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INFINITE BUFFER BULK SERVICE QUEUE WITH MULTIPLE VACATIONS

N. Sujatha*
Sr. Assistant Professor in Mathematics, Aditya Engineering College, Surampalem – 533437, Andhra Pradesh, India.

M. Srikanth
Sr. Assistant Professor in Mathematics, Aditya Engineering College, Surampalem – 533437, Andhra Pradesh, India.

M. Srinivasu
Sr. Assistant Professor in Mathematics, Sri Aditya Engineering College, Surampalem – 533437, Andhra Pradesh, India.

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ABSTRACT

In this paper, it is studied about infinite buffer bulk service queue $M/G^{(a,b)}/1$ with multiple vacations and some analytical results, relations among queue length distributions, computations of state probabilities at various epochs are included.

Keywords: *Bulk service queue, infinite buffer, multiple vacations, probability generating function, probability density function.*

1. INTRODUCTION

Queueing systems with vacations have wide applications in computer data communication systems, telecommunication engineering and manufacturing systems etc. The M/G/1 queue with vacations has been studied by several authors e.g. Fuhrmann and Cooper [1], Levy and Yechiali [5] and many others. Reddy and Anitha [6], [7] analyzed $M/G^{(a,b)}/1$ queue with 'M' different typers of vacation policies using a supplementary variable technique. Further they discussed $M/G^{(a,b)}/1$ queue with multiple vacations in which the first vacation is differently distributed from subsequent vacations. Lakshmi Srinivasan *et al* [4] have done an excellent analysis on $M/M^{(a,b)}/1$ queue with infinite buffer, multiple vacations and feed back. The infinite buffer bulk service queues such as $M/G^{(a,b)}/1$ queue with single vacation and $M/G^{(1,b)}/1$ queue with multiple vacations have been analyzed by Gupta and Sikdar [2], [3].

In this paper, we have analyzed an infinite buffer bulk service queue $M/G^{(a,b)}/1$ with multiple vacations which is an extension of $M/G^{(1,b)}/1$ queueing model with multiple vacations which have been analyzed by Gupta and Sikdar [2], [3]. In this model, customers are served by a single server in batches of maximum size b with a minimum threshold value a. When the server finishes serving a batch and finds less than a customers in the queue, the server goes away for a length of time called a vacation. This time may be utilized by the server to carry out some additional work. On return from a vacation if he finds n ($a \leq n \leq b$) customers waiting, he takes all of them for s ervice and if he finds more than b customers waiting, he takes maximum b of them for service and continue to do so until the queue length is less than a. However on return from a vacation, if he finds less than a customers waiting then he immediately proceeds for another vacation. We model the system using the supplementary variable method by considering remaining service time of the batch in service and remaining vacation time of the server as supplementary variables. The probability generating function of distributions of number of customers in the queue at service completion, vacation termination, arbitrary and departure epochs have been obtained. As relations among the queue length distributions at various epochs play an important role in the analysis of queueing models, we develop relations among arbitrary, departure, service completion and vacation termination epochs. We have also determined easily implementable procedure for computation of state probabilities at various epochs by using a combination of the supplementary variable and imbedded Markov chain technique.

*Corresponding author: N. Sujatha**
Sr. Assistant Professor in Mathematics, Aditya Engineering College, Surampalem – 533437, Andhra Pradesh, India.
E-mail: nandurisujatha@rediff.com

2. PRELIMINARIES AND BASIC NOTATIONS

2.1. Preliminaries

2.1.1 Bulk service queue: A queueing phenomenon in which customers are served in batches are called bulk service queues. For example, this type of queues occur frequently in loading and unloading of cargoes at a sea port, in traffic signal systems and in mass transportation system etc.

2.1.2 Infinite buffer: If the maximum number of customers that can be allowed into the system is infinite then the system is said to have infinite buffer queue.

2.1.3 Multiple vacations: In multiple vacation when the server finishes serving a customer and the queue is empty, he goes for a vacation. On return, if he finds one or more customers waiting, he serves the customers until the system empties then he takes another vacation. However, on return from a vacation if he finds no customer waiting, he immediately proceeds for another vacation and continues in this manner until at least one customer is waiting in the queue.

2.1.4 Probability generating function: Probability generating function of a sequence $\{a_n\}$ is denoted by A(z) and is given by $\boldsymbol{A(z) = \sum_{n=0}^{\infty} a_n z^n}$.

2.1.5 Steady State: If the system is independent of time then the system is said to be in steady state.

2.2. Basic notations

2.2.1 $\boldsymbol{f^{*j}(\lambda)}$: j^{th} derivative of $\boldsymbol{f^*(\theta)}$ at **θ=λ** where $\boldsymbol{f^*(\theta) = \int_0^{\infty} e^{-\theta x} f(x) dx}$.

2.2.2 gj: Probability that j customers arrive during a service time S of a batch. It is given by

$$\boldsymbol{g_j = \int_0^{\infty} \frac{(\lambda x)^j}{j!} e^{-\lambda x} dS(x) = \frac{(-\lambda)^j}{j!} S^{*(j)}(\lambda).}$$

2.2.3 $\mathbf{h_j}$: Probability that j customers arrive during a vacation time V of a batch. It is given by

$$\boldsymbol{h_j = \int_0^{\infty} \frac{(\lambda x)^j}{j!} e^{-\lambda x} dV(x) = \frac{(-\lambda)^j}{j!} V^{*(j)}(\lambda).}$$

2.2.4 $\boldsymbol{\phi_j}$: Probability that j customers arrive during an idle period. It is given by

$$\boldsymbol{\Phi_j = \frac{h_j}{1 - h_0}, j \geq 1.}$$

2.2.5 $\mathbf{K_{i,j}}$: Probability of having j customers at a departure epoch (t+1) of a batch if there are i customers at a departure epoch t of a batch.

2.2.6 $\mathbf{p_n^+}$: Steady state probability that n customers are left in the system at a departure epoch of a batch.

2.2.7 $\boldsymbol{\pi_n^+}$: Probability (n customers in the queue just prior to service completion epoch≥0 customers in the queue just prior to service completion or vacation termination epoch)

2.2.8 $\boldsymbol{\omega_n^+}$: Probability (n customers in the queue just prior to vacation termination epoch≥0 customers in the queue just prior to service completion or vacation termination epoch)

2.2.9 $\mathbf{p_n}$: Probability that there are n customers in the queue at an arbitrary epoch.

2.2.10 $\boldsymbol{\pi_n}$: Probability that there are n customers in the queue at service completion epoch.

2.2.11 $\boldsymbol{\omega_n}$: Probability that there are n customers in the queue at vacation termination epoch.

2.3. Some important lemma

Lemma: 2.3.1 The probability that the server is busy is given by

$$\boldsymbol{\rho^1 = \frac{E(S)k}{E(S)k + E(V)s_0}, a = 1.}$$

$$\rho^1 = \frac{E(S)k^2}{E(S)k^2 + E(V)(ks_1 + h_1 s_0)}, a = 2.$$

$$\rho^1 = \frac{E(S)k^3}{E(S)k^3 + E(V)(k^2 s_2 + kh_1 s_1 + s_0(h_2 k + {h_1}^2))}, a = 3.$$

$$\rho^1 = \frac{E(S)k^4}{E(S)k^4 + E(V)(k^3 s_3 + k^2 h_1 s_2 + ks_1(h_2 k + {h_1}^2) + s_0 m)}, a = 4.$$

where $k = 1 - h_0, s_i = \sum_{k=0}^{i} {p_k}^+, m = h_3 k^2 + 2h_1 h_2 + {h_1}^3, E(S) = -S^{*(1)}(0), E(V) = -V^{*(1)}(0)$.

Lemma: 2.3.2 The expression of σ in terms of ρ^1 is given by

$$\sigma = \frac{\rho^1 E(V) + (1-\rho^1)E(S)}{E(S)E(V)}.$$

Lemma: 2.3.3 The relation between $\{{\pi_n}^+\}$ and $\{{p_n}^+\}$ is given by

$${\pi_n}^+ = \frac{\rho^1}{\sigma E(S)} {p_n}^+, n \geq 0.$$

Here p_n^+ are obtained by solving the equations $p^+ P = p^+$ using GTH algorithm where P is one-step transition probability matrix which is given by

$$p_{ij} = \begin{cases} k_{i,j}, & 0 \leq i \leq (a-1), j \geq 0 \\ g_j, & a \leq i \leq b, j \geq 0 \\ g_{j-(i-b)}, & j \geq (i-b), i \geq (b+1), j \geq 0 \end{cases}$$

and $p_{ij} = 0$, in all the other cases. Here

$$k_{i,j} = \begin{cases} \sum_{n=a-i}^{b-i} \Phi_n g_j, & 0 \leq i \leq (a-1), j \geq 0 \\ \sum_{n=a-i}^{b-i} \Phi_n g_j + \sum_{n=b-i+1}^{b-i+j} \Phi_n g_{j-(n-b+i)}, & 0 \leq i \leq (a-1), j \geq 1 \end{cases}$$

Lemma: 2.3.4 In steady state, we have

$${\omega_n}^+ = \sum_{k=0}^{n} \frac{{\pi_k}^+}{1-h_0} h_{n-k} + \sum_{k=0}^{n-1} \frac{{\omega_k}^+}{1-h_0} h_{n-k}, 0 \leq n \leq (a-1),$$

$${\omega_n}^+ = \sum_{k=0}^{a-1} ({\pi_k}^+ + {\omega_k}^+) h_{n-k}, n \geq a.$$

Lemma: 2.3.5 Relation between $\{\omega_n\}, \{{\omega_n}^+\}, {\pi_0}^+$ is given by

$$\omega_n = \frac{\sigma}{\lambda} \sum_{i=0}^{n} {\pi_i}^+, 0 \leq n \leq (a-1),$$

$$\omega_n = \frac{\sigma}{\lambda} \left(\sum_{i=0}^{a-1} {\pi_i}^+ - \sum_{j=a}^{n} {\omega_j}^+ \right), n \geq a.$$

Lemma: 2.3.6 Relation between $\{{\pi_n}^+\}$, $\{{\omega_n}^+\}$, $\{\pi_n\}$ is given by

$$\pi_0 = \frac{\sigma}{\lambda} \left(\sum_{i=a}^{b} ({\pi_i}^+ + {\omega_i}^+) - {\pi_0}^+ \right),$$

$$\pi_n = \frac{\sigma}{\lambda} \left(\sum_{i=a}^{b+n} ({\pi_i}^+ + {\omega_i}^+) - \sum_{i=0}^{n} {\pi_i}^+ \right), n \geq 1.$$

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Lemma: 2.3.7 Relation between $\{\boldsymbol{p_n}\}$ and $\{\boldsymbol{\pi_n}^+ + \boldsymbol{\omega_n}^+\}$ is given by

$$\boldsymbol{p_n} = \frac{\boldsymbol{\sigma}}{\boldsymbol{\lambda}} \sum_{\boldsymbol{i=a}}^{\boldsymbol{b+n}} (\boldsymbol{\pi_i}^+ + \boldsymbol{\omega_i}^+), \boldsymbol{0 \le n \le (a-1)},$$

$$\boldsymbol{p_n} = \frac{\boldsymbol{\sigma}}{\boldsymbol{\lambda}} \sum_{\boldsymbol{i=n+1}}^{\boldsymbol{b+n}} (\boldsymbol{\pi_i}^+ + \boldsymbol{\omega_i}^+), \boldsymbol{n \ge a}.$$

3. PERFORMANCE MEASURES AND NUMERICAL RESULTS

3.1. Performance measures

We calculate various performance measures such as average queue length $\boldsymbol{L_q} = \sum_{\boldsymbol{n=0}}^{\boldsymbol{l}} \boldsymbol{np_n}$, average queue length when the server is busy , $\boldsymbol{L_{q1}} = \sum_{\boldsymbol{n=0}}^{\boldsymbol{l}} \boldsymbol{n\pi_n}$, average queue length when the server is on vacation $\boldsymbol{L_{q0}} = \sum_{\boldsymbol{n=0}}^{\boldsymbol{l}} \boldsymbol{n\omega_n}$, probability that the server is busy is given by $\boldsymbol{\rho^1} = \sum_{\boldsymbol{n=0}}^{\boldsymbol{l}} \boldsymbol{\pi_n}$, probability that the server is on vacation is given by $\boldsymbol{\rho^1} = \sum_{\boldsymbol{n=0}}^{\boldsymbol{l}} \boldsymbol{\omega_n}$.

3.2. Numerical results

We have performed computational analysis of the model for a combination of various service and vacation time distributions viz. exponential (M), deterministic (D), Erlang (E_k, each phase has mean 1/μ). Here, we present numerical results in the form of self explanatory tables. All calculations have been done in double precision but they have been reported here in six decimal places. Notations used in the tables are the same as one used in this paper. The probability distributions of number of customers in the queue (pmf) and their cumulative (cum) probabilities at various epochs are presented in Tables 1 – 4. We have presented state probabilities only for few values of n ($n \ge 0$) in various columns of the Tables. Also various performance measures such as average queue lengths $\mathbf{L_q}$, $\mathbf{L_{q1}}$, $\mathbf{L_{q0}}$ and the probability that the server is busy (ρ^1) are presented at the bottom of the Tables.

In Tables 3.2.1 and 3.2.2, the results are given for $M/M^{(1,4)}/1$ and $M/M^{(4,4)}/1$ where vacation time follows deterministic distribution. The results of $M/E_2^{(1,4)}/1$ and $M/E_2^{(4,4)}/1$ where vacation time is deterministically distributed are given in Tables 3.2.3 and 3.2.4.

In Figure 1, we have shown the effect of λ on average queue length in different cases of service and vacation time distributions. It can be observed that as λ increases, average queue length increases.

In Figure 2, the effect of μ on average queue length has been shown in different cases. It can be observed that as μ increases, average queue length increases.

Figure 3 shows the effect of mean vacation time on average queue length. It can be observed that as the mean vacation time increases average queue length increases.

Figure 4 shows the effect of minimum threshold value on the probability that the server is busy for different cases of service and vacation time distributions. It can be observed from this figure as a increases, ρ^1 approaches ρ and in the case when a = b, i.e., for fixed batch size ρ and ρ^1 are equal.

Table - 3.2.1: Queue length distributions at various epochs for $\mathbf{M/M^{(1,4)}/1}$ (vacation is deterministically distributed) with E(V)=2.6, **λ**=7.6, E(S)=0.345423 and ρ=0.656304.

n	p_n+(pmf)	p_n+(cum)	π_n+(pmf)	π_n+(cum)	w_n+(pmf)	w_n+(cum)
0	5.422e-02	5.422e-02	5.143e-02	5.143e-02	1.348e-10	1.348e-10
4	4.675e-02	2.508e-01	4.434e-02	2.379e-01	8.561e-07	1.059e-06
6	4.457e-02	3.409e-01	4.227e-02	3.234e-01	1.114e-05	1.558e-05
8	4.297e-02	4.276e-01	4.076e-02	4.056e-01	7.769e-05	1.247e-04
10	4.160e-02	5.115e-01	3.945e-02	4.852e-01	3.371e-04	6.324e-04
40	9.221e-04	9.952e-01	8.746e-04	9.440e-01	1.121e-06	5.143e-02
60	2.720e-05	9.999e-01	2.580e-05	9.484e-01	9.050e-15	5.143e-02
80.	8.021e-07.	1.000e-00.	7.609e-07.	9.486e-01.	8.666e-26.	5.143e-02

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n	π_n (pmf)	π_n (cum)	w_n(pmf)	w_n (cum)	p_n(pmf)	p_n(cum)
0	3.851e-02	3.851e-02	1.467e-02	1.467e-02	5.318e-02	5.318e-02
4	3.320e-02	1.781e-01	1.467e-02	7.334e-02	4.787e-02	2.515e-01
6	3.165e-02	2.421e-01	1.466e-02	1.027e-01	4.631e-02	3.448e-01
8	3.051e-02	3.037e-01	1.463e-02	1.320e-01	4.515e-02	4.356e-01
10	2.954e-02	3.632e-01	1.449e-02	1.610e-01	4.402e-02	5.243e-01
40	6.548e-04	7.068e-01	2.861e-07	2.898e-01	6.551e-04	9.966e-01
60	1.931e-05	7.100e-01	1.223e-15	2.898e-01	1.931e-05	9.999e-01
80.	5.696e-07.	7.102e-01.	1.979e-18.	2.898e-01.	5.696e-07.	1.000e-00

ρ^1= 0.710157, Lq = 11.021, $\mathbf{Lq_1}$= 8.157, $\mathbf{Lq_0}$ = 2.864.

Table - 3.2.2: Queue length distributions at various epochs for M/M $^{(4,4)}$/1 (vacation time is deterministically distributed) with E(V) = 2.6, λ=7.6, E(S)=0.345423 and ρ=0.656304.

n	p_n+(pmf)	p_n+(cum)	π_n+(pmf)	π_n+(cum)	w_n+(pmf)	w_n+(cum)
0	8.175e-03	8.175e-03	7.643e-03	7.643e-03	2.003e-11	2.003e-11
4	2.963e-02	9.921e-02	2.771e-02	9.276e-02	1.857e-07	2.249e-07
6	3.586e-02	1.681e-01	3.353e-02	1.572e-01	2.984e-06	4.023e-06
8	4.021e-02	2.465e-01	3.760e-02	2.305e-01	2.592e-05	3.934e-05
10	4.316e-02	3.315e-01	4.035e-02	3.100e-01	1.407e-04	2.436e-04
40	1.624e-03	9.916e-01	1.518e-03	9.271e-01	6.482e-06	6.504e-02
60	4.791e-05	9.998e-01	4.480e-05	9.347e-01	1.519e-13	6.505e-02
80.	1.413e-06.	1.000e-00.	1.321e-06.	9.349e-01.	3.215e-24.	6.505e-02

n	π_n (pmf)	π_n (cum)	ω_n(pmf)	ω_n (cum)	p_n(pmf)	p_n(cum)
0	5.365e-03	5.365e-03	2.044e-03	2.044e-03	7.409e-03	7.409e-03
4	1.945e-02	6.511e-02	1.739e-02	5.363e-02	3.684e-02	1.187e-01
6	2.353e-02	1.103e-01	1.739e-02	8.841e-02	4.093e-02	1.987e-01
8	2.639e-02	1.618e-01	1.738e-02	1.232e-01	4.378e-02	2.850e-01
10	2.833e-02	2.176e-01	1.733e-02	1.579e-01	4.565e-02	3.755e-01
40	1.066e-03	6.508e-01	1.742e-06	3.437e-01	1.068e-03	9.945e-01
60	3.144e-05	6.561e-01	2.055e-14	3.437e-01	3.144e-05	9.998e-01
80.	9.274e-07.	6.563e-01.	7.422e-18.	3.437e-01.	9.274e-07.	1.000e-00.

ρ^1= 0.656304, **Lq** = 13.945, $\mathbf{Lq_1}$= 9.891, $\mathbf{Lq_0}$ = 4.055.

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Table - 3.2.3: Queue length distributions at various epochs for $M/E_2^{(1,4)}/1$ (vacation time is deterministically distributed) with E(V) = 2.5, λ=7.5, E(S)=0.266667 and ρ=0.5.

n	p_n +(pmf)	p_n +(cum)	π_n + (pmf)	π_n +(cum)	ω_n + (pmf)	ω_n +(cum)
0	7.880e-02	7.880e-02	7.304e-02	7.304e-02	5.255e-10	5.255e-10
4	6.334e-02	3.940e-01	5.871e-02	3.652e-01	2.706e-06	3.386e-06
6	4.982e-02	4.992e-01	4.618e-02	4.627e-01	3.171e-05	4.525e-05
8	4.344e-02	5.887e-01	4.026e-02	5.457e-01	1.991e-04	3.293e-04
10	4.015e-02	6.704e-01	3.721e-02	6.214e-01	7.777e-04	1.522e-03
20	1.486e-02	9.524e-01	1.377e-02	8.829e-01	6.229e-03	4.884e-02
40	7.991e-06	9.999e-01	7.407e-06	9.269e-01	5.357e-07	7.304e-02
60.	5.488e-10.	1.000e-00.	5.087e-10.	9.270e-01.	1.515e-15.	7.304e-02.

n	π_n (pmf)	π_n (cum)	ω_n(pmf)	ω_n (cum)	p_n(pmf)	p_n(cum)
0	6.798e-02	6.798e-02	2.266e-02	2.266e-02	9.064e-02	9.064e-02
4	3.343e-02	2.448e-01	2.265e-02	1.133e-01	5.068e-02	3.581e-01
6	2.704e-02	3.014e-01	2.264e-02	1.586e-01	4.969e-02	4.600e-01
8	2.424e-02	3.510e-01	2.256e-02	2.038e-01	4.680e-02	5.548e-01
10	2.267e-02	3.971e-01	2.219e-02	2.484e-01	4.486e-02	6.455e-01
20	7.642e-03	5.521e-01	7.508e-03	4.055e-01	1.515e-02	9.576e-01
40	3.220e-06	5.751e-01	1.354e-07	4.249e-01	3.355e-06	9.999e-01
60.	2.181e-10.	5.751e-01.	2.153e-16.	4.249e-01.	2.181e-10.	1.000e-00.

ρ^1= 0.5751, **Lq** = 8.313, $\mathbf{Lq_1}$= 4.329, $\mathbf{Lq_0}$ = 3.983.

Table - 3.2.4: Queue length distributions at various epochs for $\mathbf{M/E_2^{(4,4)}/1}$ (vacation time is deterministically distributed) with E(V) = 2.5, λ=7.5, E(S)=0.266667 and ρ=0.5.

n	p_n +(pmf)	p_n +(cum)	π_n + (pmf)	π_n +(cum)	ω_n +(pmf)	ω_n +(cum)
0	1.114e-02	1.114e-02	1.006e-02	1.006e-02	7.239e-11	7.239e-11
4	4.454e-02	1.512e-01	4.025e-02	1.366e-01	5.798e-07	7.072e-07
6	4.978e-02	2.487e-01	4.498e-02	2.247e-01	8.644e-06	1.180e-05
8	5.186e-02	3.516e-01	4.686e-02	3.177e-01	6.949e-05	1.075e-04
10	5.238e-02	4.563e-01	4.733e-02	4.123e-01	3.478e-04	6.193e-04
20	2.856e-02	8.904e-01	2.581e-02	8.046e-01	8.658e-03	4.782e-02
40	2.961e-05	9.999e-01	2.676e-05	9.036e-01	3.731e-06	9.638e-02
60	2.106e-09.	1.000e-00.	1.903e-09.	9.036e-01.	3.097e-14.	9.639e-02.

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n	π_n (pmf)	π_n (cum)	ω_n(pmf)	ω_n (cum)	p_n(pmf)	p_n(cum)
0	8.351e-03	8.351e-03	2.784e-03	2.784e-03	1.114e-02	1.114e-02
4	2.347e-02	8.674e-02	2.667e-02	8.140e-02	5.014e-02	1.681e-01
6	2.540e-02	1.368e-01	2.667e-02	1.347e-01	5.207e-02	2.715e-01
8	2.613e-02	1.888e-01	2.664e-02	1.880e-01	5.276e-02	3.768e-01
10	2.622e-02	2.412e-01	2.650e-02	2.411e-01	5.271e-02	4.823e-01
20	1.319e-02	4.523e-01	1.344e-02	4.567e-01	2.663e-02	9.091e-01
40	1.048e-05	0.500e-00	9.414e-07	0.500e-00	1.142e-05	9.999e-01
60.	7.277e-10.	0.500e-00.	4.032e-15.	0.500e-00.	7.277e-10.	1.000e-00.

ρ^1= 0.5, **Lq** = 11.323, $\mathbf{Lq_1}$= 5.661, $\mathbf{Lq_0}$ = 5.661.

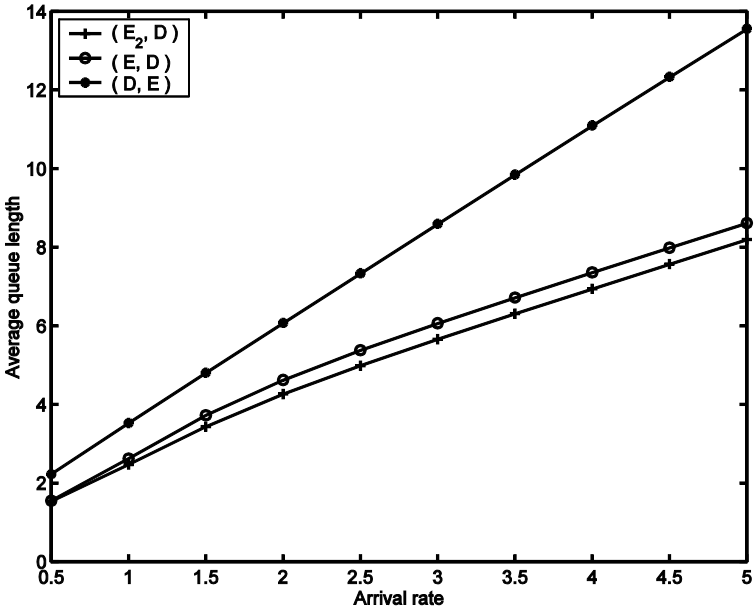

Figure - 1: Effect of λ on average queue length for b=4, ρ = 0.5

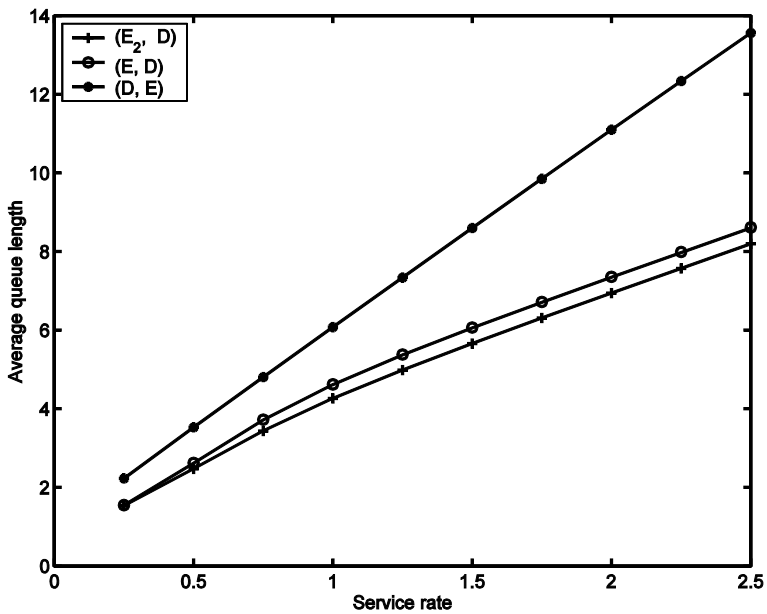

Figure - 2: Effect of μ on average queue length for b = 4 and ρ = 0.5

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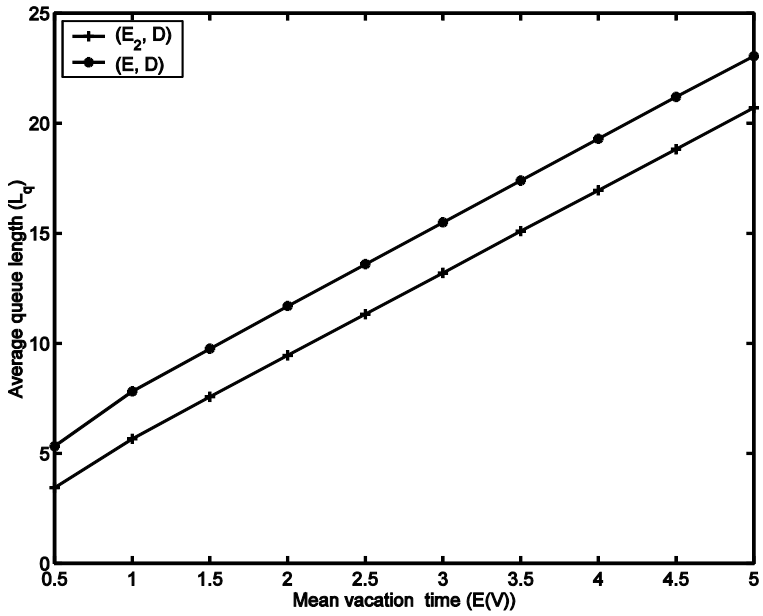

Figure - 3: Effect of mean vacation time on average queue length.

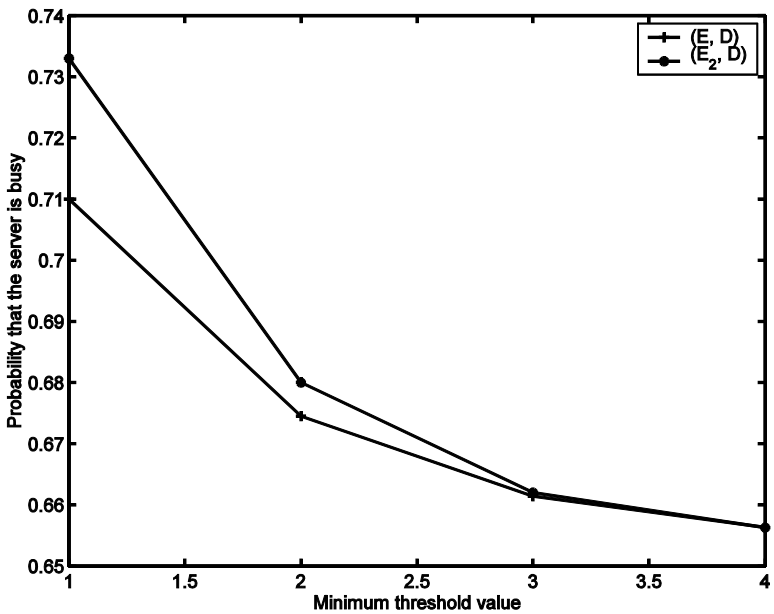

Figure - 4: Effect of minimum threshold value on the probability that the server is busy for b =4 and ρ = 0.656304.

3.3. CONCLUSIONS

In this paper, we have tried to analyze $\mathbf{M/G^{(a,b)}/1}$ queue with multiple vacations for specific values of a. Using the supplementary variable method, we obtain the probability generating function (p.g.f.) of queue length distributions at service completion, vacation termination, departure and arbitrary epochs. Also, the relations among queue length distributions at various epochs are obtained. This model can be more generalized i.e., it can be extended for higher values of a.

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