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Further results on α-labeling number of graphs

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ABSTRACT

In this note we show that the α-labeling number of a bipartite graph G is bounded, which proves a conjecture.

Keywords: *α-labeling; bipartite graph.*

INTRODUCTION

A bipartite graph (or bigraph) is a graph whose vertices can be divided into two disjoint sets u and v (that is, u and v are each independent sets) such that every edge connects a vertex in u to one in v. Equivalently, a bipartite graph is a graph that does not contain any odd-length cycles.

Definition: If the vertices of the graph are assigned values subject to certain conditions is known as *graph labeling*. Enough literature is available in printed as well as in electronic form on different types of graph labeling and more than 1000 research papers have been published so far in past four decades. A current survey of various graph labeling problems can be found in (Gallian, J., 2009).

Following three are the common features of any graph labeling problem.
(1) a set of numbers from which vertex labels are assigned;
(2) a rule that assigns a value to each edge;
(3) A condition that these values must satisfy.

A vertex *labeling* (or *valuation*) of a graph G is an assignment γ of labels to the vertices of G that induces for each edge uv a label depending on the vertex labels $\gamma(u)$ and $\gamma(v)$.A function γ, a β-*labeling* of a graph G with q Edges if γ is an injection from the vertices of G to the set $\{0; 1; \ldots q\}$ such that, when each edge uv is assigned the label $|\gamma(u) - \gamma(v)|$, the resulting edge labels are distinct. A β-labeling is now more commonly called a *graceful* labeling. A α-*labeling* is a graceful labeling with the additional property that there exists an *integer* λ such that for each edge uv either $\gamma(u) \leq \lambda < \gamma(v)$ or $\gamma(v) \leq \lambda < \gamma(u)$. Note that if G *admits* α-labeling *then G is necessarily* bipartite.

Applications in graph decompositions, α-labelings are of particular interest, $K_{m;\,n}$ *has an* α*-labelling* for all positive integers m and n and that C_m has an α-labeling if and only if $m \equiv 0 \pmod 4$. *on the other hand* one can easily show that there exist bipartite graphs which do not have α-labeling; Examples of such graphs include 2-regular graphs with $4m+2$ edges, forests with more than one component and the trees obtained by subdividing each edge in $K_{1;k}$; $k \geq 3$

Let G and H be simple graphs with G a subgraph of H. A *G-decomposition* of H is a partition of $E(H)$, the edge set of H, into subgraphs isomorphic to G. In this case we say that G *divides* H and write $G|H$.

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A bipartite graph G is said to *eventually have a* α*-labeling* provided that there exists a graph H (a '*host*' graph) which *has an* α*-labelling and* $G|H$*. The* α *-labelling number of G is defined to be* $G_\alpha = \min\{t$: there exists a host graph H of G with $|E(H)| = t \cdot |E(G)|\}$.showed that if C is a cycle of even length then $C_\alpha \leq 2$ and proposed the following conjecture.

Conjecture: 1 If G is a bipartite graph then $G_\alpha < \infty$.

In this note we shall give a short proof of the above conjecture by showing that every bipartite graph divides a complete bipartite graph.

First, if G is a regular bipartite graph then the following result shows that, the conjecture is true. Recall that $K_{m;n}$ has an x-labeling for all positive integers m and n.

Theorem: 1 Let G be a k-regular bipartite graph on 2n vertices. Then $G|K_{k2!\,n;\,k2\,!n}$.

Thus, it suffices to show that every bipartite graph divides a regular bipartite graph.

Theorem: 2 Every bipartite graph with q edges divides a q-regular bipartite graph.

Proof: Let G *be* a bipartite graph with q *edges*. Let $(A; B)$ be the bipartition of G,

where $A = \{a_1; a_2 \ldots . a_s\}$ and $B = \{b_1; b_2 \ldots b_t\}$. We shall construct a q-regular bipartite graph H *with* bipartition (A,B) such that $G|H$. Let A be the disjoint union of A1; A2… At where Ai = {ai;1; ai; 2;:::; ai; s} for i = 1; 2;:::;t and B be the disjoint union of $B_1; B_2; : : : ; B_s$ where $B_j = \{b_{j;1}; b_{j;2}; : : : ; b_{j;t}\}$ for $j = 1; 2; : : : ; s$. Then, for $1 \leq k; j \leq s$ and $1 \leq i; l \leq t$, let $a_{i;k}\, b_{j;l}$ be an edge in $E(H)$ if and only if $a_{k+j \pmod s}$ (which means here that the subscript takes values in the set $\{1; 2; : : : ; s\}$) and $b_{l+i \pmod t}$ (similarly, the subscript takes values in $\{1; 2; : : : ; t\}$) are adjacent in G. It remains to show that H is q-regular and $G|H$.

First, for each

$$a_{i;k} \in A_i \subseteq A, \quad \deg_H(a_{i;k}) = \sum_{j=1}^{s} \deg_G\left(a_{k+j \pmod s}\right) = q.$$

Similarly,

For each $b_{j;l} \in B_j \subseteq B$, $\deg_H(b_{j;l}) = \sum_{i=1}^{t} \deg_G\left(a_{l+i \pmod t}\right) = q$.

Hence H is q-regular.

By the definition of H *it is* clear that the bipartite sub graph induced by $A_i \cup B_j$ is Isomorphic to G and that $G|H$.

Combining the results we have proved

Theorem: 3 Let G be a bipartite graph; then $G_\alpha < \infty$

The bound obtained in Theorem 3 is clearly quite large. $T_\alpha \leq n$ if T is a tree with n edges. A number of results in the literature on G-decompositions of complete bipartite graphs suggest that n is a reasonable bound for G_α for any bipartite graph G with n edges. We note however that we know of no example of a bipartite graph G where $G_\alpha > 2$.

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