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IDEALS IN PRIMITIVE WEAKLY STANDARD RINGS

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ABSTRACT

In this paper we prove that if R has a maximal right ideal A $\neq 0$ which contains no two-sided ideal of other than (0), then R is associative. It is used to show that a left primitive weakly standard ring is either associative or simple with right identity element. Also we prove that the radical of A is contained in any primitive ideal P of R.

Keywords: *Commutator, associator, Nucleus, Primitive ring, Simple ring, weakly standard ring, Prime radical.*

INTRODUCTION

Paul and Saradha [1] proved that a left primitive (-1, 1) ring with commutators in the left nucleus is either associative or simple with right identity element. In a weakly standard ring R every commutator is in the nucleus and every associator commutes with every element of R. Using these properties in this paper we proved that if R has a maximal left ideal A $\neq 0$ which contains no two-sided ideal of R other than (0), then R is associative. It is used to show that a left primitive weakly standard ring is either associative or simple with right identity element. Also we proved that the radical of A is contained in any primitive ideal P of R.

PRELIMINARIES

A weakly standard ring R is a non associative ring satisfying the identities.

$$(x, y, x) = 0$$
$$((w, x), y, z) = 0$$
$$(w, (x, y), z) = 0$$

for all w, x, y, z in R, where the associator

$(x, y, z) = (xy)z – x(yz)$ and commutator $[x, y] = xy – yz$.

The nucleus N of R is defined as the set of all elements n in R such that $(n, R, R) = (R, n, R) = (R, R, n) = 0$. We define a ring R is simple if whenever A is an ideal of R, then either A = R or A = 0 and a ring R is primitive if it contains regular maximal which contains no two-sided ideal of R other than (0).

$P(R) = \cap\{P: P \text{ is an L-Prime L-ideal of } R\}$.

A left ideal A of R is called regular if there exists an element $g\in R$ such that $x-xg\in A$ for all $x\in R$. R is called left primitive if it contains a regular maximal left ideal, which contains no two-sided ideal of R other than the zero ideal. We define an ideal P of R to be a primitive ideal if the ring R/P is a primitive ring. The intersection of all regular maximal left ideals in R is called the radical of R and is denoted by rad R. Let $U = \{u \in R \mid (u, x) = 0 \text{ for all } x \text{ in } R\}$. Since every associator commutes with every element of R, we have

$$(R, R, R) \subseteq U, \tag{1}$$

where (R, R, R) is the set of all finite sums of elements of the form (x, y, z). In any ring R, (R, R, R) + (R, R, R) R is a two-sided ideal of R. First we prove the following lemma.

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Lemma: 1 *Let A be a left ideal of R. Then*
i) $S = \{s \in A: sR \subseteq A\}$ *is a two-sided ideal of R.*
ii) $(R, A, R) \subseteq S$.

Proof:
i) For any $s \in S$ and $x, y \in R$,

$$(sx)\, y = (s, x, y) + s\,(xy)$$

$$= -\,(y, x, s) + s\,(xy)$$

A is a left ideal and $s \in S$ implies $-\,(y, x, s) + s\,(xy) \in A$. Hence $(sx)\, y \in A$. Also $sR \subseteq A$ implies $sx \in A$. Hence $sx \in S$. Now $(xs)\, y = (x,s,y) + x\,(sy) = -\,(y, x, s) + (x, y, s) + x\,(sy)$, using 4.1.5. Again, using the fact that A is a left ideal and $s \in S$, in the above equation, we get $(xs)\, y \in A$. Hence $xs \in S$, so S is a two sided ideal of R.

ii) Let $x, w, z \in R$ and $a \in A$. Then

$$(w, a, x)\, z = [-(x, w, a) + (w, x, a)]z,$$

$$= -\,(x, w, a)\, z + (w, x, a)\, z$$

$$= -z\,(x, w, a) + z\,(w, x, a), \text{ Using (1).}$$

Again, since A is a left ideal,

$$-z\,(x, w, a) + z\,(w, x, a) \in A.$$

Therefore $(w, a, x)\, z \in A$. Hence $(R, A, R) \subseteq S$.

Using this lemma we prove the following theorems.

MAIN RESULTS

Theorem: 1 *If R has a maximal left ideal* $A \neq 0$ *which contains no two-sided ideal of R other than (0), then R is associative.*

Proof: By lemma 1, S is a two-sided ideal of R contained in A. Hence $S = (0)$. Since $(R, A, R) \subseteq S$, we have $(R, A, R) = (0)$. On the other hand it is easy to verify that $A+AR$ is a two-sided ideal of R. Since $A \subseteq A + AR$, we must have $A + AR = R$.

$$\begin{aligned} \text{Hence } (R, R, R) &= (R, A + AR, R) \\ &\subseteq (R, A, R) + (R, AR, R) \\ &\subseteq (R, A, R) + (R, A, R), \quad \text{since } AR \subseteq A \\ &\subseteq 2\,(R, A, R) \subseteq (R, A, R). \end{aligned}$$

But $(R, A, R) = (0)$.

Therefore $(R, R, R) = (0)$. Hence R is associative.

Theorem: 2 *If R is a left primitive weakly standard ring, then either R is associative or it is simple with a right identity element.*

Proof: Let A be a regular maximal left ideal which contains no two-sided ideal of R other than (0). If $A \neq 0$ then by theorem 1, R is associative. Thus we assume that (0) is a maximal regular left ideal of R, which contains no two sided ideal of R other than (0). (0) is maximal implies R has no proper ideal. Since (0) is regular, $\exists g \in R$ such that $x - xg \in (0)$ for all $x \in R$, i.e., $x = xg$ for all $x \in R$. This implies $RR \neq (0)$ and g is a right identity element. Hence R is simple with a right identity element.

Theorem: 3 *Let R be a weakly standard ring with characteristic* $\neq 2$. *Then rad R is contained in P for any primitive ideal P of R.*

Proof: Suppose that P is a primitive ideal of R. R/P is a primitive ring. Therefore by theorem 2 R/P is either a simple ring with an identity element or it is an associative ring. In either case rad $R/P = (0)$.

If R/P is simple then R/P has no one sided proper ideals. If R/P is associative, then the intersection of all the regular maximal left ideals of the ring R/P is zero. But the regular maximal left ideals of the ring R/P are of the form P_i/P with P_i is a regular maximal left ideal of the ring $R \supseteq P$. Let $\{P_i: i \in I\}$ be the set of all the regular maximal left ideals of the ring $R \supseteq P$. Then we have $\bigcap_{i \in I}\left(\frac{P_i}{P}\right) = (0)$ = zero ideal of $\frac{R}{P} = P$. This implies that $\frac{\bigcap_{i\in I} P_i}{P} = P$. Therefore $\bigcap_{i\in I} P_i \subseteq P$. That is, rad R is contained in P.

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