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THERMAL RADIATION EFFECT ON AN UNSTEADY MHD MIXED CONVECTIVE VISCOUS DISSIPATIVE FLUID FLOW PAST A MOVING ACCELERATED VERTICAL POROUS PLATE

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ABSTRACT

A numerical study is presented on the effect of thermal radiation on an unsteady magnetohydrodynamic mixed convective flow of a viscous incompressible fluid past a moving accelerated vertical plate embedded in a porous medium on taking viscous dissipation into account has been studied. The fluid is considered to be a gray, absorbing emitting but non – scattering medium and the Rossel and approximation is used to describe the radiative heat flux in the energy equation. The dimensionless governing coupled, non – linear boundary layer partial differential equations are solved by an efficient finite element method. The variations of the fluid velocity, temperature and concentration fields with the help of different flow parameters viz. thermal Grashof number, solutal Grashof number, Hartmann number, Permeability parameter, Suction parameter, thermal radiation parameter, Prandtl number, Schmidt number and Eckert number are presented graphically. And also, the effects these flow parameters on Skin – friction, Rate of heat and mass transfer are discussed through tabular forms. This model finds applications in geophysics, astrophysics and also in the design of high temperature industrial processing systems.

Keywords: *Thermal radiation, Free convection, Porous medium, Viscous dissipation and Finite element method.*

1. INTRODUCTION

The heat transfer from different geometries embedded in porous media has many engineering and geophysical applications such as geothermal reservoirs, drying of porous solids, thermal insulation, enhanced oil recovery, packed – bed catalytic reactors, cooling of nuclear reactors, and underground energy transport. A very significant area of research in radiative heat transfer, at the present time is the numerical simulation of combined radiation and convection or conduction transport processes. The effort has arisen largely due to the need to optimize industrial system such as furnaces, ovens and boilers and the interest in our environment and in non – conventional energy sources such as the use of salt – gradient solar ponds for energy collection and storage. In particular, natural convection induced by the simultaneous action of buoyancy forces resulting from thermal diffusion is of considerable interest in nature and in many industrial applications such as geophysics, oceanography, drying processes and solidification of binary alloy. Convective heat transfer in porous media has received considerable attention in recent years owing to its importance in various technological applications such as fibre and granular insulation, electronic system cooling, cool combustors, oil extraction, thermal energy storage and flow through filtering devices, porous material regenerative heat exchangers. Books by Nield and Bejan [1], Bejan and Kraus [2] and Ingham *et al.* [3] excellently describe the extent of the research information in this area. The viscous dissipation effects are important in geophysical flows and also in certain industrial operations and are usually characterized by the Eckert number. In the literature, extensive research work is available to examine the effect of natural convection on flow past a plate.

Callahan and Manner [4] first considered the transient free convection flow past a semi infinite plate by explicit finite difference method. Unsteady free convective flow on taking into account the mass transfer phenomenon past an infinite vertical porous plate with constant suction was studied by Soundalgekar and Wavre [5]. Kafousias *et al.* [6] have studied the effects of free convective currents on the flow field of an incompressible viscous fluid past an impulsively started infinite vertical porous plate with constant suction. However, this analysis is not applicable for other fluids whose Prandtl number is different from unity. Soundalgekar and Ganesan [7] have analyzed transient free convective

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flow past a semi infinite vertical flat plate, taking into account mass transfer by an implicit finite difference method of Crank Nicolson type. Free convection at a vertical plate with transpiration has investigated by Kolar and Sastri [8]. Yih [9] have analyzed the effect of transpiration on coupled heat and mass transfer in mixed convection over a vertical plate embedded in a saturated porous medium. Elbashbeshy [10] has investigated the mixed convection along a vertical plate embedded in non – darcian porous medium with suction and injection. Chin *et al.* [11] has studied the effect of variable viscosity on mixed convection boundary layer flow over a vertical surface embedded in a porous medium. MHD steady free convection flow from vertical surface in porous medium has been studied by Reddy [12]. Thermal radiation in fluid dynamics has become a significant branch of the engineering sciences and is an essential aspect of various scenarios in mechanical, aerospace, chemical, environmental, solar power and hazards engineering. For some industrial applications such as glass production and furnace design and in space technology applications such as cosmical flight aerodynamics rocket, propulsion systems, plasma physics and spacecraft re – entry aerothermodynamics which operate at higher temperatures, radiation effects can be significant. In view of this, Hossain and Takhar [13] have analyzed the effect of radiation on mixed convection along a vertical plate with uniform surface temperature. Pal and Talukdar [14] have studied the buoyancy and chemical reaction effects on MHD mixed convection heat and mass transfer in a porous medium with thermal radiation and ohmic heating.

Vasu *et al.* [15] have studied the radiation and mass transfer effects on transient free convection flow of a dissipative fluid past semi – infinite vertical plate with uniform heat and mass flux. Gebhart [16] has shown the importance of viscous dissipative heat in free convection flow in the case of isothermal and constant heat flux at the plate. Gebhart and Mollendorf [17] have considered the effects of viscous dissipation for external natural convection flow over a surface. Soundalgekar [18] has analyzed viscous dissipative heat on the two – dimensional unsteady free convective flow past an infinite vertical porous plate when the temperature oscillates in time and there is constant suction at the plate. Maharajan and Gebhart [19] have reported the influence of viscous dissipation effects in natural convective flows, showing that the heat transfer rates are reduced by an increase in the dissipation parameter. Israel Cookey *et al.* [20] have investigated the influence of viscous dissipation and radiation on unsteady MHD free convection flow past an infinite heated vertical plate in a porous medium with time dependent suction. Srihari *et al.* [21] have studied MHD free convection flow of an incompressible viscous dissipative fluid in an infinite vertical oscillating plate with constant heat flux. Suneetha *et al.* [22] have analyzed the effects of viscous dissipation and thermal radiation on hydromagnetic free convective flow past an impulsively started vertical plate. Suneetha *et al.* [23] have studied the effects of thermal radiation on the natural convective heat and mass transfer of a viscous incompressible gray absorbing – emitting fluid flowing past an impulsively started moving vertical plate with viscous dissipation. Ahmed and Batin [24] have obtained an analytical model of MHD mixed convective radiating fluid with viscous dissipative heat. Babu *et al.* [25] have studied the radiation and chemical reaction effects on an unsteady MHD convective flow past a vertical moving porous plate embedded in a porous medium with viscous dissipation. Kishore *et al.* [26] have analyzed the effects of thermal radiation and viscous dissipation on MHD heat and mass diffusion flow past an oscillating vertical plate embedded in a porous medium with variable surface conditions.

Motivated by the above reference work and the numerous possible industrial applications of the problem, it is of paramount interest in this study to investigate the effect of thermal radiation on an unsteady magnetohydrodynamic mixed convective flow of a viscous incompressible fluid past a moving accelerated vertical plate embedded in a porous medium on taking viscous dissipation into account has been studied. The dimensionless governing coupled, non – linear boundary layer partial differential equations are solved by an efficient finite element method. The purpose of this paper is to extend the results of Ramana Reddy *et al.* [27] to study the more general problem which includes the thermal radiation effect on an unsteady magnetohydrodynamic flow and heat transfer along an accelerated infinite vertical porous flat plate with mass transfer in presence of viscous dissipation. The variations of the fluid velocity, temperature and concentration fields with the help of different flow parameters, thermal Grashof number, solutal Grashof number, Hartmann number, Permeability parameter, Suction parameter, thermal radiation parameter, Prandtl number, Schmidt number and Eckert number are presented graphically. And also, the effects these flow parameters on Skin – friction, Rate of heat and mass transfer are discussed through tabular forms. The mathematical analysis and the method of solution of this paper have been presented in Section 2 and 3 respectively and the results are discussed in Section 4 and the conclusions are set out in Section 5.

2. MATHEMATICAL FORMULATION

Consider the effect of thermal radiation on an unsteady mixed convective mass flow of a viscous incompressible electrically conducting fluid past an accelerating vertical infinite porous flat plate with viscous dissipation.

We made the following assumptions.

1. In Cartesian coordinate system, let $x'-$axis is taken to be along the plate and the $y'-$axis normal to the plate. Since the plate is considered infinite in $x'-$direction, hence all physical quantities will be independent of $x'-$direction.

2. The wall is maintained at constant temperature (T'_w) and concentration (C'_w) higher than the ambient temperature (T'_∞) and concentration (C'_∞) respectively.
3. A uniform magnetic field of magnitude B_o is applied normal to the plate. The transverse applied magnetic field and magnetic Reynold's number are assumed to be very small, so that the induced magnetic field is negligible.
4. The homogeneous chemical reaction is of first order with rate constant $\overline{K}$ between the diffusing species and the fluid is neglected.
5. It is assumed that there is no applied voltage which implies the absence of an electric field.
6. It is assumed that the plate is accelerating with a velocity $u = U_o$ in its own plane for $t' \geq 0$
7. The fluid has constant kinematic viscosity and constant thermal conductivity and the Boussinesq's approximation have been adopted for the flow.

The magnetohydrodynamic unsteady mixed convective boundary layer equations under the Boussinesq's approximations are:

Continuity Equation:

$$\frac{\partial v'}{\partial y'} = 0 \Rightarrow v' = -v'_o \text{ (Constant)} \tag{1}$$

Momentum Equation:

$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = g\beta\left(T' - T'_\infty\right) - \nu\frac{u'}{k'} + g\beta^*\left(C' - C'_\infty\right) + \nu\frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_o^2}{\rho}u' \tag{2}$$

Energy Equation:

$$\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'} = \frac{\kappa}{\rho\, c_p}\frac{\partial^2 T'}{\partial y'^2} + \frac{\nu}{c_p}\left(\frac{\partial u'}{\partial y'}\right)^2 - \frac{\partial q_r}{\partial y'} \tag{3}$$

Concentration Equation:

$$\frac{\partial C'}{\partial t'} + v'\frac{\partial C'}{\partial y'} = D\frac{\partial^2 C'}{\partial y'^2} \tag{4}$$

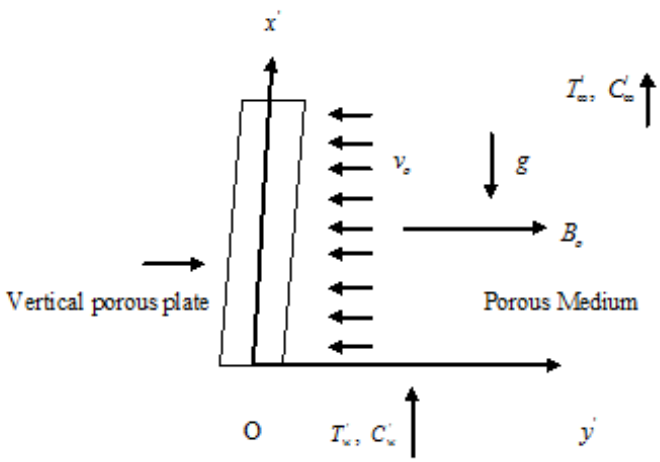

Figure 1. Physical sketch and geometry of the problem

The boundary conditions of the problem are:

$$\left.\begin{array}{l} t' \leq 0 : \left\{ u' = 0,\ v' = 0,\ T' = 0,\ C' = 0 \ \ for\ all\ y' \right. \\ t' \geq 0 : \left\{\begin{array}{l} u' = U_O,\ v' = -v'_o,\ T' = T'_w,\ C' = C'_w \ \ at\ y' = 0 \\ u' \to 0,\ T' \to T'_\infty,\ C' \to C'_\infty \ \ as\ y' \to \infty \end{array}\right. \end{array}\right\} \tag{5}$$

The radiative heat flux term is simplified by making use of the Rosseland approximation [28] as

$$q_r = -\frac{4\overline{\sigma}}{3k^*}\frac{\partial T'^4}{\partial y'} \tag{6}$$

Here $\overline{\sigma}$ is Stefan – Boltzmann constant and k^* is the mean absorption coefficient. It is assumed that the temperature differences within the flow are sufficiently small so that T'^4 can be expressed as a linear function of T' after using Taylor's series to expand T'^4 about the free stream temperature T_h' and neglecting higher – order terms. This results in the following approximation:

$$T'^4 \cong 4T_h'^3 T' - 3T_h'^4 \tag{7}$$

Using equations (6) and (7) in the last term of equation (3), we obtain:

$$\frac{\partial q_r}{\partial y} = -\frac{16\overline{\sigma}T_h'^3}{3k^*}\frac{\partial^2 T'}{\partial y'^2} \tag{8}$$

Introducing (8) in the equation (3), the energy equation becomes:

$$\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'} = \frac{\kappa}{\rho\, c_p}\frac{\partial^2 T'}{\partial y'^2} + \frac{v}{c_p}\left(\frac{\partial u'}{\partial y'}\right)^2 + \frac{16\overline{\sigma}T_h'^3}{3k^*}\frac{\partial^2 T'}{\partial y'^2} \tag{9}$$

Introducing the following non – dimensional variables and parameters,

$$\left.\begin{aligned} & y = \frac{y'v_o'}{v},\ t = \frac{t'v_o'^2}{4v},\ \omega = \frac{4v\omega'}{v_o'^2},\ u = \frac{u'}{U_o},\ M = \left(\frac{\sigma B_o^2}{\rho}\right)\frac{v}{v_o'^2},\ K = \frac{k'\,v_o'^2}{v^2}, Sc = \frac{v}{D}, \\ & \theta = \frac{T'-T_\infty'}{T_w'-T_\infty'},\ C = \frac{C'-C_\infty'}{C_w'-C_\infty'},\ \Pr = \frac{v}{k},\ Gr = \frac{vg\beta\left(T_w'-T_\infty'\right)}{U_o v_o'^3},\ Gc = \frac{g\beta^* v\left(C_w'-C_\infty'\right)}{U_o v_o'^3}, \\ & Ec = \frac{v_o'^2}{c_p\left(T_w^{'}-T_\infty^{'}\right)},\ R = \frac{\kappa k^*}{4\sigma T_h'^3} \end{aligned}\right\} \tag{10}$$

Substituting (10) in equations (2), (9) and (4) under boundary conditions (5), we get:

$$\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = (Gr)\theta + (Gc)C + \frac{\partial^2 u}{\partial y^2} - \left(M + \frac{1}{K}\right)u \tag{11}$$

$$\frac{\partial \theta}{\partial t} - \frac{\partial \theta}{\partial y} = \frac{1}{\Pr}\left(1 + \frac{4}{3R}\right)\frac{\partial^2 \theta}{\partial y^2} + (Ec)\left(\frac{\partial u}{\partial y}\right)^2 \tag{12}$$

$$\frac{\partial C}{\partial t} - \frac{\partial C}{\partial y} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} \tag{13}$$

The corresponding boundary conditions are:

$$\left.\begin{aligned} & u = 1,\ \theta = 1,\ C = 1\ at\ y = 0 \\ & u \to 0,\ \theta \to 0,\ C \to 0\ as\ y \to \infty \end{aligned}\right\} \tag{14}$$

The skin – friction, Nusselt number and Sherwood number are important physical parameters for this type of boundary layer flow. The skin – friction at the plate, which in the non – dimensional form is given by

$$\tau = \frac{\tau_w'}{\rho U_o v} = \left(\frac{\partial u}{\partial y}\right)_{y=0} \tag{15}$$

The rate of heat transfer coefficient, which in the non – dimensional form in terms of the Nusselt number is given by

$$Nu = -x\frac{\left(\frac{\partial T'}{\partial y'}\right)_{y'=0}}{T'_w - T'_\infty} \Rightarrow Nu\,\mathrm{Re}_x^{-1} = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0} \tag{16}$$

The rate of mass transfer coefficient, which in the non – dimensional form in terms of the Sherwood number, is given by

$$Sh = -x\frac{\left(\frac{\partial C'}{\partial y'}\right)_{y'=0}}{C'_w - C'_\infty} \Rightarrow Sh\,\mathrm{Re}_x^{-1} = -\left(\frac{\partial C}{\partial y}\right)_{y=0} \tag{17}$$

Where $\mathrm{Re} = \frac{U_o x}{\nu}$ is the local Reynolds number.

The mathematical formulation of the problem is now completed. Equations (11), (12) & (13) present a coupled nonlinear system of partial differential equations and are to be solved by using initial and boundary conditions (14). However, exact solutions are difficult, whenever possible. Hence, these equations are solved by the Finite element method.

3. NUMERICAL SOLUTION BY FEM

By applying Galerkin finite element method for equation (11) over the element (e), ($y_j \le y \le y_k$) is:

$$\int_{y_j}^{y_k}\left\{N^T\left[\frac{\partial^2 u^{(e)}}{\partial y^2} - \frac{\partial u^{(e)}}{\partial t} + \frac{\partial u^{(e)}}{\partial y} - Au^{(e)} + P\right]\right\}dy = 0 \tag{18}$$

Where $P = (Gr)\theta_i^j + (Gc)C_i^j$, $A = M + \frac{1}{K}$;

Integrating the first term in equation (18) by parts one obtains

$$N^{(e)T}\left\{\frac{\partial u^{(e)}}{\partial y}\right\}_{y_j}^{y_k} - \int_{y_j}^{y_k}\left\{\frac{\partial N^{(e)T}}{\partial y}\frac{\partial u^{(e)}}{\partial y} + N^{(e)T}\left(\frac{\partial u^{(e)}}{\partial t} - \frac{\partial u^{(e)}}{\partial y} + Au^{(e)} - P\right)\right\}dy = 0 \tag{19}$$

Neglecting the first term in equation (19), one gets:

$$\int_{y_j}^{y_k}\left\{\frac{\partial N^{(e)T}}{\partial y}\frac{\partial u^{(e)}}{\partial y} + N^{(e)T}\left(\frac{\partial u^{(e)}}{\partial t} - \frac{\partial u^{(e)}}{\partial y} + Au^{(e)} - P\right)\right\}dy = 0$$

Let $u^{(e)} = N^{(e)}\phi^{(e)}$ be the linear piecewise approximation solution over the element (e), ($y_j \le y \le y_k$),

where $N^{(e)} = \begin{bmatrix} N_j & N_k \end{bmatrix}$, $\phi^{(e)} = \begin{bmatrix} u_j & u_k \end{bmatrix}^T$ and $N_j = \frac{y_k - y}{y_k - y_j}$, $N_k = \frac{y - y_j}{y_k - y_j}$ are the basis functions. One obtains:

$$\int_{y_j}^{y_k}\left\{\begin{bmatrix} N_j' N_j' & N_j' N_k' \\ N_j' N_k' & N_k' N_k' \end{bmatrix}\begin{bmatrix} u_j \\ u_k \end{bmatrix}\right\}dy + \int_{y_j}^{y_k}\left\{\begin{bmatrix} N_j N_j & N_j N_k \\ N_j N_k & N_k N_k \end{bmatrix}\begin{bmatrix} \dot{u}_j \\ \dot{u}_k \end{bmatrix}\right\}dy - \int_{y_j}^{y_k}\left\{\begin{bmatrix} N_j N_j' & N_j N_k' \\ N_j' N_k & N_k' N_k \end{bmatrix}\begin{bmatrix} u_j \\ u_k \end{bmatrix}\right\}dy$$

$$+ A\int_{y_j}^{y_k}\left\{\begin{bmatrix} N_j N_j & N_j N_k \\ N_j N_k & N_k N_k \end{bmatrix}\begin{bmatrix} u_j \\ u_k \end{bmatrix}\right\}dy = P\int_{y_j}^{y_k}\begin{bmatrix} N_j \\ N_k \end{bmatrix}dy$$

Simplifying we get

$$\frac{1}{l^{(e)^2}}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix}u_j\\ u_k\end{bmatrix}+\frac{1}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}\begin{bmatrix}\dot{u}_j\\ \dot{u}_k\end{bmatrix}-\frac{1}{2l^{(e)}}\begin{bmatrix}-1 & 1\\ -1 & 1\end{bmatrix}\begin{bmatrix}u_j\\ u_k\end{bmatrix}+\frac{A}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}\begin{bmatrix}u_j\\ u_k\end{bmatrix}=\frac{P}{2}\begin{bmatrix}1\\ 1\end{bmatrix}$$

Where prime and dot denotes differentiation w.r.t y and time t respectively. Assembling the element equations for two consecutive elements $(y_{i-1}\le y\le y_i)$ and $(y_i\le y\le y_{i+1})$ following is obtained:

$$\frac{1}{l^{(e)^2}}\begin{bmatrix}1 & -1 & 0\\ -1 & 2 & -1\\ 0 & -1 & 1\end{bmatrix}\begin{bmatrix}u_{i-1}\\ u_i\\ u_{i+1}\end{bmatrix}+\frac{1}{6}\begin{bmatrix}2 & 1 & 0\\ 1 & 4 & 1\\ 0 & 1 & 2\end{bmatrix}\begin{bmatrix}\dot{u}_{i-1}\\ \dot{u}_i\\ \dot{u}_{i+1}\end{bmatrix}-\frac{1}{2l^{(e)}}\begin{bmatrix}-1 & 1 & 0\\ -1 & 0 & 1\\ 0 & -1 & 1\end{bmatrix}\begin{bmatrix}u_{i-1}\\ u_i\\ u_{i+1}\end{bmatrix}$$
$$+\frac{A}{6}\begin{bmatrix}2 & 1 & 0\\ 1 & 4 & 1\\ 0 & 1 & 2\end{bmatrix}\begin{bmatrix}u_{i-1}\\ u_i\\ u_{i+1}\end{bmatrix}=\frac{P}{2}\begin{bmatrix}1\\ 2\\ 1\end{bmatrix} \tag{20}$$

Now put row corresponding to the node i to zero, from equation (20) the difference schemes with $l^{(e)}=h$ is:

$$\frac{1}{h^2}\left[-u_{i-1}+2u_i-u_{i+1}\right]+\frac{1}{6}\left[\dot{u}_{i-1}+4\dot{u}_i+\dot{u}_{i+1}\right]-\frac{1}{2h}\left[-u_{i-1}+u_{i+1}\right]+\frac{A}{6}\left[u_{i-1}+4u_i+u_{i+1}\right]=P \tag{21}$$

Applying the trapezoidal rule, following system of equations in Crank Nicholson method are obtained:

$$A_1u_{i-1}^{n+1}+A_2u_i^{n+1}+A_3u_{i+1}^{n+1}=A_4u_{i-1}^{n}+A_5u_i^{n}+A_6u_{i+1}^{n}+P^* \tag{22}$$

Now from equations (12) and (13) following equations are obtained:

$$B_1\theta_{i-1}^{n+1}+B_2\theta_i^{n+1}+B_3\theta_{i+1}^{n+1}=B_4\theta_{i-1}^{n}+B_5\theta_i^{n}+B_6\theta_{i+1}^{n}+Q^* \tag{23}$$

$$D_1C_{i-1}^{n+1}+D_2C_i^{n+1}+D_3C_{i+1}^{n+1}=D_4C_{i-1}^{n}+D_5C_i^{n}+D_6C_{i+1}^{n} \tag{24}$$

Where

$A_1=2+Ak+3rh-6r,\ A_2=8+4Ak+12r,\ A_3=2+Ak-3rh-6r,\ A_4=2-Ak-3rh+6r,$
$A_5=8-4Ak-12r,\ A_6=2-Ak+3rh+6r,\quad B_1=2(\mathrm{Pr})+3rh(\mathrm{Pr})-6Zr,\ B_2=8(\mathrm{Pr})+12Zr,$
$B_3=2(\mathrm{Pr})-3rh(\mathrm{Pr})-6Zr,\ B_4=2(\mathrm{Pr})-3rh(\mathrm{Pr})+6Zr,\ B_5=8(\mathrm{Pr})-12Zr,$
$B_6=2(\mathrm{Pr})+3rh(\mathrm{Pr})+6Zr,\ D_1=2(Sc)+3rh(Sc)-6r,\ D_2=8(Sc)+12r,$
$D_3=2(Sc)-3rh(Sc)-6r,\ D_4=2(Sc)-3rh(Sc)+6r,\ D_5=8(Sc)-12r,$
$D_6=2(Sc)+3rh(Sc)+6r,\ P^*=12Phk=12hk(Gr)\theta_i^j+12hk(Gc)C_i^j,$

$$Q^*=12kQ=12k(\mathrm{Pr})(Ec)\left(\frac{\partial u_i^j}{\partial y}\right)^2,\ Z=1+\frac{4}{3R}$$

Here $r=\frac{k}{h^2}$ and h,k are mesh sizes along $y-$direction and tme – direction respectively. Index i refers to space and j refers to the time. In the equations (22), (23) and (24) taking $i=1(1)n$ and using boundary conditions (14), then the following system of equations are obtained:

$$A_iX_i=B_i;\ i=1(1)n \tag{25}$$

Where $A_i^{'}$s are matrices of order n and X_i, $B_i^{'}$s are column matrices having $n-$components.

The solutions of above system of equations are obtained by using Thomas algorithm for velocity, temperature and concentration. Also, numerical solutions for these equations are obtained by $C-$programme. In order to prove the convergence and stability of Galerkin finite element method, the same $C-$programme was run with smaller values of h and k and no significant change was observed in the values of u, θ and C. Hence the Galerkin finite element method is stable and convergent.

4. RESULTS AND DISCUSSIONS

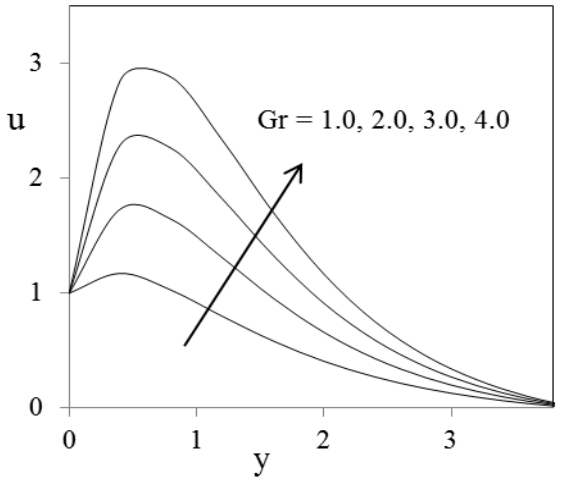

Figure - 2. Velocity profiles for different values of thermal Grashof number Gr

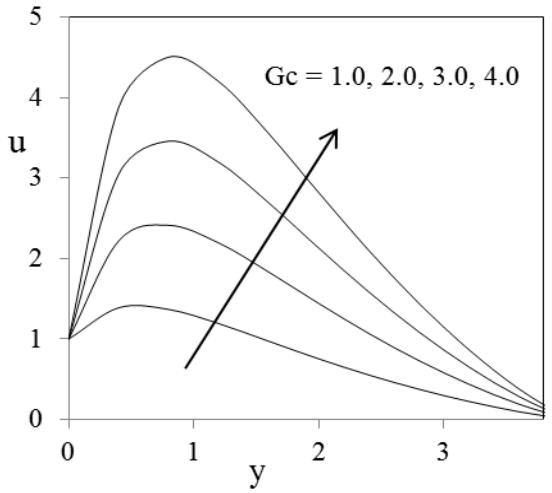

Figure - 3. Velocity profiles for different values of Solutal Grashof number Gc

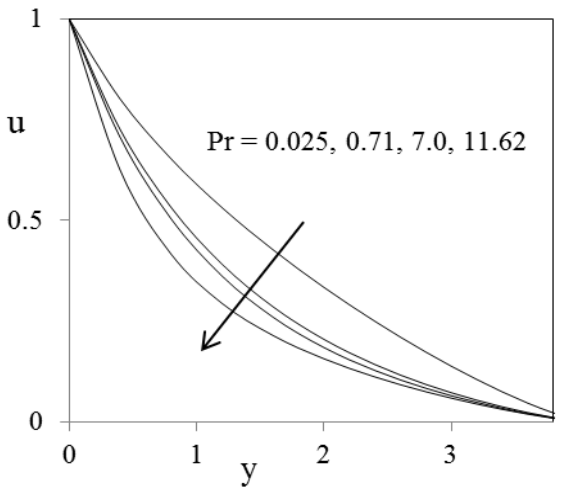

Figure - 4. Velocity profiles for different values of Prandtl number Pr

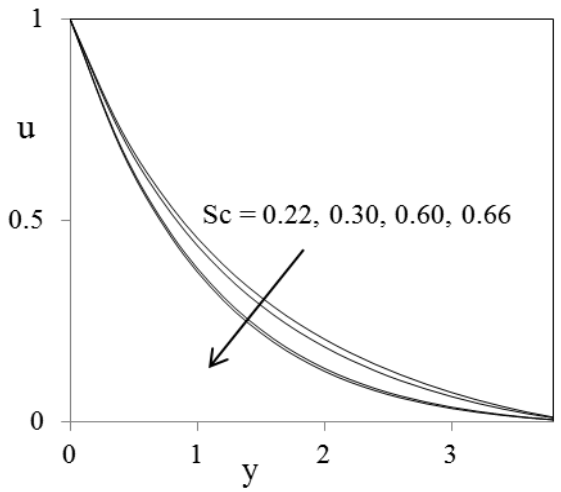

Figure - 5. Velocity profiles for different values of Schmidt number Sc

The profiles of velocity, temperature and concentration are shown in the figures from (2) to (15) respectively. Figures (2) and (3) exhibit the effect of Grashof number and Modified Grashof numbers on the velocity profile with other parameters are fixed. The Grashof number signifies the relative effect of the thermal buoyancy force to the viscous hydrodynamic force in the boundary layer. As expected, it is observed that there is a rise in the velocity due to the enhancement of thermal buoyancy force. Also, as Gr increases, the peak values of the velocity increases rapidly near the porous plate and then decays smoothly to the free stream velocity. The Modified Grashof number defines the ratio of the species buoyancy force to the viscous hydrodynamic force. As expected, the fluid velocity increases and the peak value is more distinctive due to increase in the species buoyancy force. The velocity distribution attains a distinctive maximum value in the vicinity of the plate and then decreases properly to approach the free stream value. It is noticed that the velocity increases with increasing values of the Modified Grashof number.

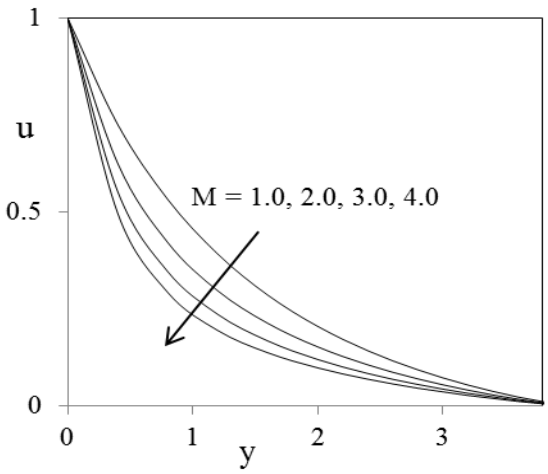

Figure - 6. Velocity profiles for different values of Hartmann number M

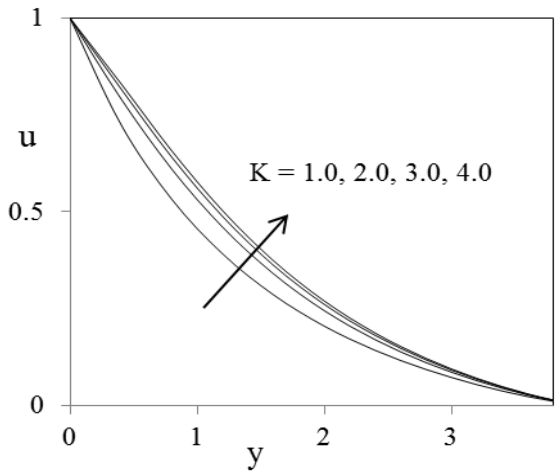

Figure - 7. Velocity profiles for different values of Permeability parameter K

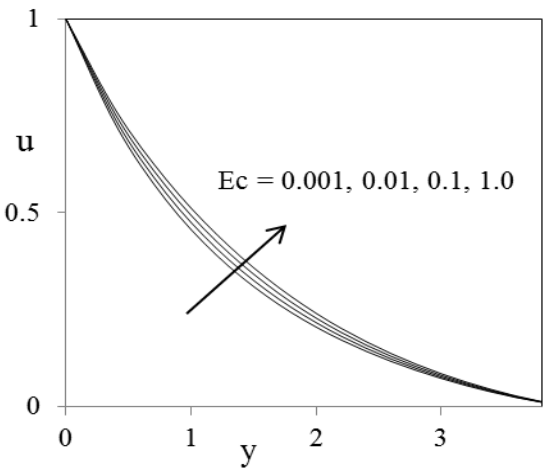

Figure - 8. Velocity profiles for different values of Eckert number Ec

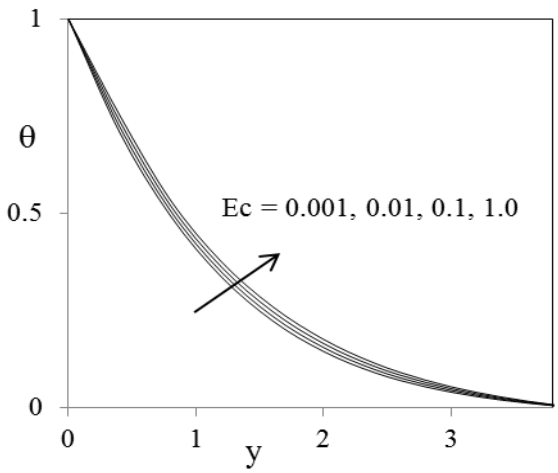

Figure - 9. Temperature profiles for different values of Eckert number Ec

Figure (4) depicts the effect of Prandtl number on velocity profiles in presence of foreign species such as Mercury (Pr = 0.025), Air (Pr = 0.71), Water (Pr = 7.00) and Water at $4^o C$ (Pr = 11.62) are shown in figure (4). We observe that from figure (4) the velocity decreases with increasing of Prandtl number $(\Pr)$. The nature of velocity profiles in presence of foreign species such as Hydrogen (Sc = 0.22), Helium (Sc = 0.30), Oxygen (Sc = 0.60) and Water – vapour (Sc = 0.66) are shown in figure (5). The flow field suffers a decrease in velocity at all points in presence of heavier diffusing species.

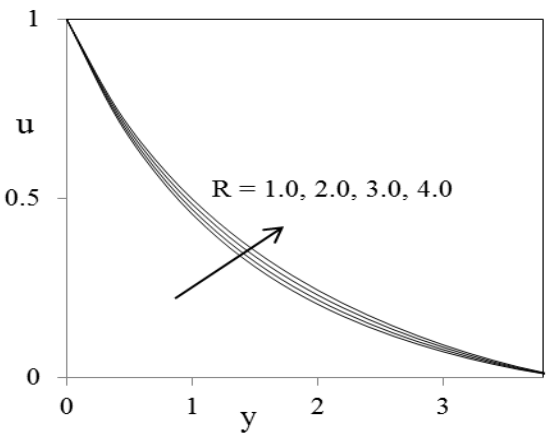

Figure - 10. Velocity profiles for different values of thermal radiation parameter R

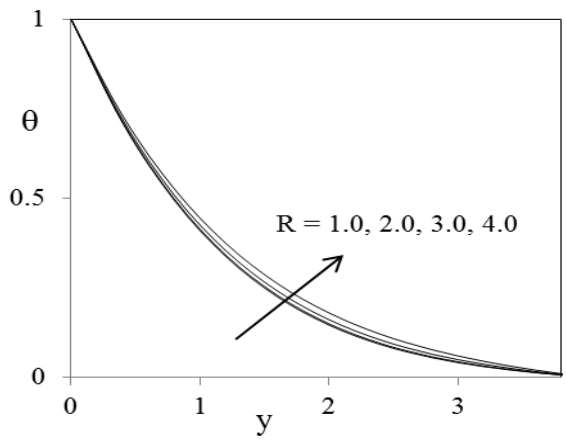

Figure - 11. Temperature profiles for different values of thermal radiation parameter R

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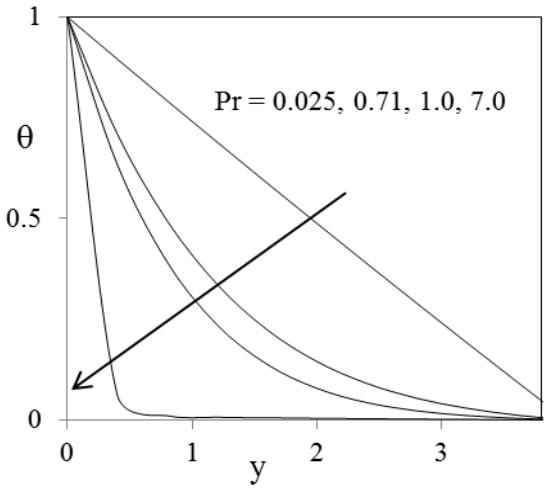

Figure - 12. Temperature profiles for different values of Prandtl number Pr

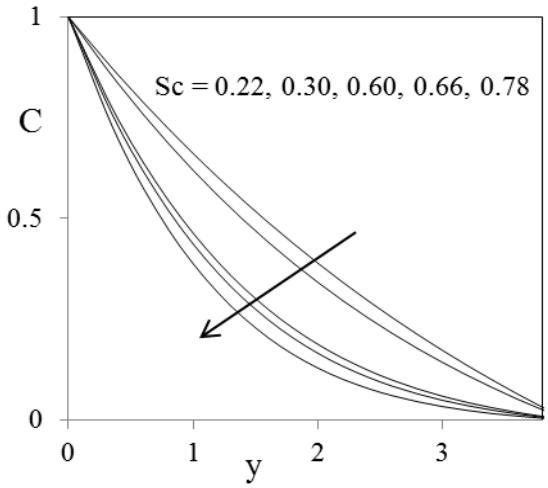

Figure - 13. Concentration profiles for different values of Schmidt number Sc

The effect of the Hartmann number (M) is shown in figure (6). It is observed that the velocity of the fluid decreases with the increase of the magnetic field number values. The decrease in the velocity as the Hartmann number (M) increases is because the presence of a magnetic field in an electrically conducting fluid introduces a force called the Lorentz force, which acts against the flow if the magnetic field is applied in the normal direction, as in the present study. This resistive force slows down the fluid velocity component as shown in figure (6). The effect of Permeability parameter (K) is presented in the figure (7). From this figure we observe that, the velocity is increases with increasing values of K. The influence of the viscous dissipation parameter i.e., the Eckert number (Ec) on the velocity and temperature are shown in figures (8) and (9) respectively. The Eckert number (Ec) expresses the relationship between the kinetic energy in the flow and the enthalpy. It embodies the conversion of kinetic energy into internal energy by work done against the viscous fluid stresses. Greater viscous dissipative heat causes a rise in the temperature as well as the velocity. This behavior is evident from figures (8) and (9). Figures (10) and (11) illustrate the variation of velocity and temperature function, for various values of the radiation absorption parameter (R). It is immediately apparent that velocity (u) as well as temperature (θ) clearly increase as R rises from 1 to 3. Velocity reaches a maximum in close proximity to the wall and then falls gradually to zero at the edge of the boundary layer. Inspection of figure (11) shows that for a small value of R ($R < 2$) temperature profile continuously decreases from the wall, while for higher values of R it increases attaining a maximum near the plate boundary and then decreases. As such there is a noticeable temperature overshoot with $R > 1$ since considerable thermal energy is imparted via the presence of a thermal radiation source to the fluid causing an elevation in temperatures near the wall.

In figure (12) we depict the effect of Prandtl number $(\Pr)$ on the temperature field. It is observed that an increase in the Prandtl number leads to decrease in the temperature field. Also, temperature field falls more rapidly for water in comparison to air and the temperature curve is exactly linear for mercury, which is more sensible towards change in temperature. From this observation it is conclude that mercury is most effective for maintaining temperature differences and can be used efficiently in the laboratory. Air can replace mercury, the effectiveness of maintaining temperature changes are much less than mercury. However, air can be better and cheap replacement for industrial purpose. This is because, either increase of kinematic viscosity or decrease of thermal conductivity leads to increase in the value of Prandtl number $(\Pr)$. Hence temperature decreases with increasing of Prandtl number $(\Pr)$. Figure (13) shows the concentration field due to variation in Schmidt number (Sc) for the gasses Hydrogen, Helium, Water – vapour, Oxygen and Ammonia. It is observed that concentration field is steadily for Hydrogen and falls rapidly for Oxygen and Ammonia in comparison to Water – vapour. Thus Hydrogen can be used for maintaining effective concentration field and Water – vapour can be used for maintaining normal concentration field. The profiles for skin – friction (τ) due to velocity under the effects of Grashof number, Modified Grashof number, Prandtl number, Schmidt number, Hartmann number, Eckert number, Thermal radiation parameter and Permeability parameter are presented in the table – 1 respectively. We observe from this table – 1, the skin – friction rises under the effects of Grashof number, Modified Grashof number, Eckert number, Thermal radiation parameter and Permeability parameter. And also falls under the effects of Prandtl number, Schmidt number Hartmann number.

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Table – 1: Skin – friction results (τ) for the values of Gr, Gc, $\Pr$, Sc, M, K, Ec and R

Gr	Gc	$\Pr$	Sc	M	K	Ec	R	τ
1.0	1.0	0.71	0.22	1.0	1.0	0.001	1.0	3.2954
2.0	1.0	0.71	0.22	1.0	1.0	0.001	1.0	4.4918
1.0	2.0	0.71	0.22	1.0	1.0	0.001	1.0	5.3946
1.0	1.0	7.00	0.22	1.0	1.0	0.001	1.0	2.1650
1.0	1.0	0.71	0.30	1.0	1.0	0.001	1.0	3.1168
1.0	1.0	0.71	0.22	2.0	1.0	0.001	1.0	2.6004
1.0	1.0	0.71	0.22	1.0	2.0	0.001	1.0	3.7905
1.0	1.0	0.71	0.22	1.0	1.0	0.100	1.0	3.2986
1.0	1.0	0.71	0.22	1.0	1.0	0.001	2.0	3.4094

Table – 2: Rate of heat transfer (Nu) values for different values of $\Pr$, Ec and R

$\Pr$	Ec	R	Nu
0.71	0.001	1.0	4.4972
7.00	0.001	1.0	1.0897
0.71	0.100	1.0	4.6270
0.71	0.001	2.0	4.6087

Table – 3: Rate of mass transfer (Sh) values for different values of Sc and Sr

Sc	Sr	Sh
0.22	1.0	6.9193
0.30	1.0	6.5249
0.22	2.0	7.0338

The profiles for Nusselt number (Nu) due to temperature profile under the effect of Prandtl number, Eckert number and Thermal radiation parameter are presented in the table – 2. From this table we observe that, the Nusselt number due to temperature profile rises under the effect of Eckert number and Thermal radiation parameter. And temperature falls under the effects of Prandtl number. The profiles for Sherwood number (Sh) due to concentration profiles under the effect of Schmidt number is presented in the table–3. We see from this table the Sherwood number due to concentration profile falls under the effect of Schmidt number. In order to ascertain the accuracy of the numerical results, the present results are compared with the previous results of Ramana Reddy *et al.* [27] for Gr = Gc = 1.0, M = 1.0, $\Pr$ = 0.71, K = 1.0 and Sc = 0.22 in table – 4. They are found to be in an excellent agreement.

Table – 4: Comparison of present Skin – friction results (τ) with the Skin – friction results (τ^*) obtained by Ramana Reddy *et al.* [27] for different values of Gr, Gc, Sc, $\Pr$, K and M

Gr	Gc	$\Pr$	Sc	M	K	τ	τ^*
1.0	1.0	0.71	0.22	1.0	1.0	2.5941	2.5705
2.0	1.0	0.71	0.22	1.0	1.0	3.0659	2.9984
1.0	2.0	0.71	0.22	1.0	1.0	3.6240	3.6191
1.0	1.0	7.00	0.22	1.0	1.0	2.1283	2.1150
1.0	1.0	0.71	0.30	1.0	1.0	1.9846	1.9716
1.0	1.0	0.71	0.22	2.0	1.0	1.5987	1.5889
1.0	1.0	0.71	0.22	1.0	2.0	3.0254	3.0018

5. CONCLUSIONS

We summarize below the following results of physical interest on the velocity, temperature and concentration distributions of the flow field and also on the skin – friction, rate of heat and mass transfer at the wall.

1. A growing Hartmann number or Prandtl number or Schmidt number retards the velocity of the flow field at all points.
2. The effects of increasing Grashof number or Modified Grashof number or Permeability parameter or Eckert number or Permeability parameter or Thermal radiation parameter are to accelerate velocity of the flow field at all points.
3. A growing Prandtl number decreases temperature of the flow field at all points and increases with increasing of Eckert number or Thermal radiation parameter.

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4. The Schmidt number decreases the concentration of the flow field at all points.
5. A growing Hartmann number or Prandtl number or Schmidt number decreases the skin – friction while increasing Grashof number or Modified Grashof number or Permeability parameter or Eckert number or Thermal radiation parameter increases the skin – friction.
6. The rate of heat transfer is decreasing with increasing of Prandtl number and increases with increasing of Eckert number and Thermal radiation parameter.
7. The rate of mass transfer is decreasing with increasing of Schmidt number.
8. On comparing the skin – friction (τ) results with the skin – friction (τ^*) results of Ramana Reddy *et al.* [27] it can be seen that they agree very well.

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