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# PROPERTIES OF MULTIPLICATIVE COUPLED FIBONACCI SEQUENCES OF FOURTH ORDER UNDER THE SPECIFIC SCHEMES 

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#### Abstract

Sequences have been fascinating topic for mathematicians for centuries. The Fibonacci sequences are a source of many nice and interesting identities. Coupled Fibonacci sequences of integers in which the elements of one sequence are part of the generalization of the other and vice versa. K.T.Atanassov was first introduced coupled Fibonacci sequences of second order in additive from. In this paper, we present some properties of multiplicative coupled Fibonacci sequences of fourth order under specific scheme.


Mathematical Subject Classification: 11B39, 11B37.
Keywords: Fibonacci sequence, Multiplicative Fibonacci sequence.

## 1. INTRODUCTION

The coupled Fibonacci sequence was first introduced by K.T.Atanassov [4]and also discussed many curious properties and new direction of generalization of Fibonacci sequences in [2], [3] and [5]. He defined and studied about four different ways to generate coupled sequences and called Fibonacci sequences. The multiplicative Fibonacci Sequences studied by P.Glaister[6] and generalized by P.Hope [7].K.T. Atanassov[4] notifies four different schemes in multiplicative form for coupled Fibonacci sequences.

Let $\left\{X_{i}\right\}_{i=1}^{\infty}$ \& $\left\{Y_{i}\right\}_{i=1}^{\infty}$ be two infinite sequences and four arbitrary real number a, b, c and d be given. The four different multiplicative schemes for 2-Fibonacci sequences are as follows

| First Scheme | $X_{n+2}=X_{n+1} X_{n}$ | $n \geq 0$ |
| :--- | :--- | :--- |
|  | $Y_{n+2}=Y_{n+1} Y_{n}$ | $n \geq 0$ |
| Second Scheme | $X_{n+2}=Y_{n+1} X_{n}$ | $n \geq 0$ |
|  | $Y_{n+2}=X_{n+1} Y_{n}$ | $n \geq 0$ |
| Third Scheme | $X_{n+2}=X_{n+1} Y_{n}$ | $n \geq 0$ |
|  | $Y_{n+2}=Y_{n+1} X_{n}$ | $n \geq 0$ |
| Fourth scheme | $X_{n+2}=Y_{n+1} Y_{n}$ | $n \geq 0$ |
|  | $Y_{n+2}=X_{n+1} X_{n}$ | $n \geq 0$ |

[^0]In this paper, I present some results on multiplicative coupled Fibonacci sequences of fourth order under specific scheme.

## 2. MULTIPLICATIVE COUPLED FIBONACCI SEQUENCES OF FOURTH ORDER

Let $\left\{X_{i}\right\}_{i=1}^{\infty}$ \& $\left\{Y_{i}\right\}_{i=1}^{\infty}$ be two infinite sequences and eight arbitrary real number a, b, c ,d, e, f, g, h be given multiplicative coupled Fibonacci sequences of fourth order are generated by the following 16 different ways:

The different schemes are as follows:

$\mathrm{T}_{4}: \begin{aligned} & X_{n+4}=X_{n+3} X_{n+2} Y_{n+1} Y_{n} \\ & Y_{n+4}=Y_{n+3} Y_{n+2} X_{n+1} X_{n}\end{aligned}, \quad \mathrm{~T}_{5}: \begin{aligned} & X_{n+4}=X_{n+3} Y_{n+2} X_{n+1} X_{n} \\ & Y_{n+4}=Y_{n+3} X_{n+2} Y_{n+1} Y_{n}\end{aligned}, \quad \mathrm{~T}_{6}: \begin{aligned} & X_{n+4}=X_{n+3} Y_{n+2} X_{n+1} Y_{n} \\ & Y_{n+4}=Y_{n+3} X_{n+2} Y_{n+1} X_{n}\end{aligned}$
$\mathrm{T}_{7}: \begin{aligned} & X_{n+4}=Y_{n+3} X_{n+2} X_{n+1} X_{n} \\ & Y_{n+4}=X_{n+3} Y_{n+2} Y_{n+1} Y_{n}\end{aligned}, \quad \mathrm{~T}_{8}: \begin{aligned} & X_{n+4}=Y_{n+3} X_{n+2} X_{n+1} Y_{n} \\ & Y_{n+4}=X_{n+3} Y_{n+2} Y_{n+1} X_{n}\end{aligned}, \quad \begin{aligned} & \mathrm{T}_{9}:\end{aligned} \begin{aligned} & X_{n+4}=X_{n+3} Y_{n+2} Y_{n+1} X_{n} \\ & Y_{n+4}=Y_{n+3} X_{n+2} X_{n+1} Y_{n}\end{aligned}$
$\mathrm{T}_{10}: \begin{aligned} & X_{n+4}=X_{n+3} Y_{n+2} Y_{n+1} Y_{n} \\ & Y_{n+4}=Y_{n+3} X_{n+2} X_{n+1} X_{n}\end{aligned}, \quad \mathrm{~T}_{11}: \begin{aligned} & X_{n+4}=Y_{n+3} Y_{n+2} X_{n+1} X_{n} \\ & Y_{n+4}=X_{n+3} X_{n+2} Y_{n+1} Y_{n}\end{aligned} \quad, \quad \begin{gathered}\mathrm{T}_{12}:\end{gathered} \begin{aligned} & X_{n+4}=Y_{n+3} Y_{n+2} X_{n+1} Y_{n} \\ & Y_{n+4}=X_{n+3} X_{n+2} Y_{n+1} X_{n}\end{aligned}$
$\begin{aligned} & X_{n+4}=Y_{n+3} X_{n+2} Y_{n+1} X_{n} \\ & \mathrm{~T}_{13} \\ & Y_{n+4}=X_{n+3} Y_{n+2} X_{n+1} Y_{n}\end{aligned}, \quad \begin{aligned} & \mathrm{T}_{14}: \\ & X_{n+4}=Y_{n+3} X_{n+2} Y_{n+1} Y_{n} \\ & Y_{n+4}=X_{n+3} Y_{n+2} X_{n+1} X_{n}\end{aligned}, \quad \begin{aligned} & X_{n+4}=Y_{n+3} Y_{n+2} Y_{n+1} X_{n} \\ & Y_{n+4}=X_{n+3} X_{n+2} X_{n+1} Y_{n}\end{aligned}$
$\begin{array}{ll} & X_{n+4}=Y_{n+3} Y_{n+2} Y_{n+1} Y_{n} \\ \mathrm{~T}_{16} \\ & Y_{n+4}=X_{n+3} X_{n+2} X_{n+1} X_{n}\end{array}$
The few terms of scheme ( $\mathrm{T}_{13}$ ) are tabulated below:

| $\mathbf{n}$ | $\mathbf{X}_{\mathbf{n}}$ | $\mathbf{Y}_{\mathbf{n}}$ |
| :---: | :---: | :---: |
| o | a | b |
| 1 | c | d |
| 2 | e | f |
| 3 | g | h |
| 4 | adeh | bcfg |
| 5 | $\mathrm{bc}^{2} \mathrm{f}^{2} \mathrm{~g}^{2}$ | $\mathrm{ad}^{2} \mathrm{e}^{2} \mathrm{~h}^{2}$ |
| 6 | $\mathrm{a}^{2} \mathrm{~d}^{3} \mathrm{e}^{4} \mathrm{~h}^{4}$ | $\mathrm{~b}^{2} \mathrm{c}^{3} \mathrm{f}^{4} \mathrm{~g}^{4}$ |

If we set $\mathrm{a}=\mathrm{b}=\mathrm{c}$ and $\mathrm{d}=\mathrm{e}=\mathrm{f}$ then the following result is true

## 3. MAIN RESULTS

Now we present some properties under scheme ( $\mathrm{T}_{13}$ )

$$
\begin{align*}
& X_{n+4}=Y_{n+3} X_{n+2} Y_{n+1} X_{n}  \tag{3.1}\\
& Y_{n+4}=X_{n+3} Y_{n+2} X_{n+1} Y_{n}
\end{align*} \quad n \geq 0
$$

Theorem 3.1: For every integers $n \geq 0$
a) $Y_{5 n} Y_{5 n+5}=X_{5 n+4}^{2}$
b) $Y_{5 n+4}^{2}=X_{5 n} X_{5 n+5}$

$$
\begin{align*}
\text { Proof: L.H.S } & =Y_{5 n} Y_{5 n+5} \\
& =Y_{5 n}\left(X_{5 n+4} Y_{5 n+3} X_{5 n+2} Y_{5 n+1}\right)  \tag{ByScheme3.1}\\
& =Y_{5 n}\left(Y_{5 n+3} X_{5 n+2} Y_{5 n+1} X_{5 n}\right) \cdot Y_{5 n+3} X_{5 n+2} Y_{5 n+1} \\
& =\left(Y_{5 n+3} X_{5 n+2} Y_{5 n+1} X_{5 n}\right) Y_{5 n+3} X_{5 n+2} Y_{5 n+1} Y_{5 n} \\
& =\left(Y_{5 n+3} X_{5 n+2} Y_{5 n+1} X_{5 n}\right) Y_{5 n+3} X_{5 n+2} Y_{5 n+1} X_{5 n} \\
& =X_{5 n+4} X_{5 n+4} \\
& =X_{5 n+4}^{2}
\end{align*}
$$

(By Scheme3.1)
(By Scheme3.1)
(By Scheme3.1)
(By induction hypo.)
(By Scheme3.1)

Hence result is true for all integers $n \geq 0$
Similar proof can be given for remaining part (b)
Theorem: 3.3 For every integer $n \geq 0$
a) $\frac{X_{n+4} X_{n+5}}{X_{n+2} X_{n+1} X_{n}}=Y_{n+4} \cdot Y_{n+3} \cdot Y_{n+2} \cdot Y_{n+1}$
b) $\frac{Y_{n+4} Y_{n+5}}{Y_{n+2} Y_{n+1} Y_{n}}=X_{n+4} \cdot X_{n+3} \cdot X_{n+2} \cdot X_{n+1}$

$$
\begin{array}{rlr}
\text { Proof: } X_{n+4} X_{n+5}=Y_{n+3} X_{n+2} Y_{n+1} X_{n} \cdot Y_{n+4} X_{n+3} Y_{n+2} X_{n+1} & \text { (By Scheme 3.1) } \\
X_{n+4} X_{n+5}=Y_{n+3} X_{n+2} Y_{n+1} X_{n}\left(X_{n+3} Y_{n+2} X_{n+1} Y_{n}\right) X_{n+3} Y_{n+2} X_{n+1} & & \text { (By Scheme 3.1) } \\
X_{n+4} X_{n+5}=Y_{n+4}\left(Y_{n+3} Y_{n+2} Y_{n+1}\right)\left(X_{n+2} X_{n+1} X_{n}\right) & & \text { (By Scheme 3.1) } \\
& \frac{X_{n+4} X_{n+5}}{}=Y_{n+4} Y_{n+3} \cdot Y_{n+2} Y_{n+1} & \tag{ByScheme3.1}
\end{array}
$$

Hence result is true for all integers $n \geq 0$
Similar proofs can be given for remaining part (b)
Theorem: 3.4 For every integer $n \geq 0$

> a) $\frac{\prod_{i=4}^{m} X_{n+i}}{\left(\prod_{i=3}^{m-3} X_{n+i}^{2}\right)\left(\left(\prod_{i=2}^{m-4} Y_{n+i}^{2}\right)\right.}=\left(Y_{n} Y_{n+1} Y_{m+n-3} Y_{m+n-2}\right)\left(X_{n+1} X_{n+2} X_{m+n-2} X_{m+n-1}\right)$
> b) $\frac{\prod_{i=4}^{m} Y_{n+i}}{\left(\prod_{i=3}^{m-3} Y_{n+i}^{2}\right)\left(\prod_{i=2}^{m-4} X_{n+i}^{2}\right)}=\left(X_{n} X_{n+1} X_{m+n-3} X_{m+n-2}\right)\left(Y_{n+1} Y_{n+2} Y_{m+n-2} Y_{m+n-1}\right)$

Theorem: 3.5 For every integer $n \geq 0$
$\left(X_{n+4} Y_{n+4}\right)^{2}=\frac{\left(X_{n} Y_{n}\right)\left(X_{n=1} Y_{n+1}\right)\left(X_{n+6} Y_{n+6}\right)}{X_{n+5} Y_{n+5}}$
Theorem: 3.6 For every integer $n \geq 0$

$$
\prod_{i=1}^{n} X_{4 i}=\prod_{i=0}^{2 n-1}\left(X_{2 i+1} Y_{2 i}\right)
$$

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## CONCLUSION

In this paper I described and extended multiplicative coupled Fibonacci sequences of fourth order under specific schemes. Similar results can be developed for other schemes.

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