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On QUASI θgs –OPEN AND QUASI θgs-CLOSED FUNCTIONS

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ABSTRACT

The aim of this paper is to introduce and study of a new type of open function called quasi θgs-open function. Also, we obtain its characterizations and its basic properties.

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1. INTRODUCTION

Functions and of course open functions stand among the most important notions in the whole of mathematical science. Many different forms of the open functions have been introduced over the years. Recently in [4] the notion of θ-generalized semi closed (briefly, θgs-closed) set was introduced and studied. In this paper we will continue the study of related functions involving θgs-open sets. We introduce and characterize the concepts of quasi θgs-open functions.

2. PRELIMINARIES

Throughout this paper (X,τ) and (Y,σ) (or simply X and Y) denote topological spaces on which no separation axioms are assumed unless explicitly stated. If A is any subset of space X, then Cl(A) and Int(A) denote the closure of A and the interior of A in X respectively.

The following definitions are useful in the sequel:

Definition: 2.1 A subset A of space X is called
(i) a semi-open set [3] if $A \subseteq Cl(Int(A))$
(ii) a semi-closed set [1] if $Int(Cl(A)) \subseteq A$

Definition: 2.2 [2] A point $x \in X$ is called a semi-θ-cluster point of A if $A \cap sCl(U) \neq \varnothing$ for each semi-open set U containing x.

The set of all semi-θ-cluster point of A is called semi- θ-closure of A and is denoted by $sCl_\theta(A)$. A subset A is called semi- θ-closed if $sCl_\theta(A) = A$. The complement of semi-θ-closed set is semi-θ-open set.

Definition: 2.3 [4] A subset A of a topological space X is called θ-generalized-semi closed (briefly, θgs-closed) if $sCl_\theta(A) \subset U$, whenever $A \subset U$ and U is open in X. The complement of θgs-closed set is θ-generalized-semi open (briefly, θgs-open). We denote the family of θgs-closed sets of X by $\theta GSC(X, \tau)$ and θgs-open sets by $\theta GSO(X, \tau)$.

Definition: 2.4 [4]
(i) The intersection of all θgs-closed sets containing a set A is called θgs-closure of A and is denoted by θgsCl(A). A set A is θgs-closed if and only if θgsCl(A) = A.

(ii) The union of all θgs-open sets contained in A is called θgs-interior of A and is denoted by θgsInt(A). A set A is θgs-open if and only if θgsInt(A) = A.

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Definition 2.5 [4] A subset A of a topological space X is called θgs-neighbourhood of a point x of X if there exists θgs-open set G containing x such that $G \subset A$.

Definition: 2.6 [6] A space X is called $T_{\theta gs}$-space if every θgs-closed set in it is closed set.

Definition: 2.7 [5] A function f: $X \rightarrow Y$ is called
(i) θ-generalized semi-continuous (in briefly, θgs-continuous), if f^{-1} (F) is θgs-closed in X for every closed set F of Y.

(ii) θ-generalized semi-irresolute (in briefly, θgs-irresolute), if f^{-1}(F) is θgs-closed in X for every θgs-closed set F of Y.

Definition: 2.8[7] A function f: $X \rightarrow Y$ is said to be θgs-open (resp., θgs-closed) if f (V) is θgs-open (resp., θgs-closed) in Y for every open set (resp., closed) V in X.

Definition: 2.9 [8] A topological space X is said to be θgs-normal if each pair of disjoint closed sets can be separated by disjoint θgs-open sets.

3. Quasi θgs-open functions

Definition: 3.1 A function f: X→Y is said to be quasi θgs-open if for each the image of every θgs-open set in X is open set in Y.

Theorem: 3.2 A function f: X→Y is quasi θgs-open if and only if for every subset U of X, $f(\theta gsInt(U)) \subset Int(f(U))$.

Proof: Let f be quasi θgs-open function. Now, we have $Int(U) \subset U$ and θgsInt(U) is a θgs-open set. Hence we obtain that $f(\theta gsInt(U)) \subset f(U)$. As f(θgsInt(U)) is open, $f(\theta gsInt(U)) \subset Int(f(U))$.

Conversely, assume that U is a θgs-open set in X. Then, $f(U)=f(\theta gsInt(U))\subset Int(f(U))$ but $Int(f(U))\subset f(U)$. Consequently, f(U) = Int(f(U)) and hence f is quasi θgs-open.

Lemma: 3.3 If a function f: X→Y is quasi θgs-open, then $\theta gsInt(f^{-1}(G)) \subset f^{-1}(Int(G))$ for every subset G of Y.

Proof: Let G be an arbitrary subset of Y. Then $\theta gsInt(f^{-1}(G))$ is a θgs-open set in X and f is quasi θgs-open, then $f(\theta gsInt(f^{-1}(G))) \subset Int(f(f^{-1}(G))$. Thus, $\theta gsInt(f^{-1}(G)) \subset f^{-1}(Int(G))$.

Theorem: 3.4 For a function f: X→Y is the following are equivalent:

(i) f is quasi θgs-open;
(ii) For each subset U of X, $f(\theta gsInt(U)) \subset Int(f(U))$;
(iii) For each x in X and each θgs-neighbourhood U of x in X, there exists a neighbourhood V of f(x) in Y such that $V \subset f(U)$.

Proof:
(i) ⇒ (ii): It follows from Theorem 3.2

(ii) ⇒ (iii): Let x in X and U be an arbitrary θgs-neighbourhood of x in X. Then there exists a θgs-open set V in X such that $x \in X \subset U$. Then by (ii), we have $f(V) = f(\theta gsInt(V)) \subset Int(f(U))$ and hence f(V) = Int(f(V)). Therefore, it follows that f(V) is open in Y such that $f(x) \in f(V) \subset f(U)$.

(iii) ⇒ (i): Let U be an arbitrary θgs-open set in X. Then for each $y \in f(U)$, by(iii), there exists a neighbourhood V_y of y in Y such that $V_y \subset f(U)$. As V_y is a neighbourhood of y, there exists an open set W_y in Y such that $y\in W_y \subset V_y$. Thus, $f(U) = \cup \{ W_y : y \in f(U)\}$ which is an open set in Y. This implies that f is quasi θgs-open function.

Theorem: 3.5 A function f: X→Y is quasi θgs-open if and only if for any subset B of Y and each θgs-closed set F of X containing f^{-1}(B), there exists a closed set G of Y containing B such that $f^{-1}(G) \subset F$.

Proof: Suppose f is quasi θgs-open. Let $B\subset Y$ and F be a θgs-closed set of X containing f^{-1}(B). Now put G = Y-f(X-F). It is clear that $f^{-1}(B) \subset F$ implies $B \subset G$. Since f is quasi θgs-open, we obtain G as a closed set of Y. Moreover, we have $f^{-1}(G) \subset F$.

Conversely, let U be a θgs-open set of X and put B = Y-f(U). Then X-U is a θgs-closed set in X containing f^{-1}(B). By, hypothesis, there exists a closed set F of Y such that $B \subset F$ and $f^{-1}(F) \subset X\text{-}F$. Hence, we obtain $f(U) \subset Y\text{-} F$. On the other hand, it follows that $B \subset F$, $Y\text{-}F \subset Y\text{-}B= f(U)$. Thus, we obtain f(U) = Y-F which is open and hence f is quasi θgs-open function.

Theorem: 3.6 A function f: X→Y is quasi θgs-open if and only if for $f^{-1}(Cl(B)) \subset \theta gsCl(f^{-1}(B))$ for every subset B of Y.

Proof: Suppose that f is quasi θgs-open. For a subset B of Y, $f^{-1}(B) \subset \theta gsCl(f^{-1}(B))$. Therefore by Theorem 3.5, there exists a closed set F in Y such that $B \subset F$ and $f^{-1}(F) \subset \theta gsCl(f^{-1}(B))$. Therefore, we obtain $f^{-1}(Cl(B)) \subset \theta gsCl(f^{-1}(B)) \subset F$. Then by Theorem 3.5, f is quasi θgs-open function.

Lemma: 3.7 Let f: X→Y and g: Y→Z be two functions and gof: X→Z is quasi θgs-open. If f is continuous injective, then f is quasi θgs-open function.

Proof: Let U be a θgs-open set in X, then (gof)(U) is open set in Z since (gof) is θgs-open. Again g is an injective continuous function, $f(U) = g^{-1}(gof(U))$ is open in Y. This shows that f is quasi θgs-open function.

4. Quasi θgs-closed functions

Definition: 4.1 A function f: X→Y is said to be quasi θgs-closed if the image of each θgs-closed set in is closed set in Y.

Remark: 4.2 Every quasi θgs-closed function is θgs-closed function. But converse need not be true in general.

Example: 4.3: Let X=Y= {a, b, c}, τ={X, Ø, {a}, {b}, {a, b}}, σ={Y, Ø, {b}, {c}, {a, b}, {a, c}} be topologies on X and Y respectively. We have θGSC(X) = {X, Ø, {a}, {a, b}, {a, c}} and θGSC(Y) = {X, Ø, {a}, {c}, {a, b}, {a, c}}. Define a function by f(a) = b, f(b) = c and f(c) = a. Then f is θgs-closed function butt it is not quasi θgs-closed because for θgs-closed set {a, b} of X, f({a, b}) ={b, c} is not closed set in Y.

Theorem: 4.4 A surjective function f: X→Y is quasi θgs-closed if and only if for any subset B of Y and for any θgs-open set G of X containing $f^{-1}(B)$, there exists an open set U of Y containing B such that $f^{-1}(U) \subset G$.

Proof: This proof is similar to that of Theorem 3.5.

Theorem: 4.5 A function f: X→Y is quasi θgs-closed if and only if $Cl(f(A)) \subset f(\theta gsCl(A))$ for every subset A of X.

Proof: Suppose that f is quasi θgs-closed function and $A \subset X$. Then θgsCl(A) is θgs-closed set in X. Therefore f(θgsCl(A)) is closed in Y. Since $f(A) \subset f(\theta gsCl(A))$, implies $Cl(f(A)) \subset Cl(f(\theta gsCl(A))) = f(\theta gsCl(A))$. This implies, $Cl(f(A)) \subset f(\theta gsCl(A))$.

Conversely, A is any θgs-closed set in X. Then θgsCl(A) = A. Therefore, f(A) = f(θgsCl(A)). By hypothesis, $Cl(f(A)) \subset f(\theta gsCl(A)) = f(A)$. Hence $Cl(f(A)) \subset f(A)$. But $f(A) \subset Cl(f(A))$ is always true. This shows, f(A) = Cl(f(A)). This implies f(A) is closed set in Y. Therefore, f is quasi θgs-closed function.

Theorem: 4.6 If f: X→Y and g: Y→ Z are two quasi θgs-closed functions, then (gof): X→ Z is quasi θgs-closed function.

Proof: Obvious

Definition: 4.7[9]: A function f: X→Y is said to be strongly θgs -closed if f(A) is θgs-closed set in Y for every θgs-closed set A in X.

Theorem: 4.8 Let f: X→Y and g: Y→Z be any two functions. Then
- (i) If f is θgs-closed function and g is quasi θgs-closed, then gof is closed.
- (ii) If f is quasi θgs-closed and g is θgs-closed, then gof is strongly θgs-closed.
- (iii) If f is strongly θgs-closed and g is quasi θgs-closed, then gof is quasi θgs-closed.

Proof: Obvious

Theorem: 4.9 If f: X→Y and g: Y→Z are two functions such that (gof):X→Z is quasi θgs-closed function.
- (i) If f is θgs-irresolute surjective, then g is closed.
- (ii) If g is θgs-continuous injective, then f is strongly θgs-closed.

Proof:
- (i) Suppose that F is an arbitrary closed set in Y. As f is θgs-irresolute, $f^{-1}(F)$ is θgs-closed. Since gof is quasi θgs-closed and f is surjective, $(gof)(f^{-1}(F)) = g(F)$, which is closed in Z. This implies that g is closed function.

(ii) Suppose F is θgs-closed set in X. Since **gof** is quasi θgs-closed, (gof)(F) is closed in Z. Again g is θgs-continuous injective function, g^{-1}(gof(F)) = f(F) which is θgs-closed in Y. This shows that f is strongly θgs-closed.

Theorem: 4.10 Let f: X→Y be a function from a space X to a $T_{\theta gs}$-space Y. Then following are equivalent

(i) f is strongly θgs-closed function.
(ii) f is quasi θgs-closed function.

Proof:
(i) ⇒ (ii): Suppose (i) holds. Let F be a θgs-closed set in X. Then f(F) is θgs-closed in Y. Since Y is $T_{\theta gs}$-space, f(F) is closed in Y. Therefore f is quasi θgs-closed function.

(ii) ⇒(i): Suppose (ii) holds. Let F be a θgs-closed set in X. Then f(F) is closed and hence θgs-closed in Y. Therefore f is strongly θgs-closed function.

Theorem: 4.10 Let X and Y be topological spaces. Then the function f: X→Y is a quasi θgs -closed if and only if g(X) is closed in Y and g(V) \ g(X\V) is open in g(X) whenever V is θgs-open in X.

Proof: Necessity: Suppose g: X→Y quasi θgs -closed function. Since X is θgs-closed, g(X) is closed in Y and g(V) \ g(X\V) = g(V) ∩ g(X) \ g(X\V) is open in g(X) when V is θgs-open in X.

Sufficiency: Suppose g(X) is closed in Y, g(V) \ g(X\V) is open in g(X) when V is is θgs-open in X and let C be a closed in X. Then g(C) = g(X) \ (g(X\C) \g(C)) is closed in g(X) and hence, closed in Y.

Corollary: 4.11 Let X and Y be topological spaces. Then a surjective function g: X→Y is quasi θgs-closed if and only if g(V) \ g(X\V) is open in Y whenever U is θgs-open in X.

Proof: Obvious.

Theorem: 4.10 Let X and Y be topological spaces with X is θgs-normal. If g: X→Y is θgs-continuous quasi θgs-closed surjective function. Then Y is normal.

Proof: Let K and M be disjoint closed subsets of Y. Then $g^{-1}(K)$, $g^{-1}(M)$ are disjoint θgs-closed subsets of X. Since X is θgs-normal, there exists disjoint open sets V and W such that $g^{-1}(K) \subset V$ and $g^{-1}(M) \subset W$. Then K ⊂ g(V) \ g(X\V) and M ⊂ g(W) \ g(X\W). Further by Corollary 4.11, g(V) \ g(X\V) and g(W) \ g(X\W) are open sets in Y and clearly g(V) \ g(X\V) ∩ g(W) \ g(X\W) =∅. This shows that Y is normal.

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