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ON - βwg^* SET AND CONTINUITY IN TOPOLOGICAL SPACES

C. Dhanapakyam
Department of Mathematics, RVS College of Arts & Science, Sulur, Coimbatore – India.

J. Subashini*
Department of Mathematics, RVS College of Arts & Science, Sulur, Coimbatore – India.

K. Indirani
Nirmala College for women, Red fields, Coimbatore – India.

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ABSTRACT

The aim of this paper is to introduce the concept of βwg^* closed and open set and tointroduce the βwg^* continuous map and their relations. Various properties and characterizations of βwg^* continuous map are discussed by using βwg^*-closure and βwg^*- interior under certain conditions and also some of the fundamental properties of this set are studied.

Keywords: *g closed set, βwg^* closed set, βwg^* open set, β^*open βwg^* continuous map.*

1. INTRODUCTION

In 1970, Levine [8] introduced the concept of generalized closed set in topological spaces. In 2000, M.K.R.S Veerakumar[19] introduced the concept of g* closed set and their continuity. Andrijevic [1] introduced semi preopen set (β- open set) in general topology. A weak form of continuous map called g continuous was introduced by Balachandran, Sundaram and Maki [2]. In this paper we study the some properties of βwg^* closed and open set and βwg^* continuity.

2. PRELIMINARIES

Definition: 2.1 A subset A of X is called generalized closed (briefly g-closed) [8] set if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open.

Definition: 2.2 A subset A of X is called regular open(briefly r-open) [12] set if $A = \text{int}((A))$ and regular closed(briefly r-closed) [6] set if $A = cl(\text{int}(A))$.

Definition: 2.3 A subset A of X is called pre-open set [14] if $A = \text{int}(cl(A))$ and pre-closed [7] set if $cl(\text{int}(A)) \subseteq A$.

Definition: 2.4 A subset A of X is called $\alpha - open$ [15] if $A \subseteq \text{int}(cl(\text{int}(A)))$ and $\alpha -$closed [11] if $cl(\text{int}(A)) \subseteq \text{A}$.

Definition: 2.5 A subset A of X is called θ-closed [20] if A= cl_θ(A), where cl_θ(A) = $x \in X : \text{int}(cl(U)) \cap A \neq U \in \tau$.

*Corresponding author: J. Subashini**
Department of Mathematics, RVS College of Arts & Science, Sulur, Coimbatore – India.
E-mail: subashini.j@rvsgroup.com

Definition: 2.6 A subset A of X is called δ-closed [20] if A=cl_δ(A), where cl_δ(A)= $x \in X : \text{int}\left(cl\left(U\right)\right) \cap A \neq U \in \tau$.

Definition: 2.7 A subset A of X is called Semi-generalized closed (briefly sg-closed) [4] if scl $\left(A\right) \subseteq U$ whenever $A \subseteq U$ and U is semi open in X.

Definition: 2.8 A subset A of X is called Generalized α-closed (briefly gα -closed) [5] if α-cl(A)⊆U whenever $A \subseteq U$ and U is α-open in X.

Definition: 2.9 A subset A of X is called Generalized semi-preclosed (briefly gsp-closed) [15] if spcl $A \subseteq U$ whenever $A \subseteq U$ and U is open in X.

Definition: 2.10 A subset A of X is called Regular generalized closed (briefly rg-closed) [16] if $clA \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.

Definition: 2.11 A subset A of X is called θ-generalized closed (briefly θ-g-closed) [7] if $cl_\theta A \subseteq U$ whenever $A \subseteq U$ and U is open in X.

Definition: 2.12 A subset A of X is called δ-generalized closed (briefly δ-g-closed) [18] if $cl_\delta A \subseteq U$ whenever $A \subseteq U$ and U is open in X.

Definition: 2.13 A subset A of X is called Strongly generalized closed (briefly g*-closed) [14] if cl $A \subseteq U$ whenever $A \subseteq U$ and U is g-open in X.

Definition: 2.14 A subset A of X is called Weakly closed (briefly w-closed) [11] if cl $A \subseteq U$ whenever $A \subseteq U$ and U is semi open in X.

Definition: 2.15 A subset A of X is called Semi weakly generalized closed (briefly swg-closed) [12] if cl(int(A)) ⊆U whenever $A \subseteq U$ and U is semi open in X.

Definition: 2.16 A subset A of X is called Regular weakly closed (briefly rw-closed) [6] if cl $A \subseteq U$ whenever $A \subseteq U$ and U is regular semi open in X.

Definition: 2.17 A subset A of X is called Regular generalized weakly closed (briefly rgw-closed) [17] if cl(int $A \subseteq U$ whenever $A \subseteq U$ and U is regular semi open in X.

Definition: 2.18 A subset A of a space (X,τ) is called regular semi open [17] if there is a regular open set U such that U⊂A⊂cl(U)

3. *ON βwg** CLOSED SET

Definition: 3.1 A subset A of a topological space (X,τ) is called *β-open if* $A \subseteq cl\left(\text{int}\left(cl\left(A\right)\right)\right)$ *whenever* $A \subseteq U$ *and U is open in X.*

Definition: 3.2 A subset A of a topological space (X,τ) is called *βwg** closed set if gcl $A \subseteq U$ whenever $A \subseteq U$ and U is *β* open in X.

Remark: 3.3 $\emptyset$ and X are *βwg** closed subset of X.

Remark: 3.4

(i). Every g closed set is *βwg** closed set, Since whenever $A \subseteq U$ gcl $A \subseteq U$

(ii). Every g* closed set is g closed. Therefore Every g* closed set is *βwg** closed set.

Theorem: 3.5 Every closed set is βwg^* closed set, but not conversely.

Proof: Let A be closed set such that $A \subseteq U$ and U is $\beta -$ open set. A=Cl $A \subseteq U \Rightarrow$ gcl $A \subseteq U$.
Hence gcl $A \subseteq U$ whenever $A \subseteq U$ and U is β^*-open. Therefore A is βwg^* closed set.

Example: 3.6 Let X= {a, b, c, d},τ={X,ϕ, {a},{b},{a, b},{a, b, c}} here A={a, d} is βwg^* closed but not closed set in X.

Remarks: 3.7

(i). Every θ-closed set is closed set. Therefore every θ-closed set is βwg^* closed.
(ii). Every π-closed is closed. Therefore every π-closed is βwg^* closed.
(iii).Every δ-closed is closed. Therefore every δ-closed is βwg^* closed

Theorem: 3.8 Every regular closed set is βwg^* closed, but not conversely.

Proof: Let A be a regular closed set, such that $A \subseteq U$ and U is β - open set, Every regular closed set is closed. By theorem3.5, A is βwg^* closed set.

Example: 3.9 Let X= {a, b, c, d}, τ={X, ϕ, {a},{b},{a, b},{a, b, c}}. Let A= {d} is βwg^* closed but not regular closed.

Theorem: 3.10 Every gr closed set is βwg^* closed.

Proof: Let A be gr closed set such that $A \subseteq U$ and U is β-open set. Every generalized regular closed set is closed. By theorem 3.5, every gr closed set is βwg^* closed set.

Theorem: 3.11 Every w-closed set is βwg^* closed set, but not conversely.

Proof: Let A be w closed set such that cl $A \subseteq U$. whenever $A \subseteq U$ and U is semi open. Then cl(A) $\subseteq$ gcl $A \subseteq U$. Therefore gcl $A \subseteq U$, whenever $A \subseteq U$ and U is semi open. Since every semi open set is β-open, therefore **e**very w-closed set is βwg* closed set.

Example: 3.12 Let X= {a, b, c, d},τ={X,ϕ, {a},{b},{a, b},{a, b, c}} here A={b, d} is βwg^* closed but not closed set in X.

Theorem: 3.13 The Union of two βwg^* closed subsets of X is also an βwg^* closed subsets of X.

Proof: Assume that A and B are βwg^* closed sets in X, such that A$\cup$B$\subset$U and U is β-open. Then $A \subseteq U$ and $B \subset U$. Since A and B are βwg^* closed set, therefore gcl$\left(A\right) \subseteq U$ and gcl(B)$\subset$U**.** Hence gcl(A$\cup$B)=gcl(A)$\cup$gcl(B)$\subset$U. That is A$\cup$B is βwg^* closed βwg^* closed set.

Theorem: 3.14 The intersection of two βwg^* closed subsets of X is also an βwg^* closed subsets of X.

Proof: Assume that A and B are βwg^* closed sets in X, such that $A \subset U$ and $B \subset U$ and U is β-open. Since A and B are βwg^* closed set, therefore gcl$\left(A\right) \subseteq U$ and gcl$\left(B\right) \subseteq U$ **.** Hence gcl(A$\cap$B)=gcl(A)$\cap$gcl(B)$\subset$U. That is A$\cap$B is βwg^* closed βwg^* closed set.

Theorem: 3.15 Let $A \subseteq U \subseteq$gcl(A) and A is a βwg^* closed subset of (X,τ) then B is also a βwg^* closed subset of (X,τ).

Proof: Since A is a βwg^* closed subset of (X,τ), So gcl$\left(A\right) \subseteq U$, whenever $A \subseteq U$. U is β open subset of X. Let $A \subseteq B \subseteq$gcl(A) . That is gcl(A)=gcl(B). Let if possible there exists an β open subset V of X such that $B \subseteq V$. So $A \subseteq V$ and A being βwg^* closed subset of X, gcl(A) $\subseteq$ V. That is gcl(B) $\subseteq$ V. Hence B is also a βwg^* closed subset of X.

Theorem: 3.16 Let A⊆ B⊆X, where B is β open in X. If A is βwg^* closed in X, then A is βwg^* closed in B.

Proof: Let A ⊆ U, where U is β open set of X. Since U=V∩B, for Some β^*open set V of X and B is β open in X. Using assumption A is βwg^* closed in X. We have gcl(A)⊆U and so gcl(A)=cl(A)∩B ⊆ U∩B ⊆ U. Hence A is βwg^* closed in B.

Theorem: 3.17 Let $A \subseteq B \subseteq X$, where B is β open and βwg^* closed in X. If A is βwg^* closed in B then A is βwg^* closed in X.

Proof: Let U be a β open set of X such that $A \subseteq U$.. Since $A \subseteq U \cap B$ where U∩B is β open in B and A is βwg^* closed in B, gcl(A) ⊆U∩B. we have gcl(A)∩B ⊆ U∩B. Since $A \subseteq B$ we have gcl(A) ⊆gcl(B).Since B is β open and βwg^* closed in X, every g closed set is βwg^* closed. Therefore B is gclosed .Therefore gcl(B)=B. Thus gcl(A) ⊆ B implies gcl(A)=gcl(A) ∩ B ⊆ U∩B ⊆ U. Hence A is βwg^* closed in X.

Theorem: 3.18 A subset A of X is βwg^* closed sets iff gcl(A) ∩ A^c contains no non-zero closed set in X.

Proof: Let A be a βwg^* closed subset of X. Also if possible let M be closed subset of X such that M ⊆gcl(A) ∩A^c. That is M ⊆gcl(A) and M ⊆ A^c. Since M is a closed subset of X, M^c is an open subset of X ⊆ A, and A being βwg^* open subset of X, gcl(A) ⊆ M^c. But M⊆gcl(A). So we get a contradiction. Therefore M=∅.So the condition is true. Conversely, let A ⊆ N, and N is a open subset of X. Then N^c ⊆A^c, and N^c is a closed subset of X. Let if possible gcl(A) ⊆ N .Then gcl(A) ∩ N^c is a nonzero closed subset of gcl(A) ∩ A^c, which is a contradiction .Hence A is a βwg^* closed subset of X.

Theorem: 3.19 A subset A of X is βwg^* closed set in X iff gcl(A)-A contain no non-empty β closed set in X.

Proof: Suppose that F is a non-empty β closed subset if gcl(A)-A. Now F ⊆gcl(A)-A. Then F ⊆gcl(A) ∩ A^c. Therefore F⊆ A^c. Since F^c is β open set and A is βwg^* closed, gcl(A) ⊆ F^c. That is F⊆$gcl(A)^c$. Hence F⊆gcl(A) ∩$[gcl(A)]^c$=∅. That is F=∅. Thus gcl(A)-A contains no non empty β closed set. Conversely assume that gcl(A)-A contains no nonempty β closed set. Let $A \subseteq U$, U is βwg^* open. Suppose that gcl(A) is not contained in U. Then gcl(A) ∩ U^c is a non-empty βwg^* closed set and contained in gcl(A)-A. which is a contradiction. Therefore gcl(A) ⊆ U and hence A is βwg^* closed set.

Example: 3.20 The figure 1 is justified with the following examples.

Let X= {a, b, c, d}, be with the topology τ={X,ϕ, {a},{b},{a, b},{a, b, c}} then

1. Closed sets in X are X, ϕ, {d},{c, d},{a, c, d},{b, c, d}
2. βwg^* closed sets in X are X,ϕ ,{d},{a, d} ,{b, d},{c, d} {a, b ,d},{a, c, d},{b, c, d}.
3. α-closed sets in X are X,ϕ ,{c},{d},{c, d},{a, c ,d},{b, c ,d}
4. Pre closed sets in X are X,ϕ ,{c},{d},{c, d},{a, c ,d},{b, c ,d}
5. Semi closed sets in X are X,ϕ ,{a},{b},{c},{d},{a, c},{a, d},{b, c},{b, d}{c, d},{a, c ,d},{b, c, d}
6. β closed sets in X are X,ϕ,{a},{b},{c},{d},{a, c},{a, d},{b, c},{b, d}{c, d},{a, c ,d},{b, c ,d}
7. Regular closed sets in X are X,ϕ, {a, c ,d},{b, c ,d}
8. g closed sets in X are X,ϕ ,{d},{a, d} ,{b, d},{c, d} {a, b ,d},{a, c, d},{b, c, d}.
9. g* closed sets in X are X,ϕ ,{d},{a, d} ,{b, d},{c, d} {a, b ,d},{a, c, d},{b, c, d}.
10. gα closed sets in X are X,ϕ ,{c},{d},{c, d},{a, c, d},{b, c, d}.
11. gsp closed sets in X are X,ϕ ,{a},{b},{c},{d},{a, c},{a, d},{b, c},{b, d},{c, d},{a, b ,d},{a, c, d},{b, c, d}.
12. sg closed sets in X are X,ϕ ,{a},{b},{c},{d},{a, c},{a, d},{b, c},{b, d},{c, d},{a, b ,d},{a, c, d},{b, c, d}.
13. swg closed sets in X are X,ϕ ,{c},{d},{c, d},{a, b ,d},{a, c, d},{b, c, d}.
14. rg closed sets in X are X,ϕ ,{c},{d},{a, b},{a. c},{a, d},{b, c},{b, d},{c, d},{a, b, d},{a, c, d},{b, c, d}.

15. gr closed sets in X are X,ϕ ,{d},{a, d} ,{b, d},{c, d},{a, b ,d},{a, c, d},{b, c, d}.
16. w closed sets in X are X,ϕ ,{d},{c, d},{a, c, d},{b, c, d}.
17. rw closed sets in X are X,ϕ ,{d},{a, b},{c, d},{a, b, c},{a, b ,d},{a, c, d},{b, c, d}.
18. rgw closed sets in X are X,ϕ ,{c},{d},{a, b},{c, d},{a, b, c},{a, b ,d},{a, c, d},{b, c, d

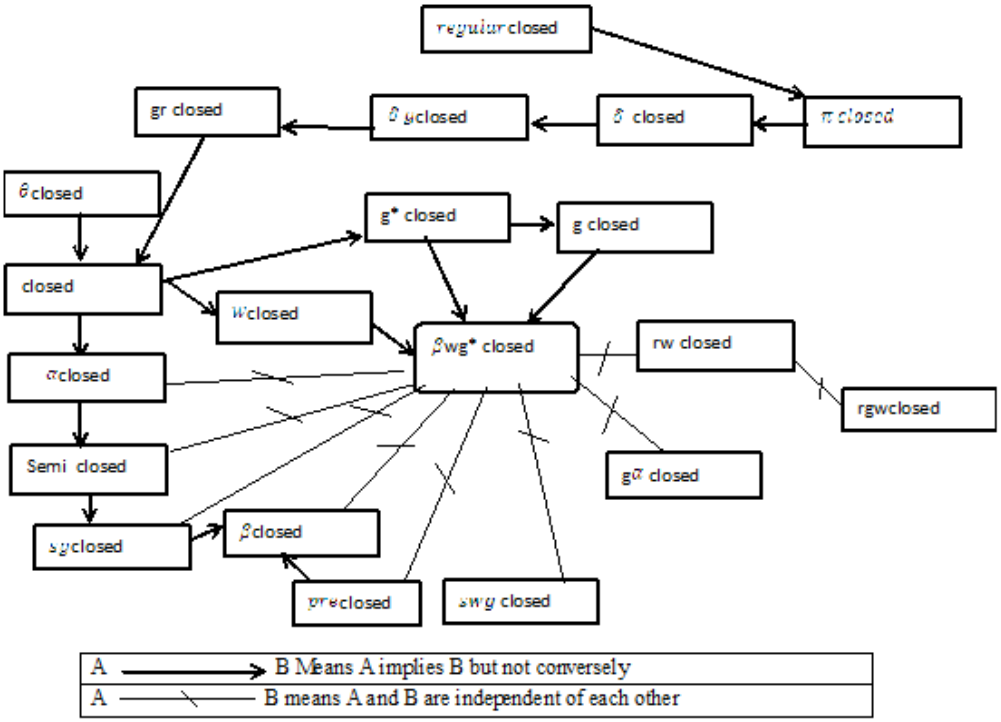

Figure - 1

4. ON βwg^* OPEN SET

Definition: 4.1 A subset A of a topological space X is called βwg^* open sets if A^c is βwg^* closed.

Theorem: 4.2 A subset A of a topological space (X,τ) is βwg^* open if and only if B $\subseteq$ gint(A) whenever B is β closed in X and B$\subseteq$A.

Proof: Necessity: Suppose B $\subseteq$ gint(A) where B is β closed in (X,τ) and B$\subseteq$A. Let $A^c \subseteq$M where M is β open. Hence $M^c \subseteq$A, where M^c is β closed. Hence by assumption $M^c \subseteq$gint(A) which implies $(gint(A))^c \subseteq$M. Therefore $gcl(A^c) \subseteq$M. Thus A^c is βwg^* closed, implies A is βwg^* open.

Sufficiency: Let A is βwg^* open in X with N $\subseteq$A, Where N is β closed. We have A^c is βwg^* closed with $A^c \subseteq N^c$ where N^c is β open. Then we have $gcl(A^c) \subseteq N^c$ implies $N \subseteq X-gcl(A^c)=gint(X-A^c)=gint(A)$

Theorem: 4.3 If gint(A) $\subseteq$B $\subseteq$A and A is βwg^* open subset of (X,τ) then B is also βwg^* open subset of (X,τ).

Proof: Let gint(A) $\subseteq$B $\subseteq$A implies $A^c \subseteq B^c \subseteq gcl(A^c)$. Given A^c is βwg^* closed. By theorem 3.15, B^c is βwg^* closed. Therefore B is βwg^* open.

Theorem: 4.4 If a subset A of a topological space (X,τ) is βwg^* open in X then F=X, whenever F is βwg^* open and gint(A) $\subseteq A^c \subseteq$ F.

Proof: Let A be a βwg^* open and F be βwg^* open, gint(A)$\cup A^c \subseteq$F. This gives $F^c \subseteq$(X-gint(A))$\cap$A=$gcl(A^c)$ $\cap$A=$gcl(A^c)-A^c$. Since F^c is βwg^* closed and A^c is βwg^* closed. By theorem 3.19, we have $F^c=\emptyset$. Thus F=X.

Theorem: 4.5 If a subset A of a topological space (X,τ) is βwg^* closed, then gcl(A)-A is βwg^* open.

Proof: Let $A\subseteq X$ be a βwg^* closed and let F be βwg^* closed such that $F\subseteq$gcl(A)-A. By theorem 3.19, we have $F=\emptyset$. So $\emptyset = F\subseteq$gint(gcl(A)-A). Therefore gcl(A)-A is βwg^* open.

Theorem: 4.6 If A and B are βwg^* open sets in X then $A\cap B$ is also βwg^* open sets in X.

Proof: Let A and B be two βwg^* open sets in X. Then A^c and B^c are βwg^* closed sets in X. By theorem3.13, $A^c\cup B^c$ is a βwg^* closed in X. That is $(A\cap B)^c$ is a βwg^* closed in X. Therefore $(A\cap B)$ is βwg^* open set in X.

Theorem: 4.7 If A and B are βwg^* open sets in X then $A\cup B$ also βwg^* open set in X.

Proof: Let A and B be two βwg^* open sets in X. Then A^c and B^c are βwg^* closed sets in X. By theorem 3.14, $A^c\cap B^c$ is a βwg^* closed in X. That is $(A\cap B)^c$ is a βwg^* closed in X. Therefore $A\cup B$ is βwg^* open sets in X.

Theorem: 4.8 $A\times B$ is a βwg^* open subset of $(X\times Y,\tau\times\sigma)$, iff A is a βwg^* open subset in (X,τ) and B is a βwg^* open subset in (Y,σ).

Proof: Let $A\times B$ be a βwg^* open subset of $(X\times Y,\tau\times\sigma)$. Let H be a closed subset of (X,τ) and G be a closed subset of (Y,σ) such that $H\subseteq A, G\subseteq B$. Then $H\times G$ is closed in $(X\times Y,\tau\times\sigma)$ such that $H\times G\subseteq A\times B$. By assumption $A\times B$ is a βwg^* open subset of $(X\times Y,\tau\times\sigma)$ and so $H\times G\subseteq$gint$(A\times B)\subseteq$gint(A) $\times$ gint(B). That is $H\subseteq$gint(A),$G\subseteq$gint(B) and hence A is a βwg^* open subset in (X,τ) and B is a βwg^* open subset in (Y,σ).Conversely, let M be a closed subset of $(X\times Y,\tau\times\sigma)$ such that $M\subseteq A\times B$. For each $(x, y)\subseteq M$, cl(X) $\times$cl(Y) $\subseteq$cl(M)=$M\subseteq A\times B$. Then the two closed sets cl(X) and cl(Y) are contained in A and B respectively. By assumption cl(X) $\subseteq$gint(A) and cl(Y) $\subseteq$gint(B) hold. This implies that for each $(x, y)\subseteq M$, $(x, y)\subseteq$gint$(A\times B)$.Thus AxB is a βwg^* open subset of $(X\times Y,\tau\times\sigma)$.

5. On βwg^* closure and βwg^* interior

Definition: 5.1 For every set $E\subset X$ we define the βwg^* closure of E to be the intersection of all βwg^* closed sets containing E. In symbols, βwg^* cl(E) $=\cap\{A:E\subset A, A\in \beta wg^* \in \beta wg^* C (X,\tau)\}$

Lemma: 5.2 For any $E\subset X$, $E\subset \beta wg^*$ cl(E) $\subset$cl(E).

Proof: Since every closed set is βwg^*closed, but not conversely.

Remark: 5.3

(i) If $A\subset B$, then βwg^* cl(A)$\subseteq \beta wg^*$ cl(B)

(ii) If E is βwg^*closed, then βwg^*cl(E)=E

Theorem: 5.4 Let A be a subset of a topological space X. For any $x\in X$, $x\in \beta wg^*$ cl(A) if and only if $U\cap A\neq\phi$ for every βwg^* open set U containing x.

Proof: Necessity: Suppose that $x\in \beta wg^*$cl(A). Let U be an βwg^*open set containing x such that $U\cap A=\phi$ and so $A\subset U^c$. But U^c is βwg^*closed and hence βwg^*cl(A) $\subset U^c$. Since $x\notin U^c$ we obtain $x\notin \beta wg^*$ cl(A) which is contrary to the hypothesis.

Sufficiency: Suppose that every βwg^* open set of X containing x meets A. If $x\notin \beta wg^*$ cl(A),then there exists an βwg^* closed F of X such that $A\subset F$ and $x\notin F$. Therefore $x\in F^c$ and F^c is an βwg^* open set containing x. But $F^c\cap A=\emptyset$. This is contrary to the hypothesis.

Definition: 5.5 For any $A\subset X$, βwg^* int(A) is defined as the union of all βwg^* open sets contained in A. That is βwg^* int(A)=$\cup$\{U:U$\subset$A and U$\in$ βwg^* O(X,τ)\}.

Note: 5.6 If A is βwg^* open, then A= βwg^* int(A).

Theorem: 5.7 Let A be a subset of a space X, then the following are true.

(i). $(\beta wg^*\text{int}(A))^c = \beta wg^*\text{ cl}(A^c)$
(ii). $\beta wg^*\text{int}(A)=(\beta wg^{**}\text{ cl}(A^c))^c$
(iii). $\beta wg^*\text{ cl}(A)=(\beta wg^*\text{ int}(A^c))^c$

Proof:

(i) Let x $\in$(βwg^*int(A))^c. Then x$\notin$ βwg^* int(A).If U be any βwg^* open set containing x then U$\not\subset$A. Therefore U$\cap A^c\neq\emptyset$. By theorem 5.4,x$\in$ βwg^* cl(A^c) and therefore (βwg^* int(A))$^c\subset$ βwg^* cl(A^c). Conversely let x$\in$ βwg^* cl(A^c), by theorem 5.4, every βwg^* open set U containing x intersects A^c. Hence U$\cap A^c\neq\emptyset$. By definition 5.5, x$\notin$ βwg^* int(A), hence x$\in$(βwg^*)^c and so βwg^* cl(A^c)$\subset$(βwg^* int(A))^c. Thus βwg^* cl(A^c)= (βwg^* int(A))^c

(ii) Follows by taking complements in (i).

(iii) Follows by replacing A by A^c in (i).

6. On βwg^* continuity

Definition 6.1: A map $f(X,\tau)\to(Y,\sigma)$ is called

1. Continuous [9] if f^{-1}(V) is closed subset in (X,τ) for every closed subset V in (Y,σ).
2. Semi continuous [9] if f^{-1}(V) is semi closed subset in (X,τ) for every closed subset V in (Y,σ).
3. g continuous [3] if f^{-1}(V) is g closed subset in (X,τ) for every closed subset V in (Y,σ).
4. αcontinuous [13] if f^{-1}(V) is α closed subset in (X,τ) for every closed subset V in (Y,σ).
5. g* continuous [19] if f^{-1}(V) is g* closed subset in (X,τ) for every closed subset V in (Y,σ).
6. Regular continuous [2] if f^{-1}(V) is r closed subset in (X,τ) for every closed subset V in (Y,σ).
7. gr continuous [10] if f^{-1}(V) is gr closed subset in (X,τ) for every closed subset V in (Y,σ).
8. w continuous [16] if f^{-1}(V) is w closed subset in (X,τ) for every closed subset V in (Y,σ).

Definition: 6.2 A function $f(X,\tau)\to(Y,\sigma)$ is called βwg^* continuous if f^{-1}(V) is βwg^* closed subset of (X,τ) for every closed subset V of (Y,σ).

Theorem: 6.3 Every continuous map is βwg^* continuous, but not conversely.

Proof: The proof follows from the fact that every closed set is βwg^* closed set.

Example: 6.4 Let X=Y= {a, b, c, d}, τ={X,ϕ,{a},{b},{a, b}, {a, b, c}} and σ ={X, ϕ, {b},{b, d}}. define a map f:X$\to$Y by f(a)=a, f(b)=b, f(c)=d, f(d)=d. This map is βwg^* continuous, but not continuous. Since for the closed set U= {a, c, d} in Y.f^{-1}(U)={a, d} is not closed in X.

Theorem: 6.5 Every regular continuous map is βwg^* continuous, but not conversely.

Proof: The proof follows from the fact that every regular closed set is βwg^* closed set.

Example: 6.6 Let X=Y={a, b, c, d}, τ={X,ϕ, {a},{b},{a, b},{a, b, c}} and σ ={X,ϕ, {b},{b, d}}. define a map f:X$\to$Y by f(a)=a, f(b)=b, f(c)=d, f(d)=d. This map is βwg^* continuous, but not regular continuous. Since for the closed set U= {b, c, d} in Y.f^{-1}(U) = {b, d} is not regular closed in X.

Remark: 6.7

(i) Every g closed set is βwg^* closed set. Therefore every g continuous map is βwg^* continuous.
(ii) Every g* closed set is βwg^* closed. Therefore every g* continuous map is βwg^* continuous.
(iii) Every gr closed set is βwg^* closed set. Therefore every gr continuous map is βwg^* continuous.
(iv) Every w closed set is βwg^* closed set. Therefore every w continuous map is βwg^* continuous.

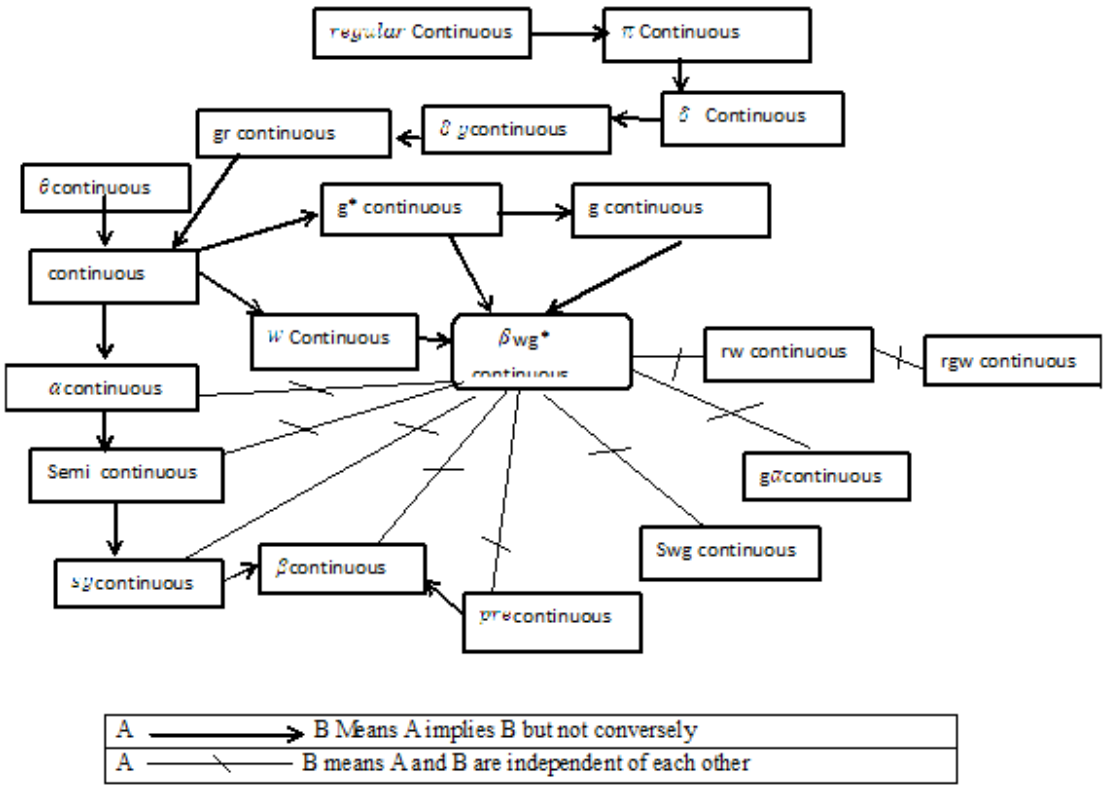

Figure - 2

Theorem: 6.8 If f: X→Y is βwg* continuous and g:Y→Z is continuous then their composition f∘g:X→Z is βwg* continuous.

Proof: Let f: X→Y is βwg^* continuous and g: Y→Z is continuous. Let U be a closed set in Z. Therefore $g^{-1}(U)$ is closed in Y and $f^{-1}(g^{-1}(U))$ is βwg^* closed in Y. ∴f∘g is βwg^* continuous.

Theorem: 6.9 Let X and Y be topological spaces. Let f:(X,τ) →(Y,σ). Then the following are equivalent.

(i). f is βwg^* continuous.

(ii). for every subset A of X, one has $f(\overline{A}) \subset \overline{f(A)}$.

(iii).for every closed set B of Y, the set $f^{-1}(B)$ is closed in X.

(iv) for each x∈ X and each neighborhood V of f(x), there is a neighborhood U of x such that f(U)⊂V.

Proof:

(i)⟹(ii): Assume that. f is βwg^* continuous. Let A be a subset of X. Let V be a neighborhood of f(x), then $f^{-1}(V)$ is an open set of X containing x, it must intersect A in some 9point y. Then V intersects f(A) in the point f(y). So that $f(x) \in \overline{f(A)}$.

(ii)⟹(iii): Let B be closed in Y and let A=$f^{-1}(B)$. Prove that, A is closed in X and we show that $\overline{A}$ = A. By elementary set theory, we have f(A)=f(f^{-1}(A)) ⊂B, If x∈ $\overline{A}$, f(x) ∈ $f(\overline{A}) \subset \overline{f(A)} \subset \overline{B}$ =B. f(x) ∈B, so that x ∈f^{-1}(B)=A. Thus $\overline{A} \subset A$, So that $\overline{A} = A$.

(iii)⟹(i): Let V be an open set of Y set B=Y-V. Then $f^{-1}(B) = f^{-1}(Y-V) = f^{-1}(Y)-f^{-1}(V)=X-f^{1}(V)$. Now B is a closed set of Y. Then $f^{-1}(B)$ is closed in X by hypothesis so that $f^{-1}(V)$ is open in X.

(i)⟹(iv): Let x∈X and let V be a neighborhood of f(x). Then the set $U=f^{-1}(V)$ is a neighborhood of x such that f(U) ⊂V.

(iv)⟹(i): Let V be an open set of Y. Let x be a point of $f^{-1}(V)$.Then f(x) ∈V, so that by hypothesis there is a neighborhood U_x of x such that $f(U_x)$ ⊂V. Then U_x ⊂$f^{-1}(V)$, and hence $f^{1}(V)$ =$\bigcup_{x\in f-1(V)} Ux$. Therefore f is continuous⟹f is βwg* continuous.

Theorem: 6.10 Let X=A∪ B, where A and B are closed in X. Let f: A→ Y and g: B→ Y be continuous. If f(x)=g(x) for every x∈ A ∩ B then f and g combine to give a βwg^* continuous function h:X→Y defined by setting h(x)=f(x) if x∈A, and h(x)=g(X) if x∈B.

Proof: Let c be a closed subset of Y. Now $h^{-1}(c)= f^{-1}(c) \cup g^{-1}(c)$. Since f is continuous, $f^{-1}(c)$ is closed in A and therefore closed in X. Similarly $g^{-1}(c)$ is closed in B and therefore closed in X. Their union $h^{-1}(c)$ is also closed in X. Therefore h is continuous. By theorem 6.3, h is βwg^* continuous.

Theorem: 6.11 A function f: $(X,\tau) \to (Y,\sigma)$ from a topological space *X* into a topological space *Y* is βwg^* continuous if and only if $f^{1}(V)$ is βwg^* open set in *X* for every open set *V* in *Y*

Proof: It is obvious

Theorem: 6.12 Let $f(X,\tau)\to(Y,\sigma)$ be a function from a topological space *X* into a topological space Y. If $f(X,\tau)\to(Y,\sigma)$ is continuous then $f(\beta wg^* cl(A)) \subseteq cl(f(A)$ for every open subset *A* of *X*.

Proof: Since $f(A)\subseteq cl(f(A)) \Rightarrow A\subseteq f^{-1}(cl(f(A)))$. Since $cl(f(A)$ is closed set in *Y* and *f* is βwg^* continuous, then $f^{-1}(cl(f(A)))$. is a βwg^* closed set in *X* containing *A*. Hence $\beta wg^* cl(A)) \subseteq f^{-1}(cl(f(A)))$.. Therefore $f(\beta wg^* cl(A) \subseteq cl(f(A))$

Theorem: 6.13 Let $f(X,\tau)\to(Y,\sigma)$ be a function from a topological space *X* into a topological space Y. Then the following statements are equivalent.

(i).For each point *x* in *X* and each open set *V* in *Y* with *f(x)*∈*V*, there is a βwg^* open set *U* in *X* such that *x*∈*U* and *f(U)*⊆ *V*

(ii).For each subset *A* of *x*, $f(\beta wg^* cl(A)) \subseteq cl(f(A)$

(iii).For each subset *B* of *y*, $\beta wg^* cl(f^{-1}(B)) \subseteq f^{-1}(cl(B))$

Proof:

(i) ⇒ (ii): Suppose that (i) holds and let y∈f(βwg^*cl(A)) and let V be any open neighborhood of y. Since y∈f(βwg^*cl(A)) ⇒ ∃ x∈ βwg*cl(A) such that f(x)=y. Since f(x) ∈ V, then by (i) ∃ a βwg^*open set U in x such that x∈U and f(U)⊆V. Since x∈ βwg^* cl(A) then by theorem 5.4, ⇒ U∩A≠ ∅ and hence f(A) ∩V≠ ∅. Therefore we have y = f(x) ∈cl(f(A)). Hence f (βwg^*cl(A)) ⊆cl(f(A)).

(ii) ⇒ (i): If (ii) holds and let x∈X and V be any open set in Y containing f(x). Let $A=f^{-1}(V^c)$ ⇒ x∉A. Since f(βwg^*cl(A)) ⊆cl(f(A)) ⊆V^c⇒ βwg*cl(A) ⊆$f^{-1}(V^c)$=A. Since x∉A ⇒ x∉ βwg^* cl(A) and by theorem 5.4, there exists a βwg^* open set U containing x such that U ∩A= ∅ and hence f(U) ⊆f(A^c) ⊆V.

(ii) ⇒(iii): Suppose that (ii) holds and let B be any subset of Y. Replacing A by $f^{-1}(B)$ we get from (ii), $f(\beta wg^{*}cl(f^{-1}(B)) \subseteq cl(f(f^{-1}(B))) \subseteq cl(B)$. Hence $\beta wg^{*}(f^{-1}(B)) \subseteq f^{-1}(cl(B))$.

(iii) ⇒(ii): Suppose that (iii) holds. Let B=f(B) where A is a subset of X. then we get from (iii) $\beta wg^{*}cl(A) \subseteq \beta wg^{*}cl(f^{-1}(f(A)) \subseteq f^{-1}(cl(f(A))$. Therefore $f(\beta wg^{*}cl(A)) \subseteq cl(f(A))$.

Theorem: 6.14 Let f: $(X,\tau) \to (Y,\sigma)$ be a function. Then the following are equivalent.

(i) f is βwg^{*} continuous.
(ii) The inverse image of each open set in Y is βwg^{*} open in X.
(iii).The inverse image of each closed set in Y is βwg^{*} closed in X.

Proof:
(i) ⇒(ii): Let G be any open set in Y. Then Y-G is closed in Y. Since f is βwg^{*} continuous, $f^{-1}(Y-G)$ is closed in X. But $f^{-1}(Y-G) = X - f^{-1}(G)$ is βwg^{*} closed in X. Therefore $f^{-1}(G)$ is βwg^{*} open in X.

(ii) ⇒(iii): and (iii) ⇒(i) are obvious.

7. βwg^{*} IRRESOLUTE FUNCTION

Definition: 7.1 A function $f(X,\tau) \to (Y,\sigma)$ from a topological space X into a topological space Y is called a βwg^{*} irresolute if $f^{-1}(V)$ is βwg^{*} closed set in X for every βwg^{*} closed set V in Y.

Theorem: 7.2 A function $f(X,\tau) \to (Y,\sigma)$ from a topological space X into a topological space Y is βwg^{*} irresolute iff $f^{-1}(V)$ is βwg^{*} open set in X for every βwg^{*} open set V in Y.

Proof: It is obvious.

Definition: 7.3 A function $f(X,\tau) \to (Y,\sigma)$ from a topplogical space X into a topological space Y is said to be contra βwg^{*} irresolute if $f^{-1}(V)$ is βwg^{*} closed set in X for every βwg^{*} open set V in Y.

Definition: 7.4 A function $f(X,\tau) \to (Y,\sigma)$ is said to be contra βwg^{*} continuous if the inverse image of each open set of Y is βwg^{*} closed in X.

Definition: 7.5 A function $f(X,\tau) \to (Y,\sigma)$ is said to be contra strongly βwg^{*} continuous if the inverse image of each βwg^{*} open set of Y is closed in X.

Theorem: 7.6 If $f(X,\tau) \to (Y,\sigma)$ and g: $(Y,\sigma) \to (Z,\delta)$ are functions, then

(i).If f:$(X,\tau) \to (Y,\sigma)$ and g: $(Y,\sigma) \to (Z,\delta)$ are both βwg^{*} irresolute function, then g∘f: $(X,\tau) \to (Z,\delta)$ is βwg^{*} irresolute.
(ii).If $f(X,\tau) \to (Y,\sigma)$ is contra βwg^{*} irresolute and g: $(Y,\sigma) \to (Z,\delta)$ is βwg^{*} irresolute, then g∘f: $(X,\tau) \to (Z,\delta)$ is contra βwg^{*} irresolute.
(iii).If $f(X,\tau) \to (Y,\sigma)$ is βwg^{*}irresolute and g: $(Y,\sigma) \to (Z,\delta)$ is βwg* continuous, then g∘f: $(X,\tau) \to (Z,\delta)$ is βwg^{*} continuous.
(iv).If $f(X,\tau) \to (Y,\sigma)$ is contra βwg^{*} irresolute and g: $(Y,\sigma) \to (Z,\delta)$ is βwg^{*} continuous, then g∘f: $(X,\tau) \to (Z,\delta)$ is βwg^{*} continuous.

Proof:
(i).Let U be a βwg^{*} open set in Z, since g is βwg^{*} irresolute, then $g^{-1}(U)$ is βwg^{*} open in Y. Since f is βwg^{*} irresolute, then $f^{-1}(g^{-1}(U))$ is βwg^{*} open in X. Since $(g\circ f)^{-1}(U) = f^{-1}(g^{-1}(U))$, then $(g\circ f)^{-1}(U)$ is a βwg^{*}open set in X. Thus g∘f is βwg^{*} irresolute.

(ii).Let U be βwg^* open set in Z, since g is βwg^* irresolute, then $g^{-1}(U)$ is βwg^*open in Y, since f is contra βwg^* irresolute, then $f^{-1}(g^{-1}(U))$ is βwg^* closed in X. Since $(g\circ f)^{-1}(U) = f^{-1}(g^{-1}(U))$, then $(g\circ f)^{-1}(U)$ is a βwg^* closed set in X. Thus $g\circ f$ is contra βwg^* irresolute.

(iii). Let F be a closed in Z, since g is βwg^* continuous, then $g^{-1}(F)$ is βwg^* closed in Y. Since f βwg^* open is βwg^* irresolute, then $f^{-1}(g^{-1}(F))$ is βwg^* closed in X. Since $(g\circ f)^{-1}(F) = f^{-1}(g^{-1}(F))$ then $(g\circ f)^{-1}(F)$ is βwg^* closed in X. Thus $g\circ f$ is βwg^* continuous.

(iv). Let U be an open set in Z, since g is βwg^* continuous, then $g^{-1}(U)$ is βwg^* open in Y. Since f is contra βwg^* irresolute, then $f^{-1}(g^{-1}(U))$ is βwg^* closed in X. Since $(g\circ f)^{-1}(U) = f^{-1}(g^{-1}(U))$, then $(g\circ f)^{-1}(U)$ is a βwg^* closed in X. Thus $g\circ f$ is contra βwg^* continuous.

Theorem: 7.7 If $f(X,\tau)\to(Y,\sigma)$ is contra strongly βwg^* continuous and $h:(Y,\sigma)\to(Z,\delta)$ is βwg^* continuous, then the composition $h\circ f: (X,\tau)\to(Z,\delta)$ is contra continuous.

Proof: Let U be open set in Z. Since h is βwg^* continuous. $h^{-1}(U)$ is βwg^* open in Y. Therefore $f^{-1}(h^{-1}(U))$ is closed in X. Since f is contra strongly βwg^* continuous. Hence $(h\circ f)^{-1}(U) = f^{-1}(h^{-1}(U))$ is closed in X. Hence $h\circ f$ is contra continuous.

Theorem: 7.8 If $f(X,\tau)\to(Y,\sigma)$ is contra βwg^* continuous and $h:(Y,\sigma)\to(Z,\delta)$ is strongly βwg^* continuous then the composition $h\circ f: (X,\tau)\to(Z,\delta)$ is contra βwg^* irresolute.

Proof: Let U be βwg^* open set in Z. Since h is strongly βwg^* continuous, $h^{-1}(U)$ is open in Y. Therefore $f^{-1}(h^{-1}(U))$ is βwg^* closed in X. Since f is contra βwg^* continuous. Hence $(h\circ f)^{-1}(U) = f^{-1}(h^{-1}(U))$ is βwg^* closed in X. Hence $h\circ f$ is contra βwg^* irresolute.

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