compatibility of pair (B, T) implies that BTv = TBv or B t =T t therefore t is a common coincidence point of A,B,S,T.

Now we show that t is a common fixed point of A, B, S, T. putting x =u, y = t in condition (ii)

$$d(t, Bt) = d(Au, Bt) \le \alpha \frac{[d(Au,Su)d(Au,Tt)+d(Bt,Tt)d(Bt,Su)}{d(Au,Tt)+d(Bt,Su)} + \beta \frac{[\{d(Au,Tt)\}^2+\{d(Bt,Su)\}^2}{d(Au,Tt)+d(Bt,Su)}$$

Putting Tt = Bt and Su = Au

$$d(t, Bt) = d(Au, Bt) \le \alpha .0 + \beta \frac{[\{d(Au,Bt)\}^2+\{d(Bt,Au)\}^2}{d(Au,Bt)+d(Bt,Au)}$$

$$d(t, Bt) = d(Au, Bt) \le \beta \frac{2\{d(Au,Bt)\}^2}{2d(Au,Bt)}$$

d (t, Bt) = d(Au, Bt) ≤ β d(Au, Bt)

$(1-\beta)\, d(Au, Bt) \leq 0$

Then $|d(Au, Bt)| = 0$

Hence Bt = Au =t but Bt =Tt so Bt =Tt =t. Hence At =Bt =St =Tt =t i.e. t is common fixed point.

If we take T(X) is closed then similar argument arises and if we take (EA) property of the pair (A, S) then similar result arise.

UNIQUENESS

Let us assume w is another common fixed point of A, B, S and T i.e. Aw = Bw = Sw = Tw = w.

Then put x=w and y=t in the condition (ii)

$$d(Aw, Bt) \leq \alpha \frac{[d(Aw,Sw)d(Aw,Tt)+d(Bt,Tt)d(Bt,St)}{d(Aw,Tt)+d(Bt,Sw)} + \beta \frac{[\{d(Aw,Tt)\}^2+\{d(Bt,Sw)\}^2}{d(Aw,Tt)+d(Bt,Sw)}$$

$$d(Aw, Bt) \leq \alpha .0 + \beta \frac{[\{d(w,t)\}^2+\{d(t,w)\}^2}{d(w,t)+d(t,w)}$$

$$d(Aw, Bt) \leq \beta \frac{2\{d(t,w)\}^2}{2d(t,w)}$$

$(1-\beta)\, d(w, t) \leq 0$

Then $|d(w, t)| = 0$

Thus w= t

Hence At =Bt= St =Tt =t is the unique common fixed point of A, B, S and T.

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