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SOFT πg-OPERATORS IN SOFT TOPOLOGICAL SPACES

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ABSTRACT

The purpose of this paper is to introduce some new concepts in soft topological spaces such as soft πg-closed set and investigate its relationship with other soft closed sets. Moreover we define and discuss the properties of soft πg-operators and the separation axiom namely soft πg - $T_{\frac{1}{2}}$-space which may be of value for further research.

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Keywords: *soft πg-closed set, soft πg-open set, soft πg-closure, soft πg- interior, soft πg-neighbourhood and soft πg - $T_{\frac{1}{2}}$-space.*

1. INTRODUCTION

Soft set theory was first initiated by Molodtsov [9] in 1999 as a general mathematical tool for dealing with uncertain sets, not clearly defined objects. In recent years the development in the field of soft set theory and its application has been taking place in a rapid pace. This is because of the general nature of parametrization expressed by a soft set. Recently Shabir and Naz [12] introduced the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters. K. Kannan **[5]** studied soft generalized closed sets in soft topological spaces along with its properties. In the present study, we introduce some new concepts in soft topological spaces such as soft πg-closed sets and soft πg-open sets and derive some of their properties.

2. PRELIMINARIES

Definition: 2.1[7] Let U be the initial universe and P (U) denote the power set of U. Let E denote the set of all parameters. Let A be a non-empty subset of E. A pair (F, A) is called a soft set over U, where F is a mapping given by F: A→P(U). In other words, a soft set over U is a parameterized family of subsets of the universe U. For $\mathcal{E} \in$ A, F ($\mathcal{E}$) may be considered as the set $\mathcal{E}$- approximate elements of the soft set (F, A).

Definition: 2.2[9] For two soft sets (F, A) and (G, B) over a common universe U, we say that (F, A) is a soft subset of (G, B) if (1) A $\tilde{\subseteq}$ B and (2) for all e ∈ A, F (e) and G(e) are identical approximations. We write (F, A) $\tilde{\subseteq}$ (G, B). (F, A) is said to be a soft super set of (G, B), if (G, B) is a soft subset of (F, A). We denote it by (F, A)$\tilde{\supseteq}$(G, B).

Definition: 2.3[7] Two soft sets (F, A) and (G, B) over a common universe U are said to be soft equal if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).

Definition: 2.4[7] The union of two soft sets of (F, A) and (G, B) over the common universe U is the soft set (H, C), where C = A ∪ B and for all e ∈ C,

$$H(e) = \begin{cases} F(e) & \text{if } e \in A - B \\ G(e) & \text{if } e \in B - A \\ F(e) \cup G(e) & \text{if } e \in A \cap B. \end{cases}$$

We write (F, A) ∪ (G, B) = (H, C).

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Definition: 2.5 [9] The intersection (H, C) of two soft sets (F, A) and (G, B) over a common universe U, denoted (F, A) ∩ (G, B), is defined as C = A ∩ B, and H(e) = F (e) ∩ G(e) for all e ∈ C.

Definition: 2.6[12] Let τ be the collection of soft sets over X, then τ is said to be a soft topology on X if (1) Φ, X belong to τ, (2) the union of any number of soft sets in τ belongs to τ, (3) the intersection of any two soft sets in τ belongs to τ.The triplet (X,τ, E) is called a soft topological space over X. Let (X, τ, E) be a soft space over X, then the members of τ are said to be soft open sets in X.

Definition: 2.7[1] A subset A of a topological space (X, τ, E) is called a soft pre open set, if (A,E) $\tilde{\subseteq}$ int(cl(A,E)). The complement of soft pre open set is soft pre closed set.

Definition: 2.8[3] A subset A of a topological space (X, τ, E) is called a soft semi open set, if (A,E) $\tilde{\subseteq}$cl(int(A,E)) .

Definition: 2.9[1] A subset (A, E) of a topological space X is called soft semi-generalized closed (soft sg-closed) if scl(A,E) $\tilde{\subseteq}$ (U,E) whenever (A,E) $\tilde{\subseteq}$ (U,E) and (U,E) is soft semi-open in X.

Definition: 2.10[1] A subset (A, E) of a topological space X is called soft β-closed, if int(cl (int(A,E))) $\tilde{\subseteq}$ (A,E)

Definition: 2.11 [5] A subset (A, E) of a topological space X is called soft generalized-closed **(**soft g **-**closed**)** if cl(A,E) $\tilde{\subseteq}$ (U,E) whenever (A,E) $\tilde{\subseteq}$ (U,E) and (U,E) is soft open in X.

Definition: 2.12[11] A subset (A, E) of a topological space X is called soft regular closed (soft r-closed), if cl(int(A,E))=(A,E). The complement of soft regular closed set is soft regular open set.

Definition: 2.13[11] The finite union of soft regular open sets is said to be soft π-open. The complement of soft π-open is said to be soft π-closed.

Definition: 2.14[11] A subset (A, E) of a topological space X is called soft πg-closed in a soft topological space (X,τ, E), if cl(A, E) $\tilde{\subseteq}$ (U, E) whenever (A, E) $\tilde{\subseteq}$ (U, E) and (U, E) is soft π-open in X.

Definition: 2.15[11] A subset (A, E) of a topological space X is called soft πsg-closed in a soft topological space (X,τ, E), if scl(A, E) $\tilde{\subseteq}$ (U, E) whenever (A, E) $\tilde{\subseteq}$ (U, E) and (U, E) is soft π-open in X.

Definition: 2.16[11] A subset (A, E) of a topological space X is called soft gpr-closed in a soft topological space (X, τ, E), if pcl(A, E) $\tilde{\subseteq}$ (U, E) whenever (A, E) $\tilde{\subseteq}$ (U, E) and (U, E) is soft regular open in X.

Definition: 2.17[1] A subset (A, E) of a topological space X is called soft regular generalised closed soft (rg-closed) if cl(A,E) $\tilde{\subseteq}$ (U, E) whenever (A, E) $\tilde{\subseteq}$ (U, E) and (U, E) is soft regular open in X.

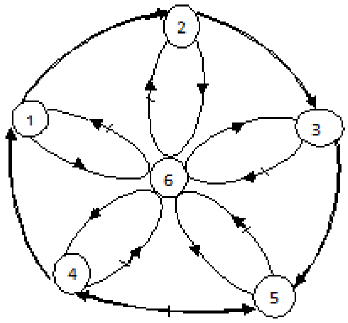
3. SOFT πg-CLOSED SET

Definition: 3.1[11] A subset (A, E) of a soft topological space (X,τ, E) is called soft πg-closed set, if cl(A, E) $\tilde{\subseteq}$ (U, E) whenever (A, E) $\tilde{\subseteq}$ (U, E) and (U, E) is soft π-open in X.

Proposition: 3.2
(i) Every soft closed set is soft πg-closed.
(ii) Every soft πg-closed set is soft gpr-closed set.
(iii) Every soft πg-closed set is soft rg-closed set.
(iv) Every soft g-closed set is soft πg-closed set.

Proof: Obvious and straightforward.

From the above we have the following implications;
1. Soft closed set
1. Soft g-closed set
2. Soft rg-closed set
3. Soft gpr-closed set
4. Soft πsg-closed set
5. Soft πg- closed set



The converse of the above are not true in general.

Example: 3.4 Let X = {a, b, c, d}, E= {e_1, e_2}.

Let F_1, F_2,…, F_6 are functions from A to P(X) and are defined as follows:
(F_1, E) = {(e_1, {c}), (e_2, {a})
(F_2, E) = {(e_1, {d}), (e_2, {b})
(F_3, E) = {(e_1, {c, d}), (e_2, {a, b})
(F_4, E) = {(e_1, {a, d}), (e_2,{b, d})
(F_5, E) = {(e_1, {b, c, d}), (e_2,{a, b, c})
(F_6, E) = {(e_1, {a, c, d}), (e_2 ,{a, b, d}).

Then τ = {$\widetilde{\emptyset}$, $\widetilde{X}$, (F_1, E),…,(F_6,E)}.

(i). Here the soft set (A, E) = {(e_1, {b, c, d}), (e_2, {a, b, c}) is soft πg-closed set but not soft closed.
(ii). Here the soft set (A, E) = {(e_1, {b, c, d}), (e_2, {a, b, c}) is soft πg-closed set but not soft rg -closed.

Proposition: 3.5 Every finite union of soft πg-closed set is soft πg-closed.

Proof: Let (A, E) and (B, E) be soft πg-closed subset of X. Let (U, E) be a soft π-open in (X,τ, E), such that (A∪B, E) $\widetilde{\subseteq}$ (U, E). Then cl(A, E) $\widetilde{\subseteq}$ (U, E) and cl(B, E) $\widetilde{\subseteq}$ (U, E). Therefore cl(A∪B, E) $\widetilde{\subseteq}$ cl(A, E) ∪ cl(B, E) $\widetilde{\subseteq}$ (U, E). This implies that cl(A∪B, E) $\widetilde{\subseteq}$(U, E). Hence (A, E) ∪ (B, E) is a soft πg-closed set.

Remark: 3.6 Finite intersection of soft πg-closed set is not soft πg-closed set.

Example: 3.7 Let X = {a, b, c}, E = {e_1, e_2}.
Let F_1, F_2,…, F_5 are functions from A to P(X) and are defined as follows:
(F_1, E) = {(e_1, {b}), (e_2, {a})}
(F_2, E) = {(e_1, {b, c}), (e_2, {a, b})}
(F_3, E) = {(e_1, {a, b}), (e_2, {a, b})}
(F_4, E) = {(e_1, {b}), (e_2, {a, c})}
(F_5, E) = {(e_1, {a, b}), (e_2, {X})}
Then τ = {$\widetilde{\emptyset}$, $\widetilde{X}$, (F_1,E),…,(F_5,E)}.
Soft πg-closed sets are {$\widetilde{\emptyset}$, $\widetilde{X}$, (F_1,E),…,(F_5,E)}.

Let A = {(e_1, {b, c}), (e_2, {a, b}) and B = {(e_1,{a, b}), (e_2, {X })}. Then A ∩ B = {(e_1, {b}), (e_2, {a, b})}
which is not soft πg-closed set.

Theorem: 3.8 If (A, E) is a soft πg-closed set of X such that (A, E) $\widetilde{\subseteq}$ (B, E) $\widetilde{\subseteq}$ cl(A, E) then (B, E) is also soft πg-closed set of X.

Proof: Let (B, E) $\widetilde{\subseteq}$ (U, E) where (U, E) is soft π-open. Then (A, E) $\widetilde{\subseteq}$ (B, E) implies (A, E) $\widetilde{\subseteq}$ (U, E). Since (A, E) is a soft πg-closed set, cl(A, E) $\widetilde{\subseteq}$ (U, E). Given (B, E) $\widetilde{\subseteq}$ cl(A, E).Hence cl(B, E) $\widetilde{\subseteq}$cl(cl(A, E)) $\widetilde{\subseteq}$ cl(A, E) $\widetilde{\subseteq}$ (U, E) which implies cl(B, E) $\widetilde{\subseteq}$ (U, E). Therefore (B, E) is a soft πg-closed set.

Theorem: 3.9 A soft set (A, E) is soft πg-closed set, then cl(A, E)−(A, E) contains no non-empty soft π-closed set.

Proof: Let (F, E) be a non-empty soft π-closed set, such that (F, E) $\widetilde{\subseteq}$ cl(A, E) −(A, E)implies that (F, E) = X−(A, E). Since (A,E) is soft πg-closed set, X−(A, E) is soft πg-open set. Since (F, E) is soft π-closed set, X−(F, E) is soft π-open set. Since cl(A, E) $\widetilde{\subseteq}$ X−(F, E), (F, E) $\widetilde{\subseteq}$ X−(A, E). Thus (F, E) $\widetilde{\subseteq}$ ∅ which is a contradiction. Therefore (F, E) = ∅. Hence cl(A, E)−(A, E) contains no non-empty soft π-closed set.

Corollary: 3.10 Let (A, E) be a soft πg-closed set in (X, τ, E) then (A, E) is soft closed if and only if cl(A, E)−(A, E) is soft π-closed.

4. SOFT πg-OPEN SET

Definition: 4.1 A subset (A, E) of a topological space X is called soft πg-open in a soft topological space (X, τ, E), if (F, E) $\tilde{\subseteq}$ int(A, E) whenever (F, E) $\tilde{\subseteq}$ (A, E) and (F, E) is soft π-closed in X.

Theorem: 4.2 If (A, E) is a soft πg-open set of X and int (A, E) $\tilde{\subseteq}$ (B, E) $\tilde{\subseteq}$ (A, E) then (B, E) is also soft πg-open set of X.

Proof: Let (A, E) be soft πg-open in X. Suppose (G, E) is a soft π -closed set such that (G, E) $\tilde{\subseteq}$ (B, E).

By assumption (B, E) $\tilde{\subseteq}$ (A, E), we have (G, E) $\tilde{\subseteq}$ (A, E). Since (A, E) is a soft πg-open set (G, E) $\tilde{\subseteq}$ int(A, E).Then int(int(A, E))$\tilde{\subseteq}$ int(B, E) implies that int(A, E) $\tilde{\subseteq}$ int(B, E). Hence (G, E) $\tilde{\subseteq}$int(A, E))$\tilde{\subseteq}$ int(B, E) implies that (G, E) $\tilde{\subseteq}$ int(B, E). Thus (B, E) is soft πg-open set.

Theorem: 4.3 If (A, E) and (B, E) are soft πg-open set then (A ∩ B, E) is also soft πg-open set.

Proof: Let (A, E) and (B, E) are soft πg-open sets. Suppose (G, E) is soft π-closed set such that (G, E) $\tilde{\subseteq}$ (A ∩ B, E). Then (G, E) $\tilde{\subseteq}$ (A, E) and (G, E) $\tilde{\subseteq}$ (B, E). Since (A, E) and (B, E) are soft πg-open sets, (G, E) $\tilde{\subseteq}$ int(A, E) and (G, E) $\tilde{\subseteq}$ int(B, E). Therefore (G, E) $\tilde{\subseteq}$ int (A, E) ∩ int (B, E). Thus (G, E) $\tilde{\subseteq}$ int(A ∩ B, E). Hence (A ∩ B, E) is soft πg-open set.

Remark: 4.4 Finite union of soft πg-open set is not soft πg-open set.

Example: 4.5 In Example: 3.9 Let A = {(e_1, {a}), (e_2, {c}) and B = {(e_1, {c}), (e_2, {c})}. Then A ∪ B = {(e_1, {a, c}), (e_2, {c})} which is not soft πg-open set.

5. SOFT πg- OPERATORS

Definition: 5.1 Let (X, τ, E) be a soft topological space and (x, E) ∈X. A subset (A, E) of X is called a soft πg-neighbourhood (Soft πg-nbhd) of (x, E), if there exists a Soft πg-open set (U, E) such that (x, E) ∈ (U, E) $\tilde{\subseteq}$ (A, E).

Definition: 5.2 Let (X, τ, E) be a soft topological space and (A, E) be a subset of X. A point (x, E) ∈ (A, E) is said to be Soft πg- interior point of (A, E), if (A, E) is a Soft πg-nbhd of (x, E).

The set of all Soft πg- interior points of (A, E) is called the Soft πg-interior of (A, E) and it is denoted by Soft πg- int (A, E).

Definition: 5.3 Let (X, τ, E) be a soft topological space and (A, E) $\tilde{\subseteq}$ X. Then Soft πg- int (A, E) is the union of all Soft πg- open sets contained in (A, E). That is largest Soft πg-open set $\tilde{\subseteq}$ (A, E).

Proposition: 5.4 Let (A, E) and (B, E) be subsets of (X, τ, E). Then
1. Soft πg- int (Φ) = Φ and Soft πg- int ($\tilde{X}$) =$\tilde{X}$.
2. Soft πg- int(A, E) $\tilde{\subseteq}$ (A, E).
3. If (B , E) is any Soft πg-open set contained in (A, E) then (B, E) $\tilde{\subseteq}$ Soft πg- int(A, E).
4. If (A, E) $\tilde{\subseteq}$ (B, E) then Soft πg- int (A, E) $\tilde{\subseteq}$ Soft πg- int (B, E).
5. Soft πg- int (Soft πg- int (A, E)) = Soft πg- int(A, E).

Proof: Obvious

Theorem: 5.5 If a subset (A, E) of (X, τ, E) is soft πg-open set, then Soft πg- int(A, E) = (A, E).

Proof: Let (A, E) be a Soft πg- open subset of (X, τ, E).We know that Soft πg- int(A, E) $\tilde{\subseteq}$ (A, E).

Since (A, E) is Soft πg- open set contained in (A, E), (A, E) $\tilde{\subseteq}$ Soft πg- int(A, E). Hence
Soft πg- int(A, E) = (A, E).

Remark: 5.6 The converse of the above is not true as seen in the following example.

Example: 5.7 Let X = $\{h_1, h_2, h_3\}$, E= $\{e_1, e_2\}$.

Let G, K, F be the mappings from A to P(X).

Then (G, E) =$\{(e_1, \{h_2\}), (e_2, \{h_1\})\}$
(K, E) = $\{(e_1, \{h_2, h_3\}), (e_2, \{h_1, h_2\})\}$
(F, E) = $\{(e_1, \{h_1, h_2\}), (e_2, \{h_1, h_3\})\}$

$\tau = \{\widetilde{\emptyset}, \widetilde{X}$, (G, E), (K, E), (F, E) } .

Soft πg-open sets are $\{\widetilde{\emptyset}, \widetilde{X}$, $(K, E)^C$ $(K, E)^C$, $(F, E)^C\}$.

Let (A, E) = (F, E) and Soft πg-int (A, E) = (F, E), but (F, E) is not a soft πg-open set in (X, τ, E).

Theorem: 5.8 If (A, E) and (B, E) are subsets of X, then Soft πg- int(A, E)∪ Soft πg- int(B, E) $\widetilde{\subseteq}$ Soft πg- int(A∪B, E).

Proof: Let (A, E) $\widetilde{\subseteq}$(A∪B, E) and (B, E) $\widetilde{\subseteq}$(A∪B, E). Then Soft πg- int (A, E) $\widetilde{\subseteq}$ Soft πg- int (A∪B, E) and Soft πg- int(B, E) $\widetilde{\subseteq}$ Soft πg- int(A∪B, E). Therefore Soft πg- int(A, E) ∪Soft πg- int(B, E) $\widetilde{\subseteq}$ Soft πg- int(A∪B, E).

Definition: 5.9 For a subset (A, E) of (X, τ, E), the Soft πg- cl(A, E) is the intersection of all Soft πg- closed sets containing (A, E).

Proposition: 5.10 Let (A, E) and (B, E) be subsets of (X, τ, E). Then
1. Soft πg- cl (Φ) = Φ and Soft πg- cl ($\widetilde{X}$) =$\widetilde{X}$.
2 (A, E) $\widetilde{\subseteq}$ Soft πg- cl (A, E).
3. If (B, E) is any Soft πg-closed set containing (A, E) then Soft πg-cl (A, E) $\widetilde{\subseteq}$(B, E).
4. If (A, E) $\widetilde{\subseteq}$ (B, E) then Soft πg- cl(A, E) $\widetilde{\subseteq}$ Soft πg- cl(B, E).
5. Soft πg- cl(A, E) = Soft πg- cl(Soft πg- cl(A, E)) .

Proof: Obvious

Proposition: 5.11 If (A, E) $\widetilde{\subseteq}$ (X, τ, E) is Soft πg-closed then Soft πg- cl(A, E) = (A, E).

Proof: Let (A, E) be a Soft πg-closed subset of (X, τ, E).We know that (A,E) $\widetilde{\subseteq}$ Soft πg- cl(A, E).
Since (A, E) is Soft πg- closed set containing (A, E), Soft πg- cl(A, E) $\widetilde{\subseteq}$ (A, E). Hence Soft πg- cl(A, E) = (A, E).

Remark: 5.12 The converse of the above is not true as seen in the following example.

Example: 5.13 Let X = $\{h_1, h_2, h_3\}$, E= $\{e_1, e_2\}$.

Let G, K, F be the mappings from A to P(X) defined by $G(e_1) = \{h_2\}$, $G(e_2) = \{h_1\}$; $K(e_1) = \{h_2, h_3\}$, $K(e_2) = \{h_1, h_2\}$; $F(e_1) = \{h_1, h_2\}$, $F(e_2) = \{h_1, h_3\}$;

Then (G, E) = $\{(e_1, \{h_2\}), (e_2, \{h_1\})\}$
(K, E) = $\{(e_1, \{h_2, h_3\}), (e_2, \{h_1, h_2\})\}$
(F, E) = $\{(e_1, \{h_1, h_2\}), (e_2, \{h_1, h_3\})\}$

$\tau = \{\widetilde{\emptyset}, \widetilde{X}$, (G, E), (K, E), (F, E)} soft πg-closed sets are = $\{\widetilde{\emptyset}, \widetilde{X}$, (G, E), (K, E), (F, E)}.

Let (A, E) = $(G,E)^C$ and soft πg-cl(A, E) =$(G,E)^C$, but$(G,E)^C$ is not a soft πg-closed set in (X, τ, E).

Proposition: 5.14 If (A, E) is a subset of X then Soft πg- cl(A, E) $\widetilde{\subseteq}$ cl(A, E).

Proof: Since every soft closed set is Soft πg-closed set. By definition cl(A, E) = ∩{(F, E):(A, E) $\widetilde{\subseteq}$ (F, E) ∈ C(X)}. If (A, E) $\widetilde{\subseteq}$ (F, E) ∈ C(X) then(A, E) $\widetilde{\subseteq}$ (F, E) ∈ Soft πgC(X). That is Soft πg-cl (A, E) $\widetilde{\subseteq}$ (F, E).Therefore Soft πg-cl(A, E)$\widetilde{\subseteq}$ ∩{(F, E) :(A, E) $\widetilde{\subseteq}$ (F, E)∈ C(X)}= cl(A, E).

Hence Soft πg- cl(A, E) $\widetilde{\subseteq}$ cl(A, E).

Lemma: 5.15 Let (A, E) be subset of (X, τ, E) and (x, E) ∈ X then (x, E) ∈ Soft πg- cl(A, E) if and only if (V, E) ∩ (A, E) ≠ ∅ for every Soft πg- open set (V, E) containing (x, E).

Proof:
Necessity: Let (x, E) ∈ Soft πg- cl (A, E). Suppose there exists a Soft πg- open set (V, E) containing (x, E) such that (V, E) ⋂ (A, E) = ∅ Since (A, E) $\tilde{\subseteq}$ X− (V, E), (x, E) ∈ Soft πg- cl(A, E) which is a contradiction. Therefore (V, E) ⋂ (A, E)≠ ∅.

Sufficiency: Suppose (x, E) ∉ Soft πg- cl(A, E). Then there exists a Soft πg-closed set (F, E) containing (A, E) such that (x, E) ∉ (F, E). Then (x, E) ∈ X− (F, E) and X− (F, E) is Soft πg-open set. Also X− (F, E) ⋂ (A, E) = ∅ which is a contradiction. Therefore (x, E) ∈ Soft πg- cl(A , E).

Lemma: 5.16 Let (A, E) be subsets of (X, τ, E). Then X−Soft πg- int(A , E) = Soft πg- cl(X− (A, E)).

Proof: Let (x, E) ∈ X−Soft πg- int(A, E). Then (x, E) ∉ Soft πg- int (A, E). That is every Soft πg-open set (B, E) containing (x, E) is such that (B,E) $\tilde{\nsubseteq}$ (A, E) which implies that every Soft πg-open set (B, E) containing (x, E) intersects X− (A, E). Therefore (x, E) ∈ Soft πg- cl(X− (A, E)). Hence X−Soft πg- int(A, E) $\tilde{\subseteq}$ Soft πg-cl(A, E).

Conversely (x, E) ∈ Soft πg- cl(X− (A, E)). Then every Soft πg-open set (B, E) containing (x, E) intersects X− (A, E). That is every Soft πg-open set (B , E) containing (x, E) is such that (B,E) $\tilde{\nsubseteq}$ (A, E) implies (x, E) ∈ Soft πg-int (A, E). Thus Soft πg- cl(X− (A, E)) $\tilde{\subseteq}$ Soft πg-int(A , E) and X−Soft πg- int(A , E) = Soft πg- cl(X− (A, E)).

Similarly X−Soft πg- cl(A , E) = Soft πg- int(X− (A, E)).

Proposition: 5.17 Let (A, E) be a Soft πg-open set and (B, E) be any set in X. If (A, E) ∩ (B, E) = ∅ then (A, E) ∩Soft πg- cl(B, E) = ∅.

Proof: Suppose (A, E) ∩ Soft πg- cl (B, E) ≠ ∅ and (x, E) ∈ (A, E) ∩ Soft πg- cl (B, E). Then (x, E) ∈ (A, E) and (x, E) ∈ (A, E) ∩ Soft πg- cl (B, E). Therefore (A, E) ∩ (B, E) ≠ ∅ which is a contradiction. Hence (A, E) ∩ Soft πg- cl (B, E) = ∅.

6. SOFT πg-$T_{\frac{1}{2}}$ –SPACES

Definition: 6.1 A soft topological space X is a soft πg - $T_{\frac{1}{2}}$-space if every soft πg -closed set is soft regular closed.

Theorem: 6.2 If the soft topological space (X, τ, E) is soft πg -$T_{\frac{1}{2}}$ –space, Every singleton of X is either soft π-closed or soft regular open.

Proof: Let (A, E) be a soft singleton set in X and let us assume that (A, E) is not soft π-closed. Then X-(A, E) is not soft π-open. Hence X-(A, E) is trivially soft πg -closed. Since every soft πg -closed set is soft regular closed, X-(A, E) is soft regular closed. Hence (A, E) is soft regular open.

Theorem: 6.3
(i) Soft RO(X,τ, E) $\tilde{\subseteq}$ Soft πGO(X, τ,E)
(ii) A soft topological space (X,τ, E) is πg -$T_{\frac{1}{2}}$ -space iff soft RO(X, τ,E) = soft πGO(X, τ, E)

Proof:
(i) Let (A, E) be soft regular open. Then X−(A, E) is soft regular closed and so soft πg -closed. Hence (A, E) is soft πg -open and soft RO(X, τ, E)$\tilde{\subseteq}$ soft πGO(X, τ,E)

(ii) **Necessity:** Let (X, τ, E) be soft πg -$T_{\frac{1}{2}}$ space. Let (A, E) ∈ $soft$ πGO(X, τ, E) . Then X−(A,E) is soft πg -closed. Since the space soft πg -$T_{\frac{1}{2}}$ -space, X−(A,E) is soft regular closed. The above implies (A, E) is soft regular open in X. Hence Soft πGO(X, τ, E) =Soft RO(X, τ,E).

Suffiency: Let soft πGO(X, τ, E) =Soft RO(X, τ, E). Let (A, E) be soft πg -closed. Then X− (A, E) is soft πg -open. Thus X−(A, E) ∈ Soft RO(X, τ,E) and hence (A,E) is soft regular closed.
Thus X−(A,E) ∈ Soft RO(X, τ,E) . Hence (A, E) is soft regular closed.

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