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ON rαg -I-CONTINUOUS IN TOPOLOGICAL SPACES

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ABSTRACT

In this paper we introduce and study the notions of rαg-closed sets and rαg-continuous and rαg -irresolute in Ideal topological spaces.

Keywords: *α-I-closed, αg-I-closed, rαg-I-closed, rαg-I-continuous and rαg-I-irresolute.*

1. INTRODUCTION AND PRELIMINARIES

An ideal I on a topological space (X,τ) is a non-empty collection of subsets of X which satisfies the following properties. (1) A∈I and B⊆A implies B∈I, (2) A∈I and B∈I implies A∪B∈I. An ideal topological space is a topological space (X,τ) with an ideal I on X and is denoted by (X, τ, I). For a subset A⊆X, A*(I,τ)={x ∈ X: A∩U ∉I for every U∈τ (X,x)} is called the local function of A with respect to I and τ [3]. We simply write A* in case there is no chance for confusion. A kuratowski closure operator cl*(.) for a topology τ*(I,τ) called the *- topology, finer than τ is defined by cl*(A) = A∪A* [9]. If A ⊆ X, cl (A) and int (A) will respectively, denote the closure and interior of A in (X, τ).

A subset A of a topological space (X,τ) is said to be regular-open [8] (resp.regular-closed [8], α-open [5], α-closed [5]), if A=int (cl(A)) (resp. A=cl(int(A)), A ⊆ int(cl(int(A))), cl(int(cl(A))) ⊆ A). A subset A of a topological space (X,τ) is αg-closed [4] (resp. rαg-closed), if αcl (A)⊆U whenever A⊆U and U is open (resp. regular open) in X. A function $f:(X,\tau)\rightarrow(Y,\sigma)$ is said to be α-continuous [6] (resp. αg-continuous [2], rαg-continuous), if $f^{-1}(A)$ is α-closed (resp. αg-closed, rαg-closed) in (X,τ) for every closed set A in (Y, σ). A subset A of an ideal space (X,τ,I) is α-I-closed [1] (resp. α-I-open [1]), if cl(int*(cl(A))) ⊆ A (resp. A⊆ int (cl*(int (A)))). A subset A of an ideal space (X,τ,I) is αg-I-closed [7], if αIcl (A) ⊆ U whenever A⊆U and U is open in X. A function $f:(X,\tau,I)\rightarrow(Y,\sigma)$ is α-I-continuous [1] (resp. αg-I-continuous [7]), if $f^{-1}(A)$ is α-I-closed (resp. αg-I-closed) in (X,τ,I) for every closed set A in (Y,σ).

2. rαg-I-closed

Definition: 2.1 A subset A of an Ideal topological space (X, τ, I) is said to be rαg-I-closed, if αIcl (A) ⊆U whenever A⊆U and U is regular-open to X.

Remark: 2.2 Suppose I= {ϕ} then the notion of rαg-I-closed set coincide with rαg-closed set.

Theorem: 2.3 Every α-I-closed set is rαg-I-closed set.

Proof: Let U be any regular open set and A⊆U. Since A is α-I-closed, αIcl(A)=A implies αIcl(A)⊆U. Therefore A is rαg-I-closed.

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Example: 2.4 Let X= {a, b, c}, τ = {ϕ, a, X} and I= {ϕ, c}. Then A= {a, b} is rαg-I-closed but not α-I-closed.

Theorem: 2.5 Every αg-I-closed set is rαg-I-closed set.

Proof: Let A be an αg-I-closed set, then αIcl(A)⊆U where A⊆U and U is open. Since every regular open set is open and hence U is regular open. Therefore A is rαg-I-closed set.

Example: 2.6 Let X= {a, b, c}, τ= {ϕ, a, X} and I= {ϕ, c}. Then A= {a} is rαg-I-closed but not αg-I-closed.

Theorem: 2.7 Every rαg-I-open set is rαg-open set.

Proof: Let A be an rαg-I-open set in (X, τ, I). Then we have U⊆α-I int(A) whenever U⊆A and U is regular closed in (X, τ, I) = A∩int(cl^*(int(A)))⊆ int(cl(int(A))) = α int(A).

This shows that A is rαg-open.

Theorem: 2.8 If A ∈ rαg-I-closed, B ∈ rαg-I-closed, then A∪B ∈ rαg-I-closed.

Proof: Let A⊆U, B⊆U and U is regular open in X. Since A is rαg-I-closed, αIcl(A)⊆U and B is rαg-I-closed αIcl(A)⊆U, also αIcl(A∪B) = αIcl(A) ∪ αIcl(B) ⊆ U. Therefore αIcl(A∪B)⊆U. Therefore A∪B is rαg-I-closed.

Theorem: 2.9 If a subset A of X is rαg-I-closed set in X. Then $\alpha Icl(A)\setminus A$ does not contain any nonempty regular open set in X .

Proof: Supose that A is rαg-I-closed set in X. We prove the result by contradicicition. Let U be a regular open set such that $\alpha Icl(A)\setminus A \supset U$ and $U \neq \phi$. Now $U \subset \alpha Icl(A)\setminus A$. Therefore $U \subset X \setminus A$ which implies $A \subset X \setminus U$. since U is regular open set, $X \setminus U$ is also regular open set in X. Since A is regular open set in X , by definition we have $\alpha Icl(A) \subset X \setminus U$. So $U \subset X \setminus \alpha Icl(A)$. Also $U \subset \alpha Icl(A)$ .Therefore $U \subset (\alpha Icl(A) \cap (X \setminus \alpha Icl(A))) = \phi$. This shows that, $U = \phi$ which contradiction is. Hence $\alpha Icl(A)\setminus A$ does not contain any nonempty regular open set in X .

Theorem: 2.10 For an element $x \in X$, the set $X \setminus \{x\}$ is rαg-I-closed or regular open.

Proof: Suppose $X \setminus \{x\}$ is not regular open set. Then X is the only regular open set containing $X \setminus \{x\}$. This implies $\alpha Icl(X \setminus \{x\}) \subset X$. Hence $X \setminus \{x\}$ is rαg-I-closed set in X.

Theorem: 2.11 If A is rαg-I-closed subset of X such that $A \subset B \subset \alpha Icl(A)$. Then B is rαg-I-closed set in X.

Proof: If A is rαg-I-closed subset of X such that $A \subset B \subset \alpha Icl(A)$. Let U be a regular open set of X such that $B \subset U$. Then $A \subset U$. Since A is rαg-I-closed we have $\alpha Icl(A) \subset U$. Now $\alpha Icl(B) \subset \alpha Icl(\alpha Icl(A)) = \alpha Icl(A) \subset U$. Therefore B is rαg-I-closed subset of X.

Theorem: 2.12 If A is both open and g-closed set in X, then it is rαg-I-closed set in X.

Proof: Let A be an open and g-closed set in X. Let $A \subset U$ and let U be a regular open set in X. Now $A \subset A$. By hypothesis $\alpha Icl(A) \subset A$. That is $\alpha Icl(A) \subset U$. Thus A is rαg-I-closed set in X.

Theorem: 2.13 Every singleton point set in a space is either rαg-I-open or regular open.

Proof: Let X be a topological space. Let $x \in X$. To prove $\{x\}$ is either rαg-I-open or regular open. That is to prove $X - \{x\}$ is either rαg-I-closed or regular open, which follows from theorem 2.10.

Remark: 2.10 For the subsets defined above we have the following implications.

$$\begin{tikzcd}
 & \alpha\text{-I-closed} \arrow[r] \arrow[d] & \alpha g\text{-I-closed} \arrow[r] \arrow[d] & r\alpha g\text{-I-closed} \arrow[d] \\
\text{Closed} \arrow[ur] \arrow[dr] & & & \\
 & \alpha\text{-closed} \arrow[r] & \alpha g\text{-closed} \arrow[r] & r\ \alpha g\text{-closed}
\end{tikzcd}$$

None of the implications is reversible.

Example: 2.11

i) Let X={a, b, c}, τ={ϕ, a,{a, b},X} and I ={ϕ, b, c,{b, c}}. Then A={b} is α-I-closed but not closed.
ii) Let X={a, b, c}, τ={ϕ, b,{b, c},X} and I={ϕ, b}. Then A={a,b} is αg-I-closed but not α-I-closed.
iii) Let X={a, b, c}, τ={ϕ, a, X} and I={ϕ, c}. Then A={a} is rαg-I-closed but not αg-I-closed.

3. rαg-I-continuous and rαg-I-irresolute

Definition: 3.12 A function f:(X, τ, I)→(Y, σ) is said to be rαg-I-continuous, if $f^{-1}(A)$ is rαg-I-open in (X, τ, I) for every open set is A is (Y, σ).

Remark: 3.13 If I = {ϕ} then the notion of rαg-I-continuous coincides with the notion of rαg-continuous.

Theorem: 3.14 For a function f: (X, τ, I)→(Y, σ), the following hold.

i) Every continuous function α-I-continuous.
ii) Every α-I-continuous function is rαg-I-continuous.
iii) Every αg-I-continuous function is rαg-I-continuous.

Proof:

i) The proof is obvious.
ii) Let f be a α-I-continuous function and V be a closed set in (Y, σ), then $f^{-1}(V)$ is α-I-closed in (X,τ,I). Since every α-I-closed set is rαg-I-closed, $f^{-1}(V)$ is rαg-I-closed in (X, τ, I). Therefore f is rαg-I-continuous.
iii) Let f be αg-I-continuous function and V be closed in (Y, σ), then $f^{-1}(V)$ is αg-I-closed in (X, τ, I). Since every αg-I-closed set is rαg-I-closed, $f^{-1}(V)$ is rαg-I-closed in (X, τ, I). Therefore f is rαg-I-continuous.

Example: 3.16

i) Let X={a, b, c}, τ={ϕ, a,{a, b},X} and I={ϕ, b, c,{b, c}} and σ={ϕ, b, X}. Let the function f: (X, τ, I)→ (Y, σ) be defined by Identity map. Then the function f is α-I-continuous but not continuous.
ii) Let X={a, b, c}, τ={ϕ, a, {a, b},X} and I={ϕ, a} and σ={ϕ,{a, c}, X}. Let the function f: (X, τ, I)→ (Y, σ) be defined by Identity map. Then the function f is αg-I-continuous but not α-I-continuous.
iii) Let X={a, b, c}, τ={ϕ, a,{a, c},X} and I={ϕ, b, c,{b, c}} and σ={ϕ, a,{a, c], X}. Let the function f: (X, τ, I)→ (Y, σ) be defined by Identity map. Then the function f is rαg-I-continuous but not αg-continuous.

Theorem: 3.17 Let f:(X, τ, I)→(Y, σ) is rαg-I-continuous and g:(Y, σ)→(Z, η) is continuous, then g∘f: (X, τ, I)→(Z, η) is rαg-I-closed.

Proof: Let g be a continuous function and V be any open set in (Z, η), then $f^{-1}(V)$ is open in (Y, σ). Since f is rαg-I-continuous $f^{-1}(g^{-1}(V)) = (g∘f)^{-1}(V)$ is rαg-I-open in (X, τ, I). Hence g∘f is rαg-I-continuous.

Theorem: 3.16 Let f: X→Y be a map. Then the following statements are equivalent:
(i) f is rαg-I-continuous.
(ii) the inverse image of each open set in y is rαg-I-open in x.

Proof: Assume that f: X→Y is rαg-I-continuous. Let G be open in y. The G^c is closed in y. Since f is rαg-I-continuous, $f^{-1}(G^c)$ is rαg-I-closed in X. But $f^{-1}(G^c)$ = X- $f^{-1}(G)$. Thus $f^{-1}(G)$ is rαg-I-open in X.

Conversely assume that the inverse image of each open set in Y is rαg-I-open in X. Let F be any closed set in Y. By assumption c is rαg-I-open in X. But $f^{-1}(F^c)$ = X- $f^{-1}(F)$. Thus X- $f^{-1}(F)$ is rαg-I-open in X and so $f^{-1}(F)$ is rαg-I-closed in X. Therefore f is rαg-I-continuous. Hence (i) and (ii) are equivalent.

Theorem: 3.17 Let X = A∪B be a topological space with topology τ and Y be a topological space with topology σ. Let f: (A, τ/A) → (Y, σ) and g: (B, τ/B) → (Y, σ) be rαg-I-continuous maps such that f(x) = g(x) for every x∈A∩B. Suppose that A and B are rαg-I-closed sets in X. Then the combination α: (X, τ, I) → (Y, σ) is rαg-I-continuous.

Proof: Let F be any closed set in Y. Clearly $\alpha^{-1}(F) = f^{-1}(F) \cup g^{-1}(F)$ = C∪D where C = $f^{-1}(F)$ and D = $g^{-1}(F)$. But C is rαg-I-closed in A and A is rαg-I-closed in X and so C is rαg-I-closed in X. Since we have proved that if B ⊆ A ⊆ X, B is rαg-I-closed in A and A is rαg-I-closedin X then B is rαg-I-closed in X. Also C∪D is rαg-I-closed in X. Therefore $\alpha^{-1}(F)$ is rαg-I-closed in X. Hence α is rαg-I-continuous.

Definition: 3.18 A function f:(X, τ, I)→(Y, σ, J) is said to be rαg-I-irresolute, if $f^{-1}(A)$ is rαg-I-open in (X, τ, I) for every rαg-I-open set in (Y, σ, I).

Theorem: 3.19 Let f: (X, τ, I) → (Y, σ, J) and g: (Y, σ, J) → (Z, η, K) be any two functions then

i) g∘f is rαg-I-continuous if g is continuous and f is αg-I-continuous
ii) g∘f is rαg-I-irresolute if g is rαg-I-irresolute and f is rαg-I-irresolute
iii) g∘f is rαg-I-continuous if g is rαg-continuous and f is rαg-I-irresolute

Proof:

i) Let V be closed set in (Z,η,K). then $g^{-1}(V)$ is closed in (Y,σ,J). Since g is continuous. rαg-I-continuity of f ⇒ $f^{-1}(g^{-1}(V))$ is rαg-I-closed in (X, τ,I). That is $(g \circ f)^{-1}(V)$ is rαg-I-closed in (X, τ, I). Hence g∘f is rαg-I-continuous.
ii) Let V be rαg-I-closed in (Z, η, K). Since g is rαg-I-irresolute, $g^{-1}(V)$ is rαg-I-closed in (Y, σ, J). As f is rαg-I-irresolute $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is rαg-I-closed in (X, τ, I). Therefore g∘f is rαg-I-irresolute.
iii) Let V be closed in (Z, η, K). Since g is rαg-I-continuous, $g^{-1}(V)$ is rαg-I-closed in (Y, σ, J). As f is rαg-I-irresolute $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is rαg-I-closed in (X, τ, I). Therefore g∘f is rαg-I-continuous.

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