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DOMINATION AND TOTAL DOMIANTION OF SPLITTED GRAPHS

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ABSTRACT

A set D of vertices in a splitted graph S(G)=(V, E) is called a dominating set if every vertex in V-D is adjacent to some vertex in D. The domination number γ [S(G)] of S(G) is minimum cardinality of a domination set of S(G) and A set D ⊆ V is a total domination set of splitted graph S(G) if every vertex in V is adjacent to some vertex in D. The total domination numberγ_t[S (G)] of S(G) is minimum cardinality of a domination set of S(G).

Keywords*: Domination, Total Domination, Splitted graph.*

Subject classification: *AMS 05C69, 05C70.*

1. INTRODUCTION

By a graph, we mean a finite simple and undirected graph. The vertex set and edge set of a graph G denoted are by V (G) and E(G) respectively. Let G be a graph. For each vertex v of a graph G, take a new vertex u. Join u to those vertices of G adjacent to v. The graph thus obtained is called the splitting graph of G. It is denoted by S(G). For a graph G, the splitting graph S of G is obtained by adding a new vertex v corresponding to each vertex u of G such that N(u) = N(v) and it is denoted by S(G) .

2. PRELIMINARIES

A set D of vertices in a splitted graph S(G) = (V,E) is called a dominating set if every vertex in V-D is adjacent to some vertex in D. The domination number γ[S(G)] of S(G) is minimum cardinality of a domination set of S(G). A set D⊆V is a total dominating set of splitted graph S(G) if every vertex in V is adjacent to some vertex in D. The total domination number γ_t [S(G)] of S(G) is minimum cardinality of a domination set of S(G).

3. DOMINATION OF SPLITTING GRAPHS

Theorem: 3.1 γ[S(P_n)] =$\lceil n/2 \rceil$ when n ≡1 (mod 2) .

Proof: let S(P_n) besplitted graph of a graph G with n is odd, n ≥3. By definition, every vertex in V-D in adjacent to some vertex in D.

Total Number of vertices = 2n

Maximum degree = 4

Number of vertices to cover = 2n/4 =$\lceil n/2 \rceil$.

A domination set of minimum cardinality is a γ -set of [S(P_n)].

$$\gamma\,[S(P_n)] = \lceil n/2 \rceil$$

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For example γ [S(P_7)] = {u_2 , u_5, u_6 , v_2}=4 as shown in the figure 3.2.

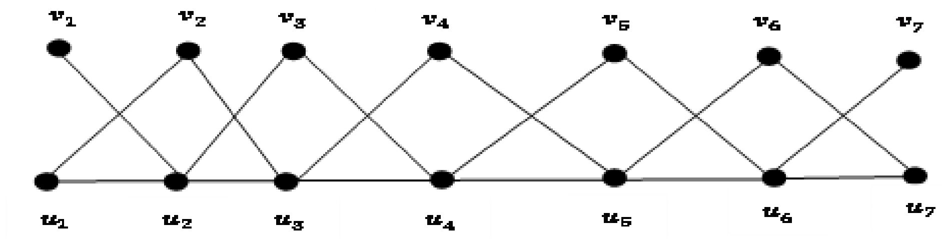

Figure - 3.2: S(P_7)

Theorem: 3.3 γ [S(P_n)] = 2 $\lceil n/4 \rceil$ when n≡0 (mod 2)

Proof: let S(P_n) be any splitted graph with n is even. By definition, every vertex in V-D in adjacent to some vertex in D.

Total Number of vertices = 2n

Maximum degree = 4 Number of vertices to cover = 2n/4 =2$\lceil n/4 \rceil$.

A domination set of minimum cardinality is a γ -set of S(P_n).

$$\gamma\,[S(P_n)] = 2\lceil n/4 \rceil$$

For example γ [S(P_6)] ={u_2, u_3, u_5, v_5} = 4 as shown in the figure 3.4

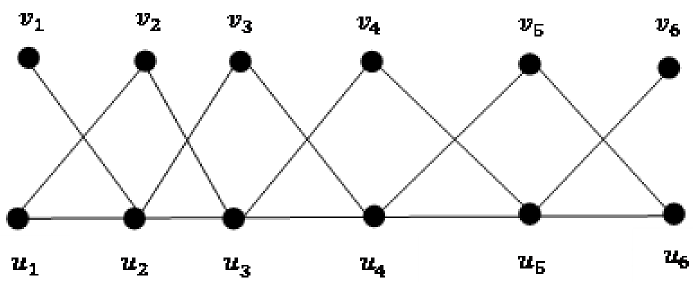

Figure - 3.4: S(P_6)

Theorem: 3.5 γ [S(H_n)] = $\begin{cases} 2\lceil n/2 \rceil & if\ n \equiv 1(mod\ 4) \\ 2 & if\quad n = 3 \end{cases}$

Proof: let S(H_n) be any splitted graph with n≡1(mod 2). By definition, every vertex in V -D in adjacent to some vertex in D.

when n=3, γ[S(H_3)] = 2.

when n > 3,By definition, every vertex in V-D in adjacent to some vertex in D.

Total Number of vertices = 2× 2n

Maximum degree = 4

Number of vertices to cover = 2×(2n/4) = 2 $\lceil n/2 \rceil$

A domination set of minimum cardinality is a γ -set of S(H_n). γ[S(H_n)] =2 $\lceil n/2 \rceil$.

Hence

γ [S(H_n)] = $\begin{cases} 2\lceil n/2 \rceil & if\ n \equiv 1(mod\ 4) \\ 2 & if\quad n = 3 \end{cases}$

For example γ[S (H_5)] = $\{u_2,\ u_3,\ u_4,\ u_2^1,\ u_3^1,\ u_4^1\}$ = 8 and γ[S(H_3)] = $\{u_2, u_2^1\}$=2 as shown in the figure 3.6 and 3.7.

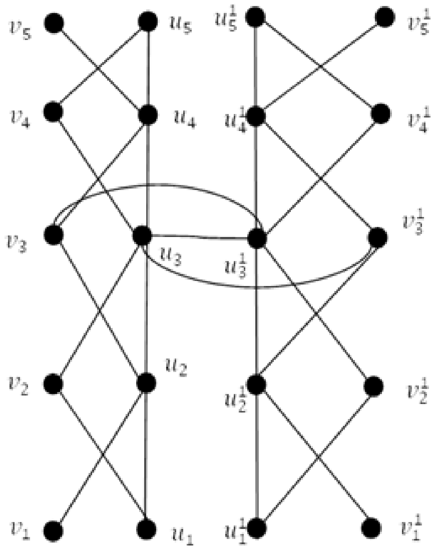

Figure - 3.6: S(H_5)

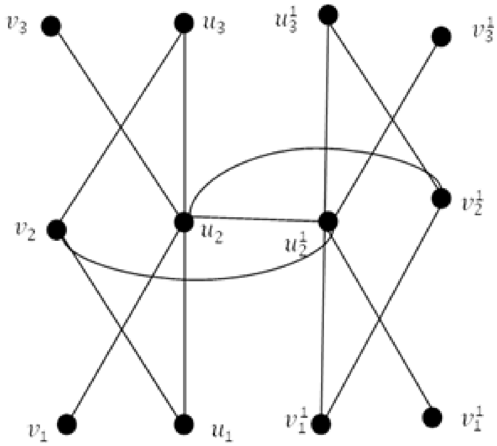

Figure - 3.7: S(H_3)

Theorem: 3.8 γ [S(H_n)] = $\begin{cases} 2\ \lceil n/2 \rceil & if\ n \equiv 7(mod\ 8) \\ 2\ \lfloor n/2 \rfloor & otherwise \end{cases}$

Proof: let S(H_n) be any splitted graph with n≡7 (mod 8). By definition, every vertex in V-D in adjacent to some vertex in D.

Total Number of vertices = 2× 2n

Maximum degree = 4

Number of vertices to cover = 2 × (2n/4) = 2 $\lceil n/2 \rceil$

A domination set of minimum cardinality is a γ -set of S(H_n).

Hence γ[S(H_n)] =2 $\lceil n/2 \rceil$.

when n > 3, By definition, every vertex in V-D in adjacent to some vertex in D.

Total Number of vertices = 2× 2n

Maximum degree = 4

Number of vertices to cover = 2 × (2n/4) =2 $\lfloor n/2 \rfloor$

A domination set of minimum cardinality is a γ -set of S(H_n).

γ[S(H_n)] =2 $\lfloor n/2 \rfloor$

Hence

γ [S(H_n)] = $\begin{cases} 2\ \lceil n/2 \rceil & if\ n \equiv 7(mod\ 8) \\ 2\ \lfloor n/2 \rfloor & otherwise \end{cases}$

For example γ [S(H_{11})] =$\{u_2, u_3, u_6, u_9, u_{10}, u_2^1, u_3^1, u_6^1 u_9^1, u_{10}^1\}$= 8

and γ[S(H_7)] =$\{u_2, u_3, u_6, v_6, u_2^1, u_3^1, u_6^1, v_6^1\}$ = 8 as shown in the figure 3.9 and 3.10.

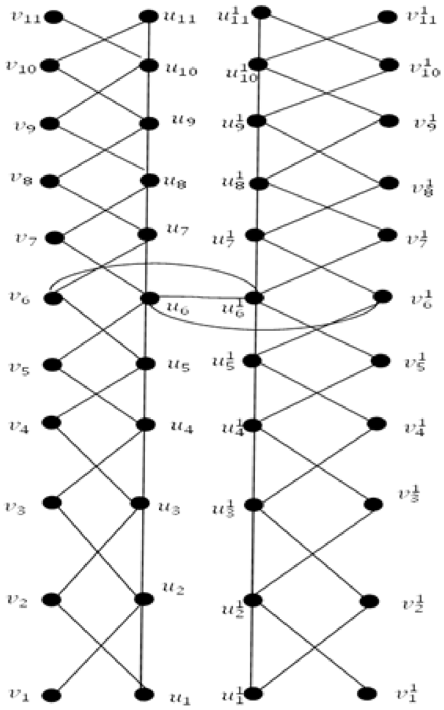

Figure - 3.10: S(H_{11})

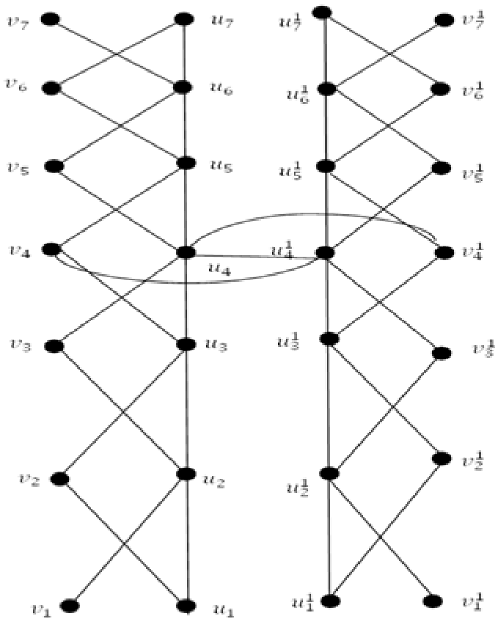

Figure - 3.11: S(H_7)

Theorem: 3.12 γ [S(H_n)] =n when n ≡0 (mod 4) .

Proof: let S(H_n) be any splitted graph with when n≡0 (mod 4) and n is even. By definition, every vertex in V -D in adjacent to some vertex in D.

Total Number of vertices = 4n

Maximum degree = 4

Number of vertices to cover = 4n/4 = n.

A domination set of minimum cardinality is a γ -set of S (H_n).

γ[S(H_n)] = n

For example γ [S (H_8)] = $\{u_2, u_3, u_6, u_7, u_2^1, u_3^1, u_6^1, u_7^1\}$=8 as shown in the figure 3.13.

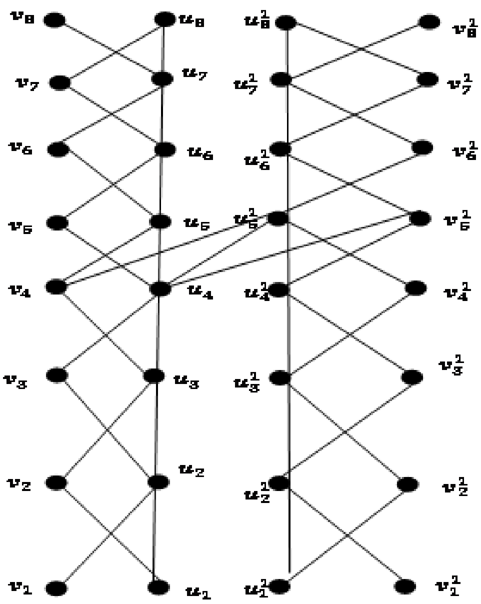

Figure - 3.13: S(H_8)

Theorem: 3.14 $\gamma\,[S(H_n)] = \begin{cases} 4\lceil n/4 \rceil & if\ i \equiv 1(mod\ 2) \\ n & if\ i \equiv 0(mod\ 2) \end{cases}$ when n=4i+2and n≡ 2(mod 4)

Proof: let $S(H_n)$ be any splitted graph with n is even and n=4i+2, n≡2(mod 4). By definition, every vertex in V -D in adjacent to some vertex in D.

when i ≡ 0(mod 2). By definition, every vertex in V-D in adjacent to some vertex in D.

Total Number of vertices = 4n

Maximum degree = 4

Number of vertices to cover = (4n/4) = n.

A domination set of minimum cardinality is a γ -set of S (H_n).

$$\gamma[S(H_n)] = n$$

when i≡ 1(mod 4).By definition, every vertex in V-D in adjacent to some vertex in D.

Total Number of vertices = 4n

Maximum degree = 4

Number of vertices to cover = (4n/4) =4 $\lceil n/4 \rceil$.

A domination set of minimum cardinality is a γ -set of S (H n).

$$\gamma[S(Hn)] = 4\lceil n/4 \rceil.$$

Hence $\gamma\,[S\,(H_n)] = \begin{cases} 4\lceil n/4 \rceil & if\ i \equiv 1(mod\ 2) \\ n & if\ i \equiv 0(mod\ 2) \end{cases}$

For example $\gamma[S(H_{10})] = \{u_2, u_5, u_8, u_9, v_2, u_2^1, u_3^1, u_6^1, u_9^1, v_9^1\} = 10$
and
$\gamma[S(H_6)] = \{u_2, u_5, v_2, v_5, u_2^1, u_5^1, v_2^1, v_5^1\}$=8 as shown in the figure 3.15 and 3.16

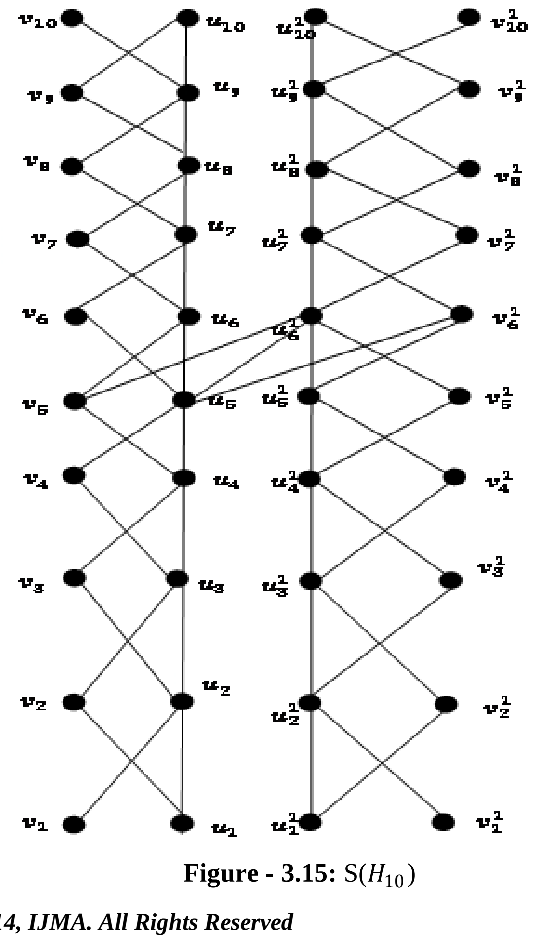

Figure - 3.15: $S(H_{10})$

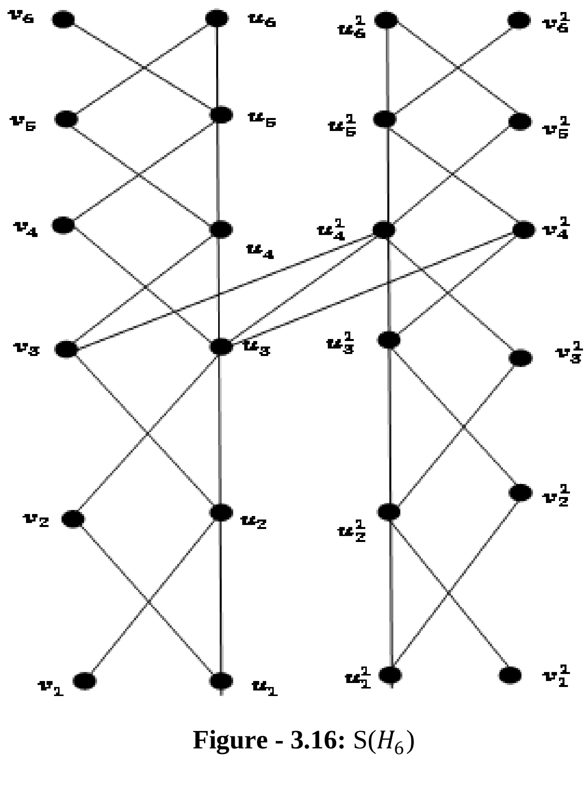

Figure - 3.16: $S(H_6)$

Theorem: 3.17 γ $[S(P_n^+)]$ = n

Proof: let $S(P_n^+)$ be any splitted graph. By definition, every vertex in V-D in adjacent to some vertex in D.

Total Number of vertices = 4n

Maximum degree = 6

Number of vertices to cover = 4n/6 =$\lceil 2/3 \rceil\, n$ = n.

A domination set of minimum cardinality is a γ -set of $S(P_n^+)$. γ$[S(P_n^+)]$ = n

For example γ $[S(P_5^+)]$ = $\{u_1, u_2, u_3, u_4, u_5\}$ = 5 as shown in the figure 3.18

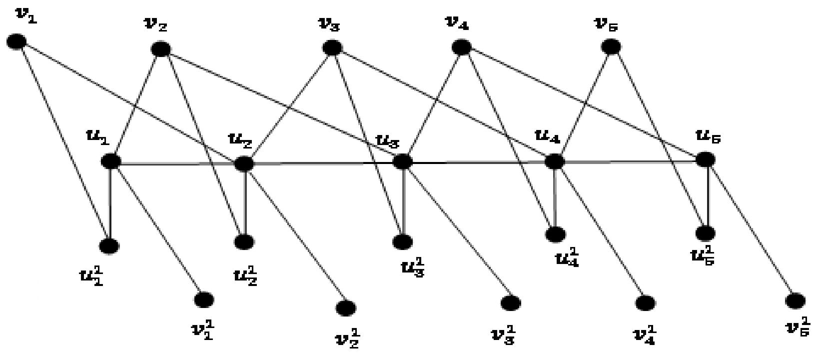

Figure -3.18:$S(P_5^+)$

Theorem: 3.19 γ[S($P_n \circ$ NK_1)] = n

Proof: let S($P_n \circ$ NK_1) be any splitted graph.

Case: 1 when N=2. By definition, every vertex in V-D in adjacent to some vertex in D.

Total Number of vertices = 6n

Maximum degree = 8

Number of vertices to cover = 6n/8 =$\lceil 6/8 \rceil n$ = n.

A domination set of minimum cardinality is a γ -set of S($P_n \circ 2K_1$).
γ[S($P_n \circ 2K_1$)] = n

Case: 2 when N=3 By definition, every vertex in V-D in adjacent to some vertex in D.

Total Number of vertices = 8n

Maximum degree = 10

Number of vertices to cover = 8n/10 =$\lceil 8/10 \rceil n$ = n.

A domination set of minimum cardinality is a γ -set of S($P_n \circ 3K_1$) .
γ[S($P_n \circ 3K_1$)] = n.

Case: 3 when N > 3 By definition, every vertex in V-D in adjacent to some vertex in D.

Total Number of vertices = 2(1+N) n

Maximum degree = 2(2+N)

Number of vertices to cover = 2(1+N) n/2(2+N) =$\lceil (1+N)/(2+\mathrm{N}) \rceil n$= n.

A domination set of minimum cardinality is a γ -set of S($P_n \circ NK_1$) .

γ[S($P_n \circ NK_1$)] = n.

For example γ[S($P_4 \circ 2K_1$)] ={u_1 , u_2 , u_3 , u_4}= 4 as shown in the figure 3.20.

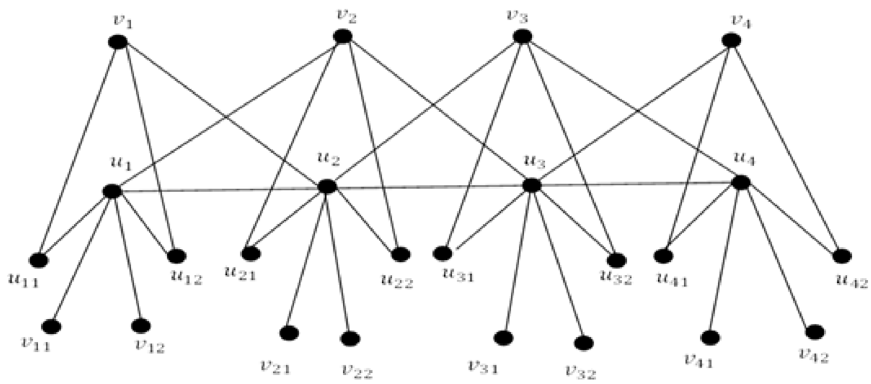

Figure -3.21: S($P_4 \circ 2K_1$)

Observation 3.22 For any splitted graph G, d is degree of vertices d(v_i) = d(u_i)/2 and d(v_1^1) = d((u_1^1)/2 1 ≤i ≤ n. for example figure 3.7 S(H_3)

d(u_1) = 2 ⇒ d(v_1) =2/2=1,

d(u_1^1) = 2 ⇒ d(v_1^1) =2/2=1.

Observation 3.23 For any splitted graph G, P < (γ(G))^2

Theorem 3.24 Let S(P_n) be a splitted graph. If a γ -set exists. Then S(P_n) has at least 4 vertices.

Proof: when n=2, Let V [S(P_n)] ={u_1 , u_2 , v_1 , v_2}, γ[S(P_n)] =2. S (P_n) has at least 4 vertices.

For example γ [S (P_2)] = {u_1,v_1} = 2 as shown in the figure 3.25.

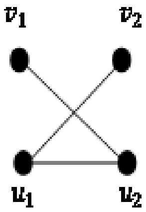

Figure - 3.25: S(P_2)

Theorem: 3.26 Let S (H_n) be a splitted graph. If a γ -set exists. Then S(H_n) has at least 8 vertices.

Proof: when n = 2, Let V [S(H_n)] = {$u_i, u_i^1, v_i, v_i^1;\ 1 \le i \le 2$ } γ[S(H_n)] =2.S(H_n) has at least 8 vertices. For example γ [S(H_2)] ={u_1,u_2^1}= 2 as shown in the figure 3.22.

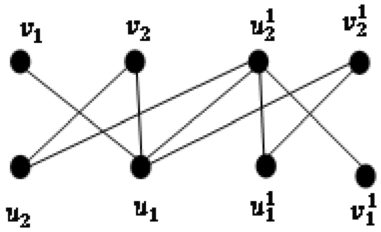

Figure - 3.27: S(H_2)

Theorem: 3.28 Let$S(P_n^+)$ be a splitted graph. If a γ -set exists. Then $S(P_n^+)$ has at least 8 vertices.

Proof: when n=2, Let V [$S(P_n^+)$] = {$u_i, u_i^1, v_i, v_i^1;\ 1 \le i \le 2$ }
γ[$S(P_n^+)$] =2. $S(P_n^+)$ has at least 8 vertices. For example γ [$S(P_2^+)$] = {u_1 ,u_2} = 2 as shown in the figure 3.24.

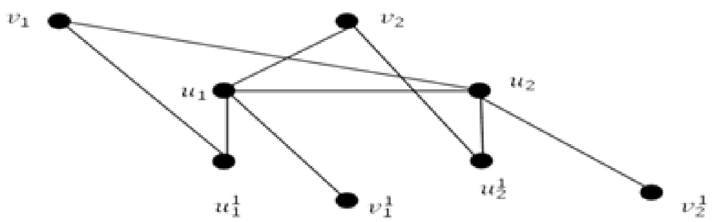

Figure - 3.29:$S(P_2^+)$

Theorem: 3.30 Let S(P_n ∘ NK_1) be a splitted graph. If a γ -set exists. Then S(P_n ∘ NK_1) has at least 4(1+N) vertices for all N is natural number.

Proof:

Case: 1 N=2 ⇒S(P_n ∘ $2K_1$)

when n = 2, Let V [S(P_n ∘ $2K_1$)] = $\{u_i, v_i, u_{ij},\ v_{ij}\ ;\ 1 \le\ i \le 2\ , 1 \le j\ \le 2\ \}$, γ[S($P_n$ ∘ $2K_1$)] =2. S(P_n ∘ $2K_1$) has at least 12 vertices.

Case: 2 N=3 ⇒S (P_n ∘ $3K_1$)

when n=2, Let V [S(P_n∘$3K_1$)]=$\{u_i, v_i,\ 1 \le\ i \le 2, (\ u_{ij}\ , v_{ij}\ );\ 1 \le\ i \le 3, 1 \le j\ \le$3 }, γ[S($P_n$ ∘ $3K_1$)] =2. S(P_n ∘ $3K_1$) has at least 16 vertices.

Case: 3 N > 3 ⇒S(P_n ∘ NK_1)

when n=2, Let V [S(P_n ∘ NK_1)] = $\{u_i, v_i,\ 1 \le\ i \le 2, (\ u_{ij}\ , v_{ij}\ );\ 1 \le\ i \le N, 1 \le j\ \le$ N}, γ[S(P_n ∘ NK_1)] =2.
S(P_n ∘ NK_1) has at least 4(1+N) vertices.

For example γ [S(P_2 ∘ $3K_1$)] =$\{u_1, u_2\}$= 2 as shown in the figure 3.31.

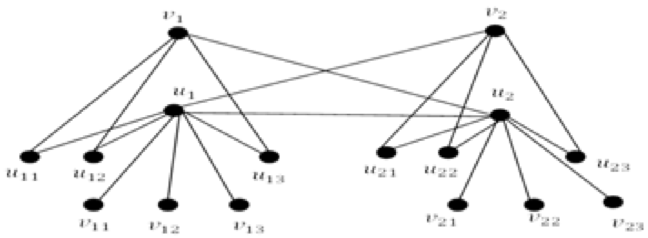

Figure - 3.31: S(P_2 ∘ $3K_1$)

Theorem: 3.32 For any splitted graph G.$\lfloor P/\Delta(G)\rfloor$ ≤ γ(G).

Proof: Let G be any splitted graph .P is total number of vertices, Δ(G) is Maximum degree of G, γ(G) is domination number of G. Number of vertices to cover = P/ Δ(G).

$$\lfloor P/\Delta(G)\rfloor \le \gamma(G)$$

Observation: 3.33 For Total Domination number the following results are observed

(1) γ_t [S(P_n)] =γ [S(P_n)] = $\lceil n/2\rceil$ when n ≡ 1 (mod 2)
(2) γ_t [S(P_n)] = γ [S(P_n)] = 2 $\lceil n/4\rceil$ when n ≡ 0 (mod 2)
(3) γ_t [S(H_n)] =γ [S(H_n)] = $\begin{cases} 2\lceil n/2\rceil & if\ n \equiv 1 (mod\ 4) \\ 2 & if\ \ \ \ n = 3 \end{cases}$
(4) γ_t [S(H_n)] = γ [S(H_n)] = $\begin{cases} 2\lceil n/2\rceil & if\ n \equiv 7 (mod\ 8) \\ 2\ \lfloor n/2\rfloor & otherwise \end{cases}$
(5) γ_t [S(H_n)] = γ [S(H_n)] = n when n ≡0 (mod 4)
(6) γ_t [S(H_n)] = γ [S(H_n)] = $\begin{cases} 4\lceil n/4\rceil & if\ i \equiv 1 (mod\ 2) \\ n & if\ i \equiv 0 (mod\ 2) \end{cases}$ when n = 4i+2 and n ≡ 2 (mod 4)
(7) γ_t [$S(P_n^+)$] =γ [$S(P_n^+)$] = n
(8) γ_t [S(P_n ∘ NK_1)] =γ [S(P_n ∘ NK_1)] = n

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