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VARIABLE VISCOSITY EFFECTS ON PENETRATIVE CONVECTION IN SUPERPOSED FLUID AND POROUS LAYERS

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ABSTRACT

The variable viscosity effects on the onset of penetrative convection simulated via internal heating in a two-layer system in which a layer of fluid overlies and saturates a layer of porous medium is studied. The Beavers–Joseph slip condition is applied at the interface between the fluid and the porous layers and dependence of viscosity is assumed to be exponential. The boundaries are considered to be rigid, however permeable, and insulated to temperature perturbations. The eigen value problem is solved using a regular perturbation technique with wave number as a perturbation parameter. The ratio of fluid layer thickness to porous layer thickness, ζ, the viscosity parameter B, and the presence of volumetric internal heat source in fluid and/or porous layer play a decisive role on the stability characteristics of the system. It is observed that both stabilizing and destabilizing factors can be enhanced because of the simultaneous presence of a volumetric heat source and variable viscosity effects so that a more precise control (suppress or augment) of thermal convective instability in a layer of fluid or porous medium is possible.

Key words: *composite layer; penetrative convection; Variable viscosity.*

1. INTRODUCTION

The problem of fluid flow over a porous medium is encountered in a wide range of industrial and geophysical applications, such as the extraction of oil from underground reservoirs, the manufacturing of composite materials used in the aircraft and automobile industries, flow of water under the earth's surface and growing of compound films in thermal chemical vapour deposition reactors. A detailed review is given by Nield and Bejan (2006) with current highly relevant literature including (Chen 1990; Chen and Jay W. Lu 1992; Carr 2004; Chang (2004; 2005; 2006); Shivakumara *et al.* (2011; 2012) and Hill and Straughan 2009).

The mechanism of internal heating in a flowing fluid is relevant to the thermal processing of liquid foods through ohmic heating, where the internal heat generation serves for the pasteurization/sterilization of the food Ruan et al. (2004). Other important applications of flows with internal heat generation are relative to nuclear reactors, as well as to the geophysics of the earth's mantle. In both cases, the internal heating is due to the radioactive decay. For nuclear reactors, processes of natural convection with internal heating are extremely important in the analysis of severe accident conditions. As pointed out by Generalis and Busse (2008), flows with volumetric heating are relevant for the physics of the atmosphere, in connection with the absorption of solar radiation. Due to the wide range of industrial and geophysical applications, extensive literature has been recently produced on this subject; see e.g. (Carr 2004, Carr and Putter 2003, Hill 2004, Straughan 2008 and Zhang and Schubert 2002).

It is important to note that the viscosity of a liquid is usually strongly dependent on temperature (cf. Capone & Gentile 1994, 1995; Galiano 2000). Convection problems for which the viscosity or conductivity is a function of temperature has received much recent attention in the literature (e.g. Payne & Straughan 2000; Manga *et al.* 2001; Shevtsova *et al.* 2001), making this work particularly timely.

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Variable viscosity effects on the convective stability in superposed fluid and porous layers configuration were investigated by Chen et al. (1992). They chose the viscosity-temperature model as exponential and the onset of the convection are obtained from observing the streamline patterns at the onset of instability. The stability of convection in a two-layer system in which a layer of fluid with a temperature-dependent viscosity overlies and saturates a highly porous material is studied by Antony *et al.* (2009). The intent of the present study is to obtain the criterion for the onset of instability thresholds of penetrative convection via internal heating in a two-layer system in which a fluid layer overlies a layer of fluid saturated porous medium with variable viscosity effects. This is achieved by performing the linear stability analysis. The boundaries are considered to be insulated to temperature perturbations. A regular perturbation technique with wave number as a perturbation parameter is used to solve the eigen value problem in a closed form. A wide-ranging parametric study is undertaken to explore their impact on the stability characteristics of the system.

2. MATHEMATICAL FORMULATION

We consider penetrative convection via internal heating in a system consisting of an infinite horizontal fluid layer of thickness *d* overlying a layer of porous medium of thickness d_m as shown in Fig.1. A Cartesian coordinate system (x, y, z) is chosen with the origin at the interface and the z-axis vertically upward. The gravity acts in the vertical direction with constant acceleration g . The top and bottom boundaries are assumed to be rigid-permeable and are maintained at uniform but different temperatures T_l and $T_u\,(<T_l)$ respectively.

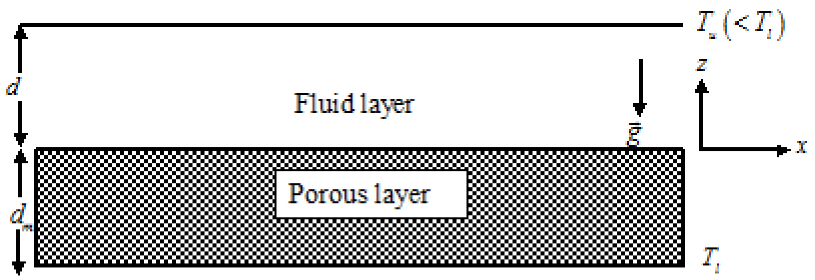

Fig. - 1. Physical configuration

The governing equations for the fluid and the porous layers are:

Fluid layer:

$$\nabla\cdot\vec{V}=0 \tag{1}$$

$$\rho_0\left(\vec{V}\cdot\nabla\right)\vec{V}=-\nabla p+\rho_0\vec{g}\left[1-\alpha\left(T-T_0\right)\right]+\nabla.\left[\mu\left(T\right)D\right] \tag{2}$$

$$\left(\vec{V}\cdot\nabla\right)T=\kappa\nabla^2T+q_f\,. \tag{3}$$

Porous layer:

$$\nabla\cdot\vec{V}_m=0 \tag{4}$$

$$-\nabla p_m+\rho_0\vec{g}\left[1-\alpha\left(T_m-T_0\right)\right]-\frac{\mu\left(T\right)}{K}\vec{V}_m=0 \tag{5}$$

$$\left(\vec{V}_m\cdot\nabla_m\right)T_m=\kappa_m\nabla^2T_m+q_m\,. \tag{6}$$

In the above equations, $\vec{V}=(u,v,w)$ is the velocity vector, p is the pressure, T is the temperature, q_f is the heat source in the fluid layer, the deviatoric strain tensor $D\ is\ \nabla V+\nabla V^T$ and κ is the thermal diffusivity, while $\vec{V}_m$, p_m , T_m , q_m and κ_m are the corresponding quantities in the porous layer

The boundary conditions are formally the same as those in Chen (1990). At the upper boundary $z=d$, $T=T_u$ and at the lower boundary $z_m=-d_m$, $T=T_l$. At the interface $z=0$, the continuity of normal velocity, temperature, heat flux and the normal stress are assumed. That is,

$$w = w_m \tag{7}$$

$$T = T_m \tag{8}$$

$$k\frac{\partial T}{\partial z} = k_m \frac{\partial T_m}{\partial z_m} \tag{9}$$

$$-p + 2\mu\frac{\partial w}{\partial z} = -p_m \tag{10}$$

where k and k_m are the thermal conductivities for the fluid and the porous medium, respectively.

As the fifth condition, the Beavers and Joseph (1967) slip condition in which the slip in the tangential velocity is proportional to the vertical gradient of the tangential velocity in the fluid is used.

The basic steady state is assumed to be quiescent and temperature distributions are found to be

$$T_b(z) = T_0 - \left[\left(\frac{(T_0 - T_u)}{d} - \frac{q_f\, d}{2\kappa}\right)z + \frac{q_f}{2\kappa}z^2\right] \quad 0 \le z \le d \tag{11}$$

$$T_{mb}(z_m) = T_0 - \left[\left(\frac{(T_l - T_0)}{d_m} - \frac{q_m\, d_m}{2\kappa_m}\right)z_m + \frac{q_m}{2\kappa_m}z_m^2\right] \quad -d_m \le z_m \le 0 \tag{12}$$

Where T_0 is the temperature at the inter face. In order to investigate the stability of the basic solution, infinitesimal disturbances are introduced in the form

$$\vec{V} = \vec{V}', \quad T = T_b(z) + T', \quad p = p_b(z) + p' \tag{13}$$

$$\vec{V}_m = \vec{V}_m', \; T_m = T_{mb}(z) + T_m', \; p_m = p_{mb}(z) + p_m' \tag{14}$$

where the primed quantities are the perturbations and assumed to be small. Eqs. (13) and (14)are substituted in Eqs. (1)-(6) and linearized in the usual manner. The pressure term is eliminated from Eqs. (2) and (5) by taking curl twice on these two equations and only the vertical component is retained. The variables are then nondimensionalized using $d, d^2/\kappa, \kappa/d$ and $T_0 - T_u$ as the units of length, time, velocity, and temperature in the fluid layer and $d_m, d_m^2/\kappa_m, \kappa_m/d_m$ and $T_l - T_0$ as the corresponding characteristic quantities in the porous layer. Note that separate length scales are chosen for the two layers so that each layer is of unit depth. In this manner, the detailed flow fields in both the fluid and porous layers can be clearly discerned for all depth ratios $\zeta = d/d_m$ and the non-dimensional disturbance equations are now given by

$$f(z)\nabla^4 w + 2f'(z)\nabla^2\frac{\partial w}{\partial z} + f''(z)\left(2\nabla_h^2 w - \nabla^2 w\right) = -R\nabla_h^2 T \tag{15}$$

$$\nabla^2 T = -w\left[1 - Ns(1-2z)\right] \tag{16}$$

$$f(z_m)\nabla_m^2 w_m + f'(z_m)\frac{\partial w_m}{\partial z_m} = R_m \nabla_{hm}^2 T_m \tag{17}$$

$$\nabla_m^2 T_m = -w_m\left[1 + Ns_m(1+2z_m)\right] \tag{18}$$

The boundary conditions are

$$w = \frac{\partial T}{\partial z} = \frac{\partial w}{\partial z} = 0 \quad at \quad z = 1 \tag{19}$$

$$w_m = \frac{\partial T_m}{\partial z_m} = 0 \quad at \quad z = -1. \tag{20}$$

At the interface (i.e., z = 0) the continuity of velocity, temperature, heat flux, normal stress and the Beavers and Joseph (1967) slip conditions are imposed. Accordingly, the conditions are:

$$w = \frac{\zeta}{\varepsilon_T} w_m \tag{21}$$

$$T = \frac{\varepsilon_T}{\zeta} T_m \tag{22}$$

$$\frac{\partial T}{\partial z} = \frac{\partial T_m}{\partial z_m} \tag{23}$$

$$f(0)\left(3\nabla_h^2 + \frac{\partial^2}{\partial z^2}\right)\frac{\partial w}{\partial z} + f'(0)\left(-\nabla_h^2 + \frac{\partial^2}{\partial z^2}\right)w = -\frac{\zeta^4}{Da\varepsilon_T} f(0)\frac{\partial w_m}{\partial z_m} \tag{24}$$

$$\frac{\partial^2 w}{\partial z^2} = \frac{\beta\zeta}{\sqrt{Da}}\frac{\partial w}{\partial z} - \frac{\beta\zeta^3}{\sqrt{Da}\,\varepsilon_T}\frac{\partial w_m}{\partial z_m} \tag{25}$$

Since the principle of exchange instabilities holds for thermal convection either in fluid layer or a porous layer. It is reasonable to assume that it holds good even for the present configuration as well. Then performing a normal mode expansion of the dependent variables in both fluid and porous layers as

$$(W, T) = \left[W(z), \theta(z)\right]\exp\left[i(lx + my)\right] \tag{26}$$

$$(W_m, T_m) = \left[W_m(z), \theta_m(z)\right]\exp\left[i(\tilde{l}x + \tilde{m}y)\right] \tag{27}$$

and substituting them in Eqs. (15) – (18), we obtain the following ordinary differential equations

$$f(z)\left(D^2 - a^2\right)^2 W + 2f'(z)\left(D^2 - a^2\right)DW + f''(z)\left(D^2 + a^2\right) = Ra^2\Theta \tag{28}$$

$$\left(D^2 - a^2\right)\ \Theta = -W\left[1 - Ns(1 - 2z)\right] \tag{29}$$

$$f(z_m)\left(D_m^2 - a_m^2\right)w_m + f'(z_m)D_m w_m = -a_m^2 R_m \Theta_m \tag{30}$$

$$\left(D_m^2 - a_m^2\right)w_m = -w_m\left[1 + Ns_m(1 + 2z_m)\right] \tag{31}$$

Where W is the amplitude of perturbed vertical velocity and Θ is the amplitude of perturbed temperature in the fluid layer, while W_m and Θ_m are the corresponding quantities in the porous medium. In the above equations, $D = d/dz$. $a = \sqrt{l^2 + m^2}$ and $a_m = \sqrt{\tilde{l}^2 + \tilde{m}^2}$ are correspondingly the overall horizontal wave numbers in the fluid and porous layers.

The boundary conditions are:

$$W = D\Theta = DW = 0 \qquad at\ z = 1 \tag{32}$$

$$W_m = D_m\Theta_m = 0 \qquad at\ z_m = -1. \tag{33}$$

And those at the interface as

$$W = \frac{\zeta}{\varepsilon_T} W_m \tag{34}$$

$$\Theta = \frac{\varepsilon_T}{\zeta}\Theta_m \tag{35}$$

$$D\Theta = D_m\Theta_m \tag{36}$$

$$f(0)\left(D^2 - 3a^2\right)DW + f'(0)\left(D^2 + a^2\right) = \frac{-\zeta^4}{\varepsilon_T Da} f(0)D_m W_m \tag{37}$$

$$\left[D^2 - \frac{\beta\zeta}{\sqrt{Da}}D\right]W = \frac{-\beta\zeta^3}{\varepsilon_T\sqrt{Da}}D_m W_m. \tag{38}$$

where $\varepsilon_T = \kappa / \kappa_m$ is the ratio of thermal diffusivities, $\zeta = d/d_m$ is the ratio of fluid layer to porous layer thickness and β is the Beavers-Joseph slip parameter. Thus, the problem is reduced to an eigen value problem consisting of a sixth order ordinary differential equation in the fluid layer and a fourth order ordinary differential equation in the porous layer, subject to 10 boundary conditions. If matching of the solutions in the two layers is to be possible, the wave numbers must be the same for the fluid and porous layers, so that we have $a / d = a_m / d_m$ and hence $\zeta = a / a_m$

3. METHOD OF SOLUTION

Since the critical wave number is exceedingly small for the assumed temperature boundary conditions (Nield and Bejan 2006), the eigen value problem is solved using a regular perturbation technique with wave number $a$as a perturbation parameter. Accordingly, the dependent variables are expanded in powers of a^2in the form

$$\left(W,\Theta\right) = \sum_{i=0}^{N}\left(a^2\right)^i\left(W_i,\Theta_i\right) \tag{39}$$

$$\left(W_m,\Theta_m\right) = \sum_{i=0}^{N}\left(\frac{a^2}{\zeta^2}\right)^i\left(W_{mi},\Theta_{mi}\right). \tag{40}$$

Substitution of Eqs. (39) and (40)into Eqs. (28)-(31) and the boundary conditions (31)-(37) yields a sequence of equations for the unknown functions $W_i(z),\Theta_i(z),W_{mi}(z_m)$ and $\Theta_{mi}(z_m)$ for $i = 0,1,2,3......$.

At the leading order in a^2 Eqs. (28)-(31) become, respectively,

$$f(z)D^4W_0 + 2f'(z)D^3W_0 + f''(z)D^2W_0 = 0 \tag{41}$$

$$D^2\theta_0 = -N(z)W_0 \tag{42}$$

$$f\left(z_m\right)D_{m0}^2 w_{m0} + f'\left(z_m\right)D_{m0}w_{m0} = 0 \tag{43}$$

$$D_m^2\Theta_{m0} = W_{m0}N(z_m) \tag{44}$$

where

$$N(z) = \left[1 - Ns_f(1-2z)\right], \quad N(z_m) = \left[1 + Ns_m(1+2z_m)\right] \tag{45}$$

and the boundary conditions (32)-(38) become

$$W_0 = 0, \quad D\Theta_0 = 0, \quad DW_0 = 0 \ \text{ at } \ z = 1 \tag{46}$$

$$W_{m0} = 0, \quad D_m\Theta_{m0} = 0 \ \text{ at } \ z_m = -1 \tag{47}$$

and at the interface $z = 0$,

$$W_0 = \frac{\zeta}{\varepsilon_T}W_{m0} \tag{48}$$

$$\Theta_0 = \frac{\varepsilon_T}{\zeta}\Theta_{m0} \tag{49}$$

$$D\Theta_0 = D_m\Theta_{m0} \tag{50}$$

$$f(0)D^3W_0 + f'(0)D^2W_0 = -\frac{\zeta^4}{\varepsilon_T Da}f(0)D_mW_{m0} \tag{51}$$

$$D^2W_0 - \frac{\beta\zeta}{\sqrt{Da}}DW_0 = -\frac{\beta\zeta^3}{\varepsilon_T\sqrt{Da}}D_mW_{m0}. \tag{52}$$

The solution to the zeroth order Eqs. $(41)-(44)$is given by

$$W_0 = 0, \quad \Theta_0 = \frac{\varepsilon_T}{\zeta} \tag{53}$$

$$W_{m0} = 0, \quad \Theta_{m0} = 1. \tag{54}$$

At the first order in a^2 Eqs. (28) - (31) then reduce to

$$f(z)D^4W_1 + 2f'(z)\,D^3W_1 + f''(z)\,D^2W_1 = R\,\Theta_0 \tag{55}$$

$$D^2\theta_0 - \theta_0 = -N(z)W_1 \tag{56}$$

$$f\left(z_m\right)D_{m1}^2 w_{m0} + f'\left(z_m\right)D_{m0}w_{m1} = -R_m\Theta_{m0} \tag{57}$$

$$D_m^2\Theta_{m1} - \Theta_{m0} = W_{m1}N(z_m) \tag{58}$$

and the boundary conditions (32)-(38) become

$$W_1 = 0,\ D\Theta_1 = 0,\ DW_1 = 0 \quad at \quad z = 1 \tag{59}$$

$$W_{m1} = 0,\ D_m\Theta_{m1} = 0, \quad at \quad z_m = -1. \tag{60}$$

And at the interface($i.e\ z = 0$)

$$W_1 = \frac{1}{\zeta\varepsilon_T}W_{m1} \tag{61}$$

$$\Theta_1 = \frac{\varepsilon_T}{\zeta^3}\Theta_{m1} \tag{62}$$

$$D\Theta_1 = \frac{1}{\zeta^2}D_m\Theta_{m1} \tag{63}$$

$$f(0)D^3W_1 + f'(0)D^2W_1 = -\frac{\zeta^4}{\varepsilon_T Da}f(0)D_mW_{m1} \tag{64}$$

$$D^2W_1 - \frac{\beta\zeta}{\sqrt{Da}}DW_1 = -\frac{\beta\zeta^3}{\varepsilon_T\sqrt{Da}}D_mW_{m1}. \tag{65}$$

The general solution of Eqs.(55) and (57) are

$$W_1 = R\left[C_1 + C_2 z + C_3 e^{-Bz} + C_4\, z e^{-Bz} + \frac{\varepsilon_T z^2}{2\zeta B^2}Exp[-B\left(z - 1/2\right)]\right] \tag{66}$$

$$W_{m1} = R\left[C_5 + C_6 e^{-Bz_m} + \frac{\varepsilon_T^2 Da\, z_m}{\zeta^4 B}Exp[-B\left(z_m - 1/2\right)]\right] \tag{67}$$

Where C_1, C_2, C_3, C_4, C_5 and C_6 are constants and they have to determined using the appropriate boundary conditions.

$$C_1 = -\frac{e^{-B/2}}{2\sqrt{Da}\zeta^2}\left(\frac{e^{-B} + \zeta^4 - \varepsilon_T^2 Da}{L_1 - B^2 L_3}\right),\ C_2 = \frac{Da\, e^{-B/2}}{\zeta^4}\left(\frac{2 - 2e^B + B + Be^B}{L_2 - B^2\zeta\sqrt{Da}e^B - \zeta^2}\right)$$

$$C_3 = \frac{e^{B/2}}{2B^2}\frac{\left(\varepsilon_T\zeta - \varepsilon_T\zeta C_3 + C_6\right)}{L_1\varepsilon_T\zeta}, \qquad C_4 = \frac{e^{-B/2}}{2B^2}\frac{\left(\zeta^2 - \varepsilon_T\zeta C_2 + C_5\right)}{\varepsilon_T\zeta^4},\ C_5 = \frac{e^{B/2}}{B^2}\frac{\left(\varepsilon_T\zeta - \varepsilon_T\zeta C_1\right)}{L_4},$$

$$C_6 = \frac{e^{B/2}}{2B^2}\frac{\left(\varepsilon_T\zeta - \varepsilon_T\zeta C_4\right)}{L_1 + L_4 B^2 e^B}, \qquad L_1 = \left(5\zeta^2 B\sqrt{Da} + \beta\zeta^3\right) + Da\,\varepsilon_T^2,$$

$$L_2 = \frac{\left(B^2\zeta^2\sqrt{Da} - \beta\zeta^3 DaB\right)}{\beta Da\zeta^3\left(1 - e^B\right)}, \qquad L_3 = \left(L_2 B\sqrt{Da} - \frac{L_1\beta\zeta^3}{\varepsilon_T B^2}\right),\ L_4 = \left(\frac{B\varepsilon_T\beta\zeta^3}{Da} - \frac{\beta B\zeta^3 L_3}{\varepsilon_T\sqrt{Da}}\right)$$

Equations (56) and (58) involving $D^2\Theta_1$ and $D_m^2\Theta_{m1}$ respectively provide the solvability requirement which is given by

$$\int_0^1 N(z)W_1\,dz + \frac{1}{\zeta^2}\int_{-1}^0 N(z_m)W_{m1}\,dz = \frac{\varepsilon_T}{\zeta} + \frac{1}{\zeta^2}. \tag{68}$$

The expressions for W_1 and W_{m1} are back substituted into Eq. (68) and integrated to yield an expression for the critical Rayleigh number R_m^c, which is given by

$$R_m^c = \frac{\left(\frac{\varepsilon_T}{\zeta} + \frac{1}{\zeta^2}\right)\left(\frac{Da\,\varepsilon_T^2}{\zeta^4}\right)}{\left(k_1C_1 + k_2C_2 + k_3C_3 + k_4C_4 + k_5\right) + \frac{1}{\zeta^2}\left(C_5 - k_6C_6 + k_7\right)} \qquad (69)$$

where

$$k_1 = \left(\frac{BNs_f}{e^{-B/2}} + B^3\right),\ k_2 = \left[\frac{e^{-B/2}Ns_f}{B^2} + \frac{\left(B+5Ns_f\right)}{\left(1-e^{-B}\right)}\left(\frac{e^{-B/2}}{B} - \frac{e^{B/2}}{B^2}\right)\right]$$

$$k_3 = \left[\frac{e^{-B/2}Ns_f}{4B^2} + \frac{\left(B+2Ns_f\right)}{\left(1-e^{-B/2}+B\right)}\left(\frac{e^{B/2}}{B} - \frac{2e^B}{B^2} + \frac{2e^{-B/2}}{B^3}\right)\right]$$

$$k_4 = \left[\frac{2Ns_f}{B} + \frac{\left(B+2Ns_f\right)}{\left(B+B^2-e^{-B/2}\right)}e^{-B/2}\right], k_5 = \frac{2\varepsilon_T}{DaB\zeta}\left[\frac{2Ns_f}{B^2} + \frac{\left(B+4Ns_f\right)}{B^3}\left(\frac{e^{-B/2}}{B} - \frac{6e^{-B/2}}{B^2} + DaB^3\varepsilon_T\right)\right]$$

$$k_6 = \left[\frac{-Ns_m}{B} + \frac{\left(B+2Ns_m\right)}{\left(B-e^B\right)}\left(\frac{-e^{-B}}{B^2} - \frac{\left(e^{B/2}-B^3\right)}{B}\right)\right],\ k_7 = \left[\frac{5Ns_m}{5B^2} + \frac{e^{-B}}{B} + \frac{2e^{-B}}{B} + \frac{2\left(e^{-B}-1\right)}{B^3}\right].$$

The expression for R_m^c is evaluated for different values of various physical parameters and the results are discussed in detail in the next section.

4. RESULTS AND DISCUSSION

The variable viscosity effects on the onset of penetrative convection via internal heating is considered in a system consisting of a fluid layer overlying a porous layer. In the calculation, we have chosen the value of $\phi = 0.389$, C_b =209.25 and $\sqrt{Da} = 3.04\times10^{-3}$ which correspond to 3 cm deep porous layer consisting of 3mm diameter glass beads (Chen(1990)). The results are discussed for different depth ratios ζ .

We focus on the variable viscosity effects on the stability characteristics of the motionless fluid in the superposed layers configuration. The exponential model is applied because of its wider use for hydrogen-bonded liquids than the other models (Stengel et al. (1982); Chen and Pearlstein(1988) and Wooding(1957)). The exponential model is in the form

$$\mu(T) = B_0 \exp\left[-B\,T_B\right] \qquad (70)$$

where $B = \left(\frac{\nu_{\max}}{\nu_{\min}}\right)$ and T_B, is the dimensionless basic state temperature. We choose T_0, the basic state temperature at the interface, to be the reference temperature. And the following three different cases of internal heating pattern is considered for discussion namely,

Case - (i): internal heat source in the porous layer alone (i.e., $Ns_f = 0$, $Ns_m = 5$)

Case - (ii): internal heat sources in both fluid and porous layers (i.e., $Ns_f = 5$, $Ns_m = 5$) and

Case - (iii): internal heat source in the fluid layer alone (i.e., $Ns_f = 5$, $Ns_m = 0$).

4.1. Depth Ratio $\zeta >> 1$

This is the case of a pure fluid layer and the stability characteristic of the system is measured by the Rayleigh number. In the absence of internal heating ($Ns_f = 0$) and constant viscosity ($B = 0$), we recover the known exact value $R^c = 720$ (Sparrow et al.1964) is retrieved. It is observed that the in the absence of internal heat source ($Ns_f = 0$) the critical Rayleigh number R_c increases initially, with B reaches maximum and then decreases with further increase in the value of B. As a result of Fig. 2 some unusual behaviours are observed namely, (i) increasing variable viscosity parameter shows some stabilizing effect initially and (ii) increasing internal heat source strength causes stabilizing effect initially. Thus B increases, the bulk viscosity of the upper fluid layer increases relative to the fixed interfacial viscosity. It is known that larger viscosity makes the motionless state more stable, since viscosity is a stabilizing factor.

4.2. Depth Ratio $\zeta << 1$

This is the case of a pure porous layer and the stability characteristic of the system is measured by R_m. In the absence of internal heating ($Ns_m = 0$) and constant viscosity ($B = 0$), we recover the known exact value $R_m^c = 12$ (Nield and Bejan 2006). It is observed that the increasing both internal heat source strength and variable viscosity parameter causes destabilizing effect always (see Fig.3) because the lower porous layer predominates the system by convection hence system becomes less stable as B increases.

4.3. Depth Ratio $\zeta = 0.1$

The stability of the system is characterized by R_m^c. Figure.4 exhibit plot of R_m^c as a function of B for the above mentioned three cases of internal heating pattern. From Fig. 4 it is seen that for all the cases of internal heating pattern considered, the variable viscosity destabilizes the system because the lower porous layer predominates the system by convection hence system becomes less stable as B increases.

4.4. Depth Ratio $\zeta > 0.1$

The stability of the system is characterized by R_m^c. Figure 5 exhibit plot of R_m^c as a function of B for the above mentioned three cases of internal heating pattern for ζ = 0.2. From Figs. 4, it is observed that the system is stabilizing when $0 \le B \le 0.8$ and $0 \le B \le 0.7$ for $Ns_f = 5$ and $Ns_m = 0$ and $Ns_f = 5$ and $Ns_m = 5$, respectively and for higher values of B the system is destabilizing. For $Ns_f = 0$ and $Ns_m = 5$, the system is always destabilizing as the value of R_m^c decreases with B.

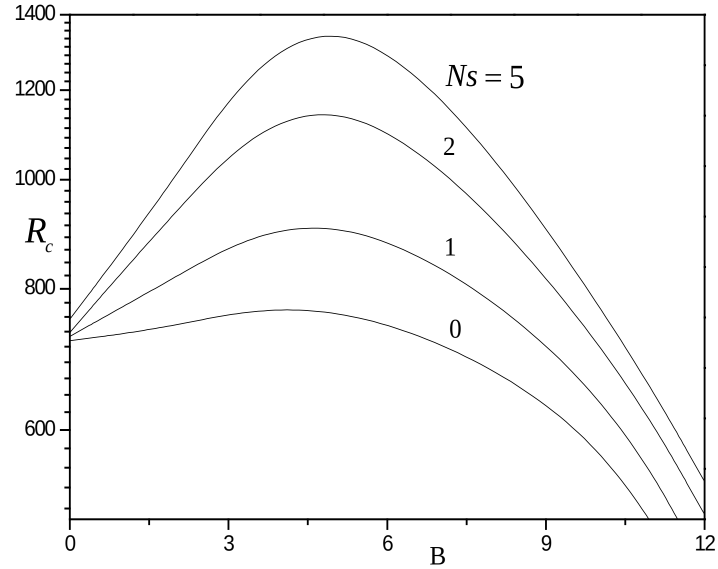

Fig. – 2. Critical Rayleigh number versus B for different values of Ns for $\zeta >> 1$

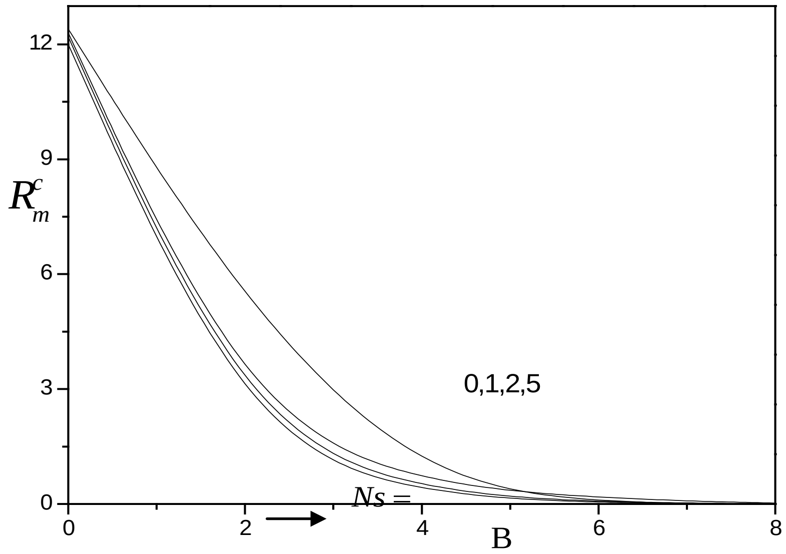

Fig. – 3. Critical Rayleigh number versus B for different values of Ns for $\zeta << 1$

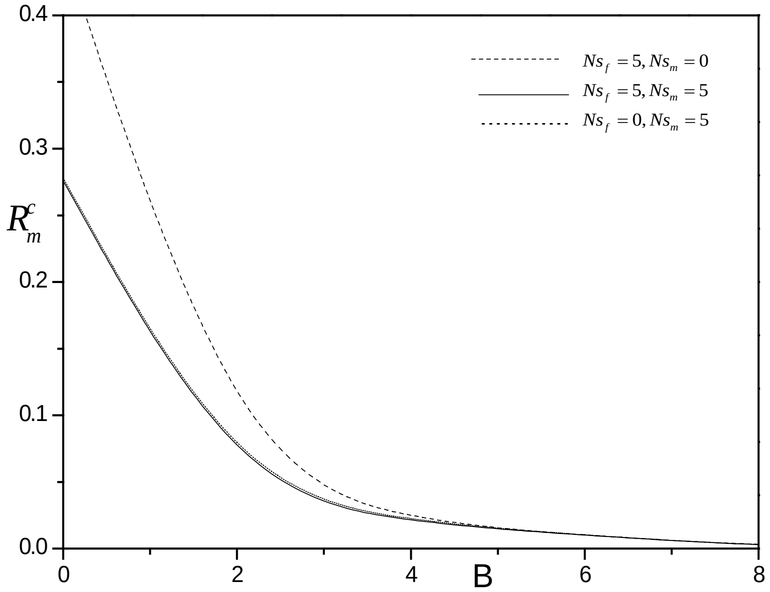

Fig. – 4. Critical Rayleigh number versus B for different values of Ns_f and Ns_m with $\zeta = 0.1$

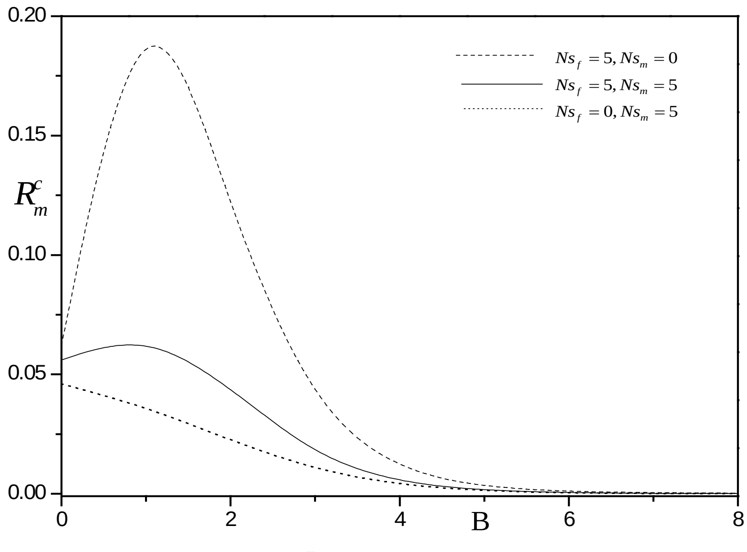

Fig. – 5. Critical Rayleigh number versus B for different values of Ns_f and Ns_m with $\zeta = 0.2$.

The findings for ζ =0.5,1,2 shown in Figs. 6, 7 and 8 for three cases of internal heating pattern are found to be similar to those of ζ =0.2 except the variation in the ranges of B in which the system gets stabilized. From Fig. 6 it is seen that the system is stabilizing when $0 \le B \le 2.4$ and $0 \le B \le 1.8$ for $Ns_f = 5$ and $Ns_m = 0$ and $Ns_f = 5$ and $Ns_m = 5$, respectively and for higher values of B the system is destabilizing. From Fig. 7, it is observed that the system is stabilizing when $0 \le B \le 5.8$ and $0 \le B \le 3.7$ for $Ns_f = 5$ and $Ns_m = 0$ and $Ns_f = 5$ and $Ns_m = 5$, respectively and for higher values of B the system is destabilizing. From Fig.8, it is observed that the system is stabilizing when $0 \le B \le 6.7$ and $0 \le B \le 5.8$ for $Ns_f = 5$ and $Ns_m = 0$ and $Ns_f = 5$ and $Ns_m = 5$, respectively and for higher values of B the system is destabilizing. For $Ns_f = 0$ and $Ns_m = 5$, the system is always destabilizing as the value of R_m^c decreases with B for all depth ratios considered.

Figure.9 depicts the perturbed vertical velocity eigen functions W and W_m for different values of Ns_f and Ns_m for $\zeta = 1$. The presence of volumetric heating has no noticeable influence on W_m and the presence of internal heating in the fluid layer alone and both fluid and porous layers is to accelerate W compared to its presence internal heating in the porous layer alone and absence of internal heating in both fluid and porous layers.

Figure.10. depicts the perturbed vertical velocity eigen functions W and W_m for $Ns_f = 0$ and $Ns_m = 0$ for different values of B with $\zeta = 1$. The presence and absence of B has no noticeable influence on W_m and the small values of B (*i.e* $B = 1$ and $B = 2$) is to accelerate W compared to its $B = 0$ and larger values of B (*i.e* $B = 5$).

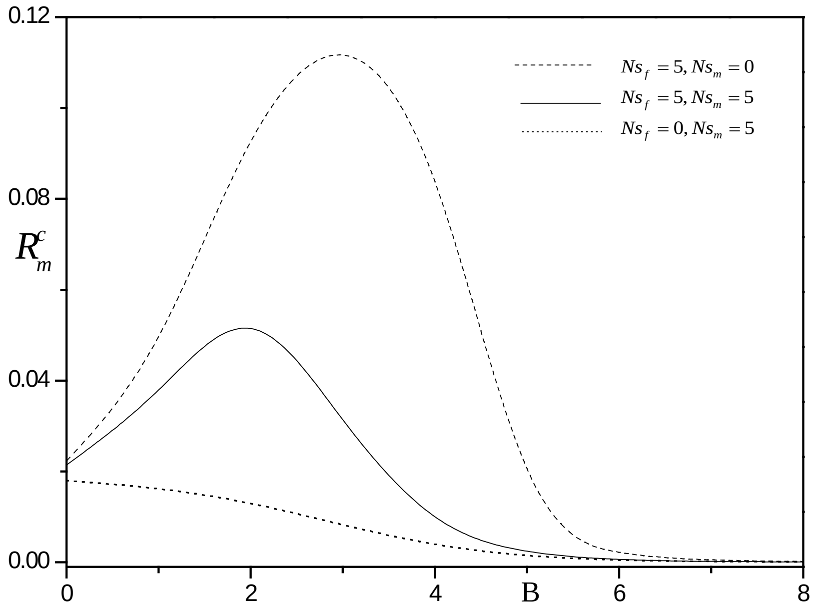

Fig. – 6. Critical Rayleigh number versus B for different values of Ns_f and Ns_m with $\zeta = 0.5$

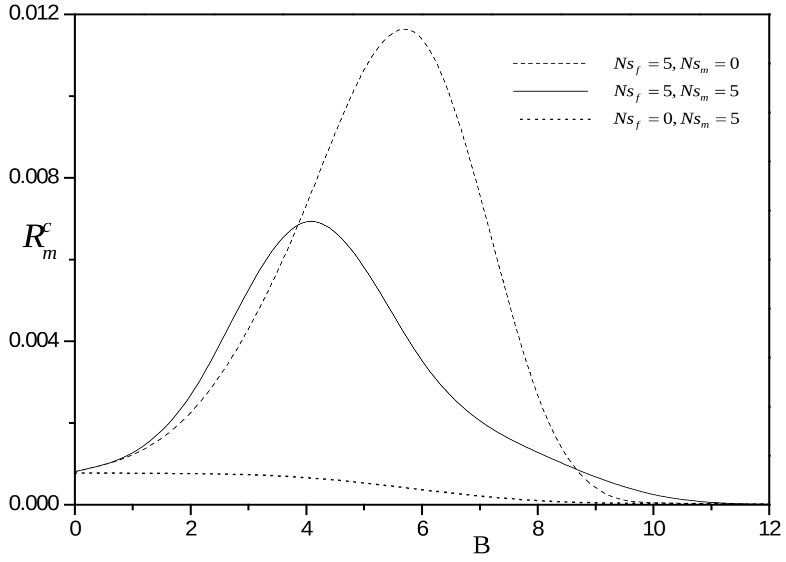

Fig.- 7. Critical Rayleigh number versus B for different values of Ns_f and Ns_m with $\zeta = 1$.

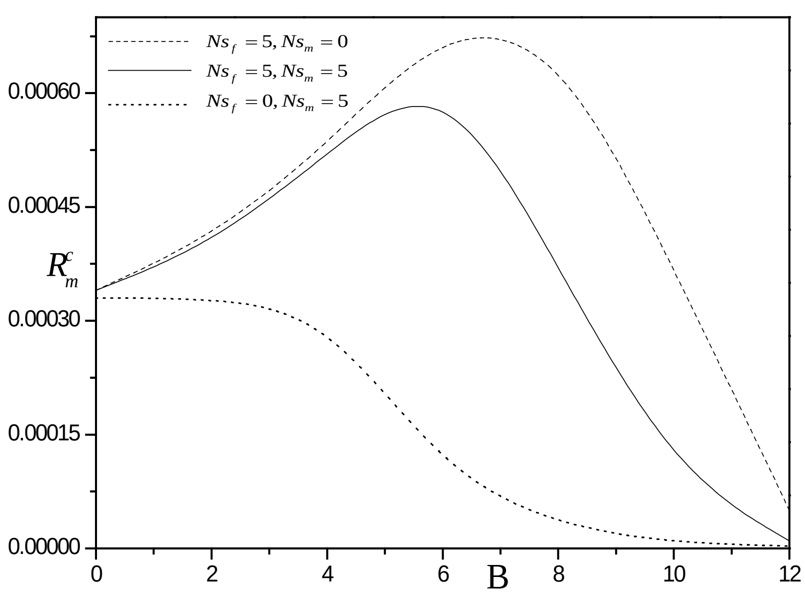

Fig. – 8. Critical Rayleigh number versus B for different values of Ns_f and Ns_m with $\zeta = 2$.

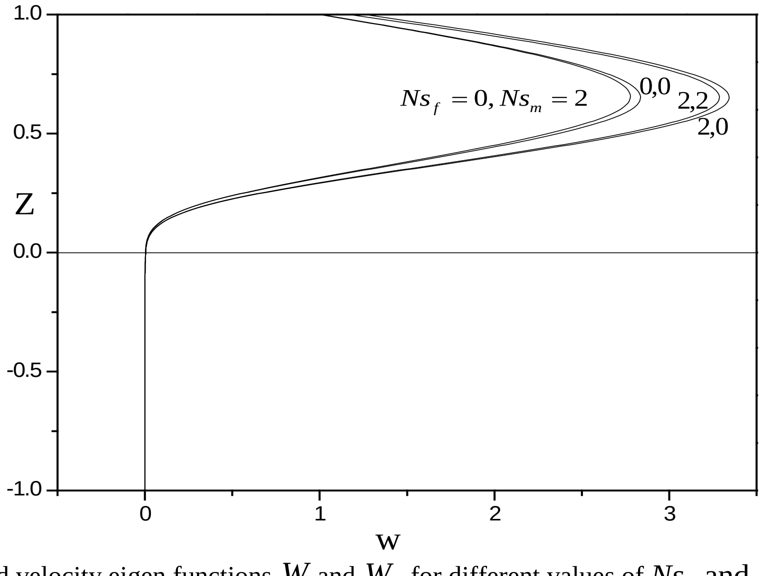

Fig. - 9. Perturbed velocity eigen functions W and W_m for different values of Ns_f and Ns_m with $\zeta = 1$.

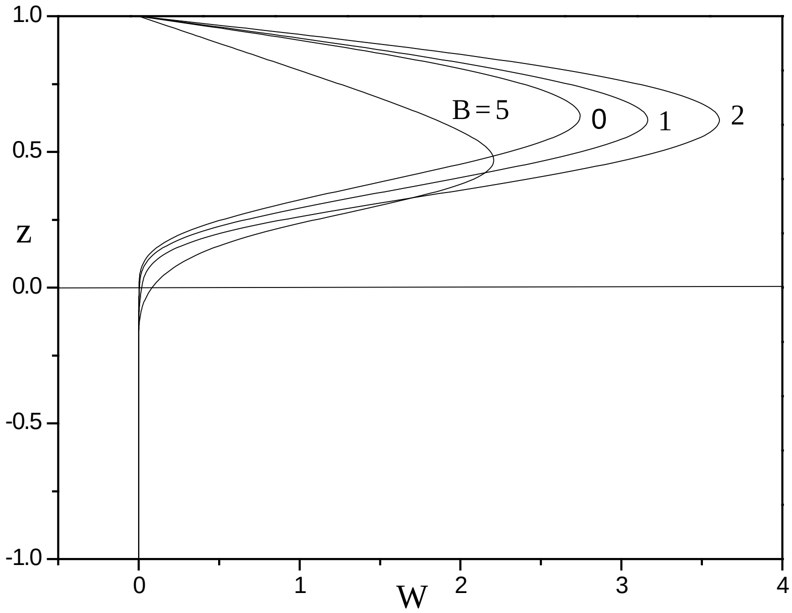

Fig. - 10. Perturbed velocity eigen functions W and W_m for different values of B with $\zeta = 1.$

5. CONCLUSIONS

The onset of penetrative convection via internal heating in superposed fluid and porous layers system is studied in the presence of a variable viscosity effects. From the foregoing analysis, it is observed that the stability characteristics of the configuration depend crucially on (i) the presence of internal heating in fluid and/or porous layer, (ii) depth ratio ζ and (iii) variable viscosity effects. For ζ = 0.1, the system is destabilizing for all the cases of internal heating pattern considered. To the contrary, the system is found to be more stabilizing if the fluid layer alone is heated internally and least stable if the both the layers are heated internally when ζ =0.2, 0.5,1and 2. In this case, the system is destabilizing for porous layer heated alone. Thus we note that the problem considered provide more precise control of thermal convective instability arising either in a porous layer or in a fluid layer by changing the internal heating pattern or the depth ratio ζ or the variable viscosity effects or considering all the effects together because both the stabilizing and destabilizing factors can be enhanced more in the combined porous and fluid layers system than for a single layer system.

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