

**EFFECT OF RADIATION ABSORPTION ON UNSTEADY CONVECTIVE HEAT AND MASS  
TRANSFER FLOW OF A VISCOUS FLUID THROUGH A POROUS MEDIUM  
IN A HORIZONTAL WAVY CHANNEL WITH OSCILLATORY FLUX AND HEAT SOURCES**

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**ABSTRACT**

*We investigate the effect of radiation absorption on unsteady free convective heat and mass transfer flow of a viscous, chemically reacting fluid through a porous medium in a horizontal wavy channel. The unsteadiness in the flow is due to the oscillatory flux in the flow region. The coupled equations governing the flow, heat and mass transfer have been solved by using a perturbation technique with the slope  $\delta$  of the wavy wall as the perturbation parameter. The velocity, the temperature, the concentration, the rate of heat and mass transfer are derived and are analysed for different variations of the governing parameters  $\beta, N, \alpha, k, Q_1$  and  $\gamma$ .*

**Keywords:** Oscillatory flux, Chemical reaction, wavy channel, Heat sources.

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**1. INTRODUCTION**

Combined heat and mass transfer by free convection under boundary layer approximations has been studied by Bejan and Khair [2], Lai and Kulacki [10] and Murthy and Singh [14]. Coupled heat and mass transfer by mixed convection in Darcian fluid-saturated porous media has been analysed by Lai [10]. The free convection heat and mass transfer in a porous enclosure has been studied recently by Angirasa *et al* [1]. The combined effects of thermal and mass diffusion in channel flows has been studied in recent times by a few authors, notably Nelson and Wood [16,17], Lee *et al* [11] and others [20,24].

In recent years, energy and material saving considerations have prompted an expansion of the efforts at producing efficient heat exchanger equipment through augmentation of heat transfer. It has been established [6] that channels with diverging – converging geometries augment the transportation of heat transfer and momentum. As the fluid flows through a tortuous path viz., the dilated – constricted geometry, there will be more intimate contact between them. The flow takes place both axially (primary) and transversely (secondary) with the secondary velocity being towards the axis in the fluid bulk rather than confining within a thin layer as in straight channels. Hence it is advantageous to go for converging – diverging geometries for improving the design of heat transfer equipment. Vajravelu and Nayfeh [21] have investigated the influence of the wall waviness on friction and pressure drop of the generated Couette flow. Vajravelu and Sastry [23] have analysed the free convection heat transfer in a viscous, incompressible fluid confined between long vertical wavy walls in the presence of constant heat source. Later Vajravelu and Debnath [22] have extended this study to convective flow in a vertical wavy channel in four different geometrical configurations. This problem has been extended to the case of wavy walls by McMichael and Deutsch [13], Deshikachar *et al* [5], Rao *et al* [18a] and Sree Ramachandra Murthy [19]. Hyan Goo Kwon *et al* [8] have analyzed that the Flow and heat/mass transfer in a wavy duct with various corrugation angles in two dimensional flow regimes. Comini *et al* [3] have analyzed the Convective heat and mass transfer in wavy finned-tube exchangers.

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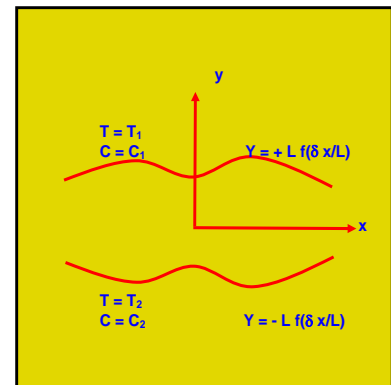
In many chemical engineering processes, there does occur the chemical reaction between a foreign mass and the fluid in which the plate is moving. These processes take place in numerous industrial applications viz., polymer production, manufacturing of ceramics or glassware and food processing .Das et al[4] have studied the effects of mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction. Muthukumara-swamy[15] has studied the effects of reaction on a long surface with suction. Recently Gnaneswar [7] has studied radiation and mass transfer on an unsteady two-dimensional laminar convective boundary layer flow of a viscous incompressible chemically reacting fluid along a semi-infinite vertical plate with suction by taking into account the effects of viscous dissipation. Kandaswamy *et al* [9] have discussed the effects of chemical reaction, heat and mass transfer on boundary layer flow over a porous wedge with heat radiation in the presence of suction or injection. Madhusudan Reddy [12] has analysed the effect of chemical reaction on double diffusive heat transfer flow of a viscous fluid in a wavy channel.

In this paper we discuss the effect of chemical reaction and radiation absorption on unsteady free convective heat and mass transfer flow through a porous medium in a horizontal wavy channel the unsteadiness in the flow is due to the oscillatory flux in the flow region. The coupled equations governing the flow, heat and mass transfer have been solved by using a perturbation technique with the slope  $\delta$  of the wavy wall as the perturbation parameter. The expression for the velocity, the temperature, the concentration, the rate of heat and mass transfer are derived and are analysed for different variations of the governing parameters  $\beta, N, \alpha, Q_1, k$  and  $\gamma$ .

## 2. FORMULATION OF THE PROBLEM

We consider the effect of chemical reaction on the unsteady motion of viscous, incompressible fluid through a porous medium in a horizontal channel bounded by wavy walls. The thermal buoyancy in the flow field is created by an oscillatory flux in the fluid region. The walls are maintained at constant temperature and concentration. The Boussinesq approximation is used so that the density variation will be considered only in the buoyancy force. The viscous and Darcy dissipations are neglected in comparison with heat by conduction and convection in the energy equation. Also the kinematic viscosity the thermal conducting  $k$  are treated as constants. We choose a rectangular Cartesian system  $O(x, y)$  with  $x$ -axis in the horizontal direction and  $y$ -axis normal to the walls. The walls of the channel are at

$$y = \pm Lf\left(\frac{\delta x}{L}\right)$$



The flow is maintained by an oscillatory volume flux for which a characteristic velocity is defined as

$$q(1 + k e^{i\omega t}) = \frac{1}{L} \int_{-L}^{L} u dy. \quad (1)$$

The boundary conditions for the velocity and temperature fields are

$$u = 0, v = 0, T = T_1, C = C_1 \text{ on } y = -Lf\left(\frac{\delta x}{L}\right)$$

$$u = 0, v = 0, T = T_2, C = C_2 \text{ on } y = +Lf\left(\frac{\delta x}{L}\right) \quad (2)$$

In view of the continuity equation we define the stream function  $\psi$  as

$$u = -\psi_y, v = \psi_x \quad (3)$$

Eliminating pressure  $p$  from momentum equations and using the equations governing the flow in terms of  $\psi$  are

$$[(\nabla^2 \psi)_t + \psi_x (\nabla^2 \psi)_y - \psi_y (\nabla^2 \psi)_x] = \nu \nabla^4 \psi - \beta g (T - T_0)_y - \beta^* g (C - C_0)_y - \left(\frac{\mu}{k}\right) \nabla^2 \psi \quad (4)$$

$$\rho_e C_p \left( \frac{\partial T}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial T}{\partial y} \right) = \lambda \nabla^2 T - Q(T - T_0) + Q_1(C - C_e) \quad (5)$$

$$\left( \frac{\partial C}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial C}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial C}{\partial y} \right) = D \nabla^2 C - k_1 (C - C_o) \quad (6)$$

Introducing the non-dimensional variables in (2.9) & (2.10) as

$$x' = x/L, \quad y' = y/L, \quad t' = t\omega, \quad \Psi' = \Psi/\nu, \quad \theta = \frac{T - T_2}{T_1 - T_2}, \quad C' = \frac{C - C_2}{C_1 - C_2} \quad (7)$$

The governing equations in the non-dimensional form (after dropping the dashes) are

$$R \left( \gamma^2 (\nabla^2 \psi)_t + \frac{\partial(\psi, \nabla^2 \psi)}{\partial(x, y)} \right) = \nabla^4 \psi + \left( \frac{G}{R} \right) (\theta_y + N C_y) - D^{-1} \nabla^2 \psi \quad (8)$$

$$P \left( \gamma^2 \frac{\partial \theta}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial y} \right) = \nabla^2 \theta - \alpha \theta + Q_1 C \quad (9)$$

$$Sc \left( \gamma^2 \frac{\partial C}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial C}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial C}{\partial y} \right) = \nabla^2 C - KC \quad (10)$$

where

$$R = \frac{UL}{\nu} \quad (\text{Reynolds number}), \quad G = \frac{\beta_0 g \Delta T_e L^3}{\nu^2} \quad (\text{Grashof number})$$

$$P = \frac{\mu c_p}{k_1} \quad (\text{Prandtl number}), \quad Sc = \frac{\nu}{D_1} \quad (\text{Schmidt Number}),$$

$$\alpha = \frac{QL^2}{\lambda} \quad (\text{Heatsourceparameter}), \quad \gamma_1 = \frac{K_1 L^2}{D_1} \quad (\text{Chemical reaction parameter}), \quad \gamma^2 = \frac{\omega L^2}{\nu} \quad (\text{Wormesly Number})$$

$$Q_1 = \frac{Q_1' (C_1 - C_2)_R L_1^2}{(T_1 - T_2) \lambda C_p} \quad (\text{Radiation Absorption parameter})$$

$$N = \frac{\beta^* \Delta C}{\beta \Delta T} \quad (\text{Buoyancy ratio}), \quad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

The corresponding boundary conditions are

$$\psi(+1) - \psi(-1) = 1$$

$$\frac{\partial \psi}{\partial x} = 0, \quad \frac{\partial \psi}{\partial y} = 0 \quad \text{at } \eta = \pm 1 \quad (11)$$

$$\theta(x, y) = 1, c = 1 \quad \text{on } \eta = -1$$

$$\theta(x, y) = 0, C = 0 \quad \text{on } \eta = 1$$

$$\frac{\partial \theta}{\partial y} = 0, \quad \frac{\partial C}{\partial y} = 0 \quad \text{at } \eta = 0 \quad (12)$$

The value of  $\psi$  on the boundary assumes the constant volumetric flow in consistent with the hypothesis (1). Also the wall temperature varies in the axial direction in accordance with the prescribed arbitrary function  $t$ .

### 3. METHOD OF SOLUTION

The main aim of the analysis is to discuss the perturbations created over a combined free and forced convection flow due to traveling thermal wave imposed on the boundaries. The perturbation analysis is carried out by assuming that the aspect ratio  $\delta$  to be small.

Introduce the transformation such that

$$\bar{x} = \delta x, \frac{\partial}{\partial x} = \delta \frac{\partial}{\partial \bar{x}}$$

Then

$$\frac{\partial}{\partial x} \approx O(\delta) \rightarrow \frac{\partial}{\partial \bar{x}} \approx O(1)$$

For small values of  $\delta \ll 1$ , the flow develops slowly with axial gradient of order  $\delta$

And hence we take  $\frac{\partial}{\partial \bar{x}} \approx O(1)$

Using the above transformation the equations (2.15-2.17) reduces to

$$\delta R \left( \gamma^2 (\nabla_1^2 \psi)_t + \frac{\partial(\psi, \nabla_1^2 \psi)}{\partial(x, y)} \right) = \nabla_1^4 \psi + \left( \frac{G}{R} \right) (\theta_y + N C_y) - D^{-1} \nabla^2 \psi \quad (13)$$

$$\delta P \left( \gamma^2 \frac{\partial \theta}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial y} \right) = \left( \frac{\partial^2 \theta}{\partial y^2} + \delta^2 \frac{\partial^2 \theta}{\partial x^2} \right) - \alpha \theta + Q_1 C \quad (14)$$

$$\delta S c \left( \gamma^2 \frac{\partial C}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial C}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial C}{\partial y} \right) = \nabla_1^2 C - K C \quad (15)$$

where

$$\nabla_1^2 = \delta^2 \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

Introducing the transformation

$$\eta = \frac{y}{f(\bar{x})}$$

The equations (3.1-3.3) reduces to

$$\delta R f \left( \gamma^2 (F^2 \psi)_t + \frac{\partial(\psi, F^2 \psi)}{\partial(\bar{x}, \eta)} \right) = F^4 \psi + \left( \frac{G f^3}{R} \right) (\theta_\eta + N C_\eta) - (D^{-1} f^2) F^2 \psi \quad (16)$$

$$\delta P \left( \gamma^2 \frac{\partial \theta}{\partial t} + f \left( \frac{\partial \psi}{\partial \eta} \frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial \eta} \right) \right) = F^2 \theta - (\alpha f^2) \theta + (Q f^2) C \quad (17)$$

$$\delta S c \left( \gamma^2 \frac{\partial C}{\partial t} + f \left( \frac{\partial \psi}{\partial \eta} \frac{\partial C}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial C}{\partial \eta} \right) \right) = F^2 C - K C \quad (18)$$

where

$$F^2 = \delta^2 \frac{\partial^2}{\partial \bar{x}^2} + \frac{\partial^2}{\partial \eta^2}, M_1^2 = M^2$$

We adopt the perturbation scheme and write

$$\begin{aligned} \psi(x, \eta, t) &= \psi_0(x, \eta, t) + k e^{it} \bar{\psi}_0(x, \eta, t) + \delta (\psi_1(x, \eta, t) + k e^{it} \bar{\psi}_1(x, \eta, t)) + \dots \\ \theta(x, \eta, t) &= \theta_0(x, \eta, t) + k e^{it} \bar{\theta}_0(x, \eta, t) + \delta (\theta_1(x, \eta, t) + k e^{it} \bar{\theta}_1(x, \eta, t)) + \dots \\ C(x, \eta, t) &= C_0(x, \eta, t) + k e^{it} \bar{C}_0(x, \eta, t) + \delta (C_1(x, \eta, t) + k e^{it} \bar{C}_1(x, \eta, t)) + \dots \end{aligned} \quad (19)$$

On substituting (19) in (16) - (18) and separating the like powers of  $\delta$  the equations and respective conditions to the zeroth order are

$$\psi_{0,\eta\eta\eta\eta} - (M_1^2 f^2) \psi_{0,\eta\eta} = - \left( \frac{Gf^3}{R} \right) (\theta_{0,\eta} + N C_{0,\eta}) \quad (20)$$

$$\theta_{0,\eta\eta} - (\alpha f^2) \theta_0 = -(Q_1 f^2) C_0 \quad (21)$$

$$C_{0,\eta\eta} - (K f^2) C_0 = 0 \quad (22)$$

with

$$\psi_{0(+1)} - \psi_{0(-1)} = 1, \quad \psi_{0,\eta} = 0, \quad \psi_{0,x} = 0 \quad \text{at } \eta = \pm 1 \quad (23)$$

$$\theta_0 = 1, \quad C_0 = 1 \quad \text{on } \eta = -1 \quad (24)$$

$$\theta_0 = 0, \quad C_0 = 0 \quad \text{on } \eta = 1 \quad (25)$$

$$\bar{C}_{0,\eta\eta} - (K \gamma^2 f^2) \bar{C}_0 = 0 \quad (26)$$

$$\bar{\psi}_{0,\eta\eta\eta\eta} - ((M_1^2 + i\gamma^2) f^2) \bar{\psi}_{0,\eta\eta} = - \left( \frac{Gf^3}{R} \right) (\bar{\theta}_{0,\eta} + N \bar{C}_{0,\eta}) \quad (27)$$

$$\bar{\theta}_0(\pm 1) = 0, \quad \bar{C}_0(\pm 1) = 0$$

$$\bar{\psi}_0(+1) - \bar{\psi}_0(-1) = 1, \quad \bar{\psi}_{0,\eta}(\pm 1) = 0, \quad \bar{\psi}_{0,x}(\pm 1) = 0 \quad (28)$$

The first order equations are

$$\psi_{1,\eta\eta\eta\eta} - (M_1^2 f^2) \psi_{1,\eta\eta} = - \left( \frac{Gf^3}{R} \right) (\theta_{1,\eta} + N C_{1,\eta}) + (Rf) (\psi_{0,\eta} \psi_{0,x\eta\eta} - \psi_{0,x} \psi_{0,\eta\eta\eta}) \quad (29)$$

$$\theta_{1,\eta\eta} - (\alpha_1 f^2) \theta_1 = (PRf) (\psi_{0,x} \theta_{0,\eta} - \psi_{0,\eta} \theta_{0,x}) - Q_1 C_1 \quad (30)$$

$$C_{1,\eta\eta} - (K f^2) C_1 = (Scf) (\psi_{0,x} C_{0,\eta} - \psi_{0,\eta} C_{0,x}) \quad (31)$$

$$\bar{\psi}_{1,\eta\eta\eta\eta} - ((M_1^2 + i\gamma^2) f^2) \bar{\psi}_{1,\eta\eta} = - \left( \frac{Gf^3}{R} \right) (\bar{\theta}_{1,\eta} + N \bar{C}_{1,\eta}) + (Rf) (\bar{\psi}_{0,\eta} \bar{\psi}_{0,x\eta\eta} + \bar{\psi}_{0,\eta} \bar{\psi}_{0,x\eta\eta} - \bar{\psi}_{0,x} \bar{\psi}_{0,\eta\eta\eta} - \bar{\psi}_{0,x} \bar{\psi}_{0,\eta\eta\eta}) \quad (32)$$

$$\bar{\theta}_{1,\eta\eta} - ((iP_1 \gamma^2 + \alpha) f^2) \bar{\theta}_1 = (PRf) (\bar{\psi}_{0,\eta} \bar{\theta}_{0,x} + \bar{\psi}_{0,\eta} \bar{\theta}_{0,x} - \bar{\psi}_{0,x} \bar{\theta}_{0,\eta} - \bar{\psi}_{0,x} \bar{\theta}_{0,\eta}) - (Q_1 N_2) \bar{C}_1 \quad (33)$$

$$\bar{C}_{1,\eta\eta} - ((K + i\gamma^2) f^2) \bar{C}_1 = (Scf) (\bar{\psi}_{0,\eta} \bar{C}_{0,x} + \bar{\psi}_{0,\eta} \bar{C}_{0,x} - \bar{\psi}_{0,x} \bar{C}_{0,\eta} - \bar{\psi}_{0,x} \bar{C}_{0,\eta}) \quad (34)$$

with

$$\psi_{1(+1)} - \psi_{1(-1)} = 0$$

$$\psi_{1,\eta} = 0, \quad \psi_{1,x} = 0 \quad \text{at } \eta = \pm 1 \quad (35)$$

$$\theta_1(\pm 1) = 0, \quad C_1(\pm 1) = 0$$

$$\bar{\theta}_1(\pm 1) = 0, \quad \bar{C}_1(\pm 1) = 0$$

$$\bar{\psi}_1(+1) - \bar{\psi}_1(-1) = 1, \quad \bar{\psi}_{1,\eta}(\pm 1) = 0, \quad \bar{\psi}_{1,x}(\pm 1) = 0 \quad (36)$$

The equations (20) - (34) are solved algebraically subject to the relevant boundary conditions (23, 24, 35 & 36).

#### 4. NUSSELT NUMBER and SHERWOOD NUMBER

The local rate of heat transfer coefficient (Nusselt number Nu) on the walls has been calculated using the formula

$$Nu = \frac{1}{\theta_m - \theta_w} \left( \frac{\partial \theta}{\partial y} \right)_{\eta=\pm 1}$$

where

$$\theta_m = 0.5 \int_{-1}^1 \theta d\eta$$

The local rate of mass transfer coefficient (Sherwood Number Sh) on the walls has been calculated using the formula

$$Sh = \frac{1}{C_m - C_w} \left( \frac{\partial C}{\partial y} \right)_{y=\pm 1}$$

where  $C_m = 0.5 \int_{-1}^1 C dy$

#### 5. DISCUSSION OF THE RESULTS

In this analysis we investigate the effect of chemical reaction and radiation absorption on unsteady convective heat and mass transfer flow of a viscous flow through a porous medium in a horizontal wavy channel with oscillatory flux. The non-linear coupled equations governing the flow heat and mass transfer have been solved by employing the perturbation technique with the slope  $\delta$  wavy wall as a perturbation parameter.

The axial velocity  $u$  shows in figure 1-9 for different values of  $\beta$ ,  $N$ ,  $K$ ,  $\alpha$ ,  $Q_1$ ,  $\gamma$ . Fig. (1) shows the variation of  $u$  with buoyancy ratio  $N$ . It is found that when the molecular buoyancy force dominates over the thermal buoyancy force  $|u|$  enhances when the buoyancy forces act in the same direction and for the forces acting in opposite directions it depreciates in the flow region. The effect of wall waviness of  $u$  is exhibited in figure (2). Higher the constriction of the channel walls smaller  $|u|$  in the region. The variation of  $u$  with chemical reaction parameter  $k$  with  $|u|$  enhances with increasing  $k \leq 1.5$  and  $|u|$  depreciates with  $k \geq 2.5$ . Also  $|u|$  depreciates with the strength of heat source  $\alpha$  with higher  $\alpha \geq 6$  (fig.3). An increasing the radiation absorption parameter  $Q_1$  leads to depreciates in  $|u|$ . Also  $|u|$  enhances with increases worm sly  $|\gamma|$  (fig.4).

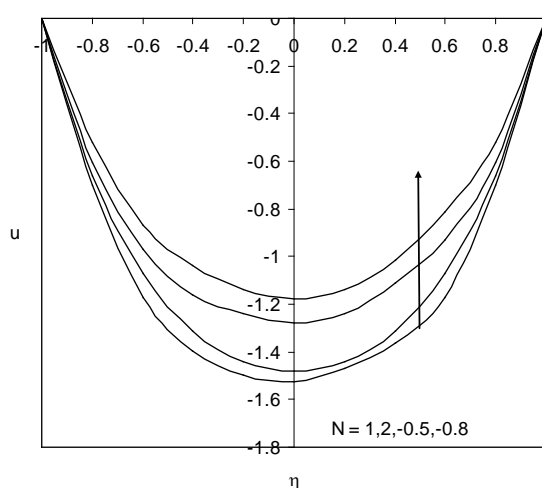


Fig. 1 : Variation of  $u$  with  $N$   
 $G = 1000, D^* = 100, R = 35, Sc = 1.3,$   
 $\beta = -0.3, k = 0.5, \alpha = 2, Q_1 = 1, \gamma = 0.5, x = \pi/4, t = \pi/4$

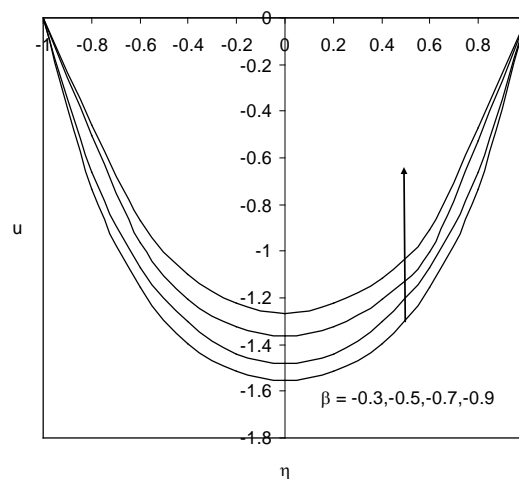


Fig. 2 : Variation of  $u$  with  $\beta$   
 $G = 1000, D^* = 100, R = 35, Sc = 1.3, N = 1,$   
 $k = 0.5, \alpha = 2, Q_1 = 1, \gamma = 0.5,$   
 $x = \pi/4, t = \pi/4$

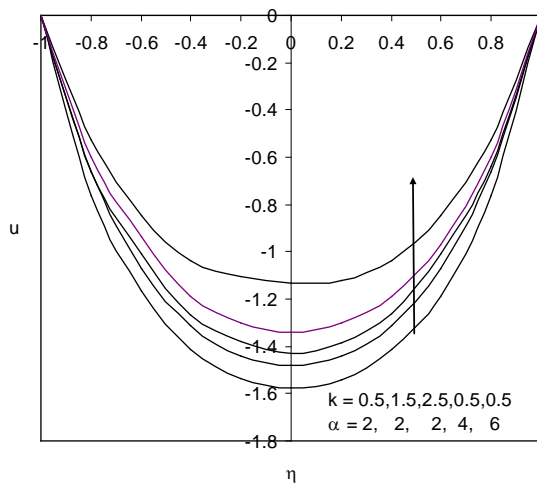


Fig.3 : Variation of u with k &  $\alpha$   
 $G = 1000, D^{-1}=100, R = 35, Sc=1.3, N = 1,$   
 $\beta=-03, Q_1 = 1, \gamma = 0.5, x = \pi/4, t = \pi/4$

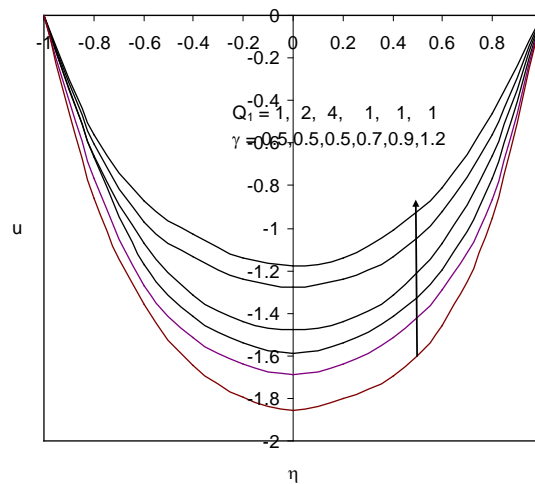


Fig. 4 : Variation of u with  $Q_1$  &  $\gamma$   
 $G = 1000, D^{-1}=100, R = 35, Sc=1.3, N = 1,$   
 $\beta=-03, k = 0.5, \alpha = 2, Q_1 = 1, \gamma = 0.5,$   
 $x = \pi/4, t = \pi/4$

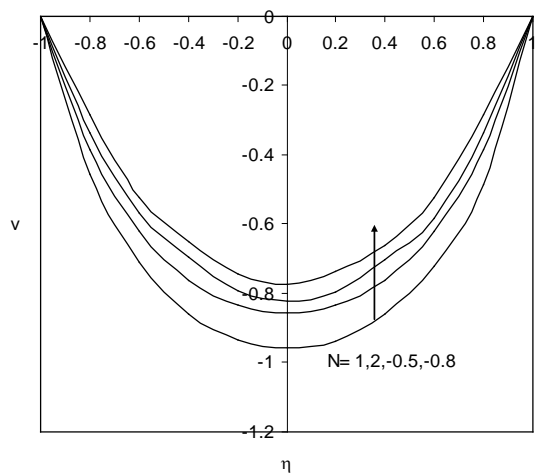


Fig. 5 : Variation of v with N  
 $G = 1000, D^{-1}=100, R = 35, Sc=1.3,$   
 $\beta=-03, k = 0.5, \alpha = 2, Q_1 = 1, \gamma = 0.5, N_1 = 1.5,$   
 $x = \pi/4, t = \pi/4$

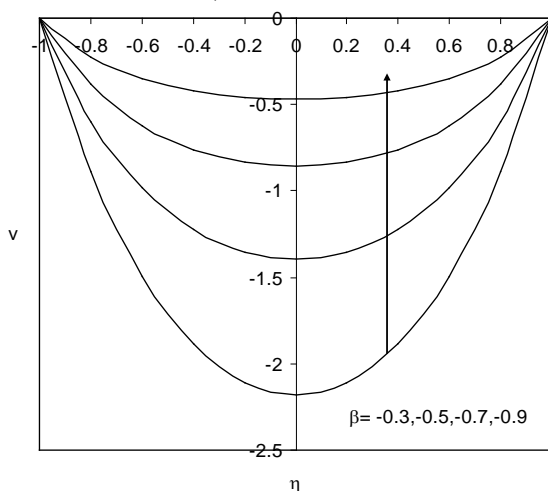


Fig. 6 : Variation of v with  $\beta$   
 $G = 1000, D^{-1}=100, R = 35, Sc=1.3, N = 1,$   
 $k = 0.5, \alpha = 2, Q_1 = 1, \gamma = 0.5$

The secondary velocity  $v$  which arraigns due to the wavy boundaries is show in figs 5-8 for different parametric values. Fig.8 shows the variation of  $v$  with  $N$ . It is found that, when the molecular buoyancy force dominates over the thermal buoyancy force  $|v|$  enhances in flow region irrespective of directions of the buoyancy forces. The effect of wall waviness on  $v$  is shown in fig (6). It is observed that higher the constriction of channel walls larger  $|v|$  is entire flow region. The variation of  $v$  with chemical reaction  $k$  shows that, magnitude of  $v$  experiences an enhancement with increasing chemical reaction  $k$ . With respect to  $\alpha$ , we find that higher the strength of the heat source smaller  $|v|$  in the flow region and for further higher  $\alpha$  we notice an enhancement with  $|v|$  (fig.7). Fig (8) represents  $v$  with radiation absorption parameter  $Q_1$  and Womersly number  $\gamma$ , it is found that  $|v|$  epreciates with increasing  $Q_1$ . Also it increases with  $\gamma$ .

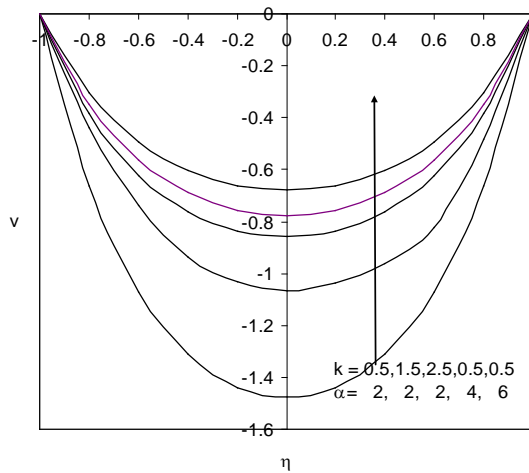


Fig. 7 : Variation of  $v$  with  $k$  &  $\alpha$   
 $G = 1000, D^{-1}=100, R = 35, Sc=1.3, N = 1,$   
 $\beta=-03, Q_1 = 1, \gamma = 0.5$

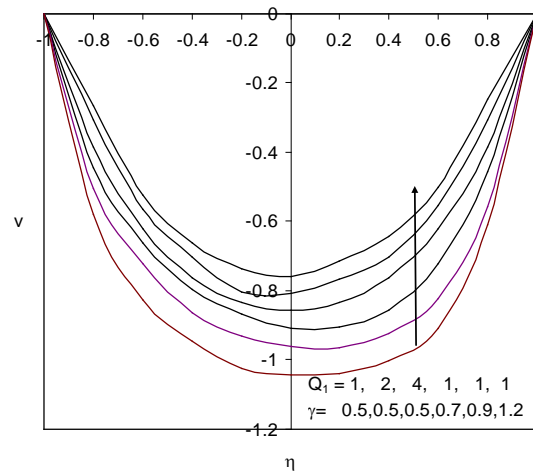


Fig. 8 : Variation of  $v$  with  $Q_1$  &  $\gamma$   
 $G = 1000, D^{-1}=100, R = 35, Sc=1.3, N = 1,$   
 $\beta=-03, k = 0.5, \alpha = 2, Q_1 = 1, \gamma = 0.5,$

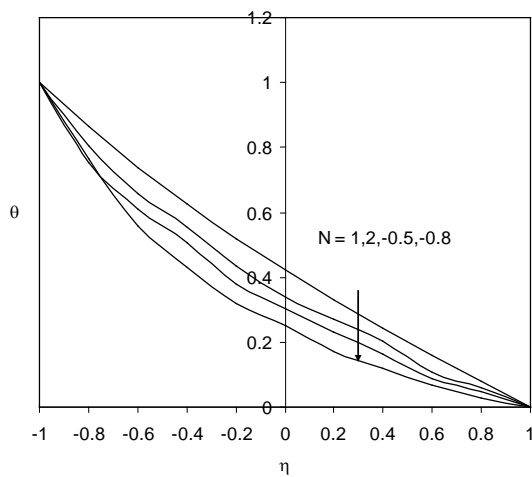


Fig.9 : Variation of  $\theta$  with  $N$   
 $G = 1000, D^{-1}=100, R = 35, Sc=1.3,$   
 $\beta=-03, k = 0.5, \alpha = 2, Q_1 = 1, \gamma = 0.5,$

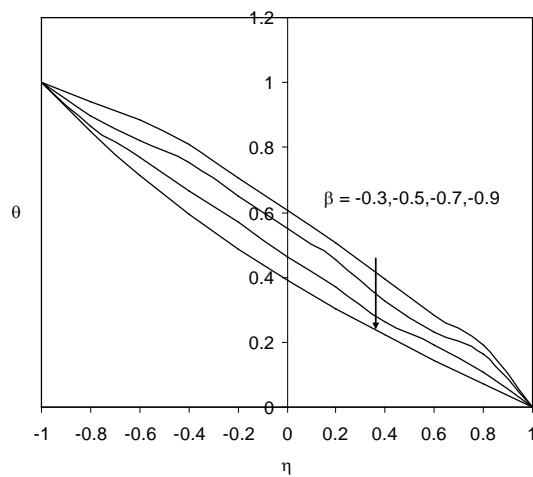


Fig.10 : Variation of  $\theta$  with  $\beta$   
 $G = 1000, D^{-1}=100, R = 35, Sc=1.3,$   
 $k = 0.5, \alpha = 2, Q_1 = 1, \gamma = 0.5$

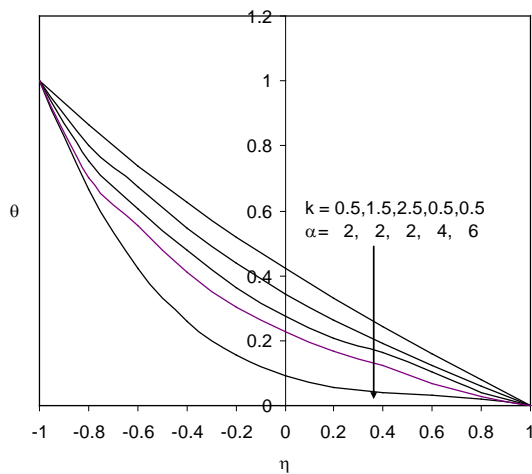


Fig. 11 : Variation of  $\theta$  with  $k$  &  $\alpha$   
 $G = 1000, D^{-1}=100, R = 35, Sc=1.3, N = 1,$   
 $\beta=-03, Q_1 = 1, \gamma = 0.5$

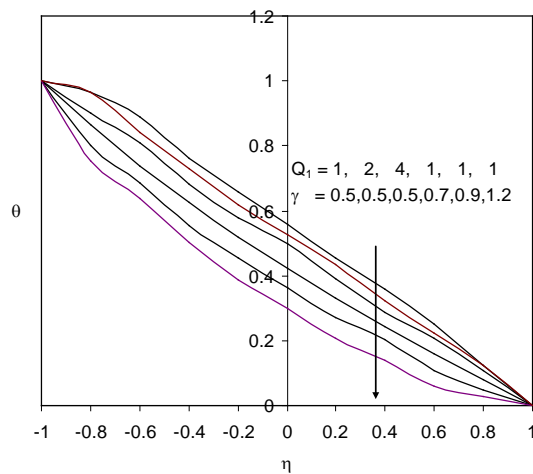


Fig.12 : Variation of  $\theta$  with  $Q_1$  &  $\gamma$   
 $G = 1000, D^{-1}=100, R = 35, Sc=1.3, N = 1,$   
 $k = 0.5, \alpha = 2, \beta=-03$



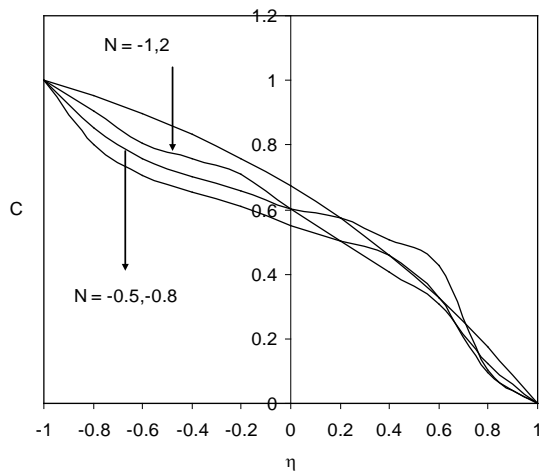


Fig. 13 : Variation of C with N  
G = 1000, D<sup>-1</sup>=100, R = 35, Sc=1.3,  
β=-03, k = 0.5, α = 2, Q<sub>1</sub> = 1, γ = 0.5

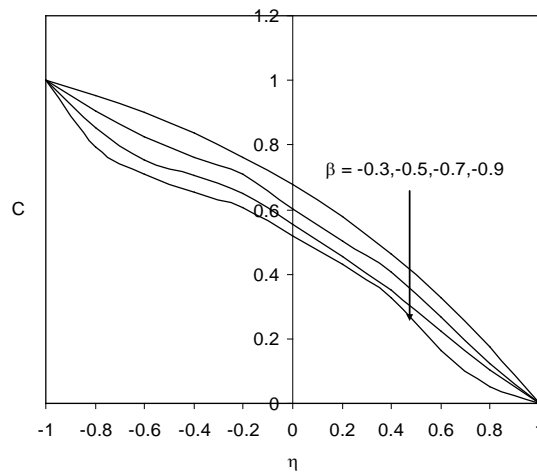


Fig. 14 : Variation of C with β  
G = 1000, D<sup>-1</sup>=100, R = 35, Sc=1.3,  
k = 0.5, α = 2, Q<sub>1</sub> = 1, γ = 0.5,

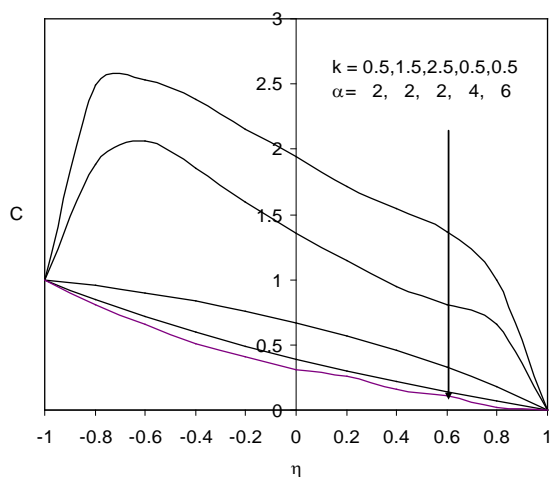


Fig. 15 : Variation of C with k & α  
G = 1000, D<sup>-1</sup>=100, R = 35, Sc=1.3, N = 1,  
β=-03, Q<sub>1</sub> = 1, γ = 0.5

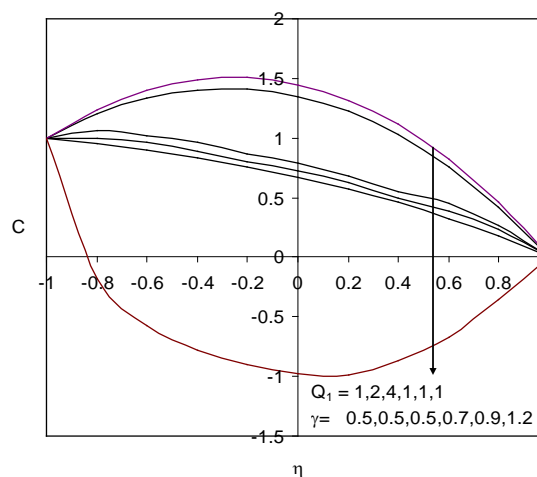


Fig. 16 : Variation of C with Q<sub>1</sub> & γ  
G = 1000, D<sup>-1</sup>=100, R = 35, Sc=1.3, N = 1,  
k = 0.5, α = 2, β=-03,

The non-dimensional temperature  $\theta$  is exhibited in figures 9-12 for different parameters. It is found that the non-dimensional temperature  $\theta$  positive for all variations. When the molecular buoyancy force dominates over the thermal buoyancy force the actual temperature depreciates irrespective of the directions of buoyancy forces (fig.9). The actual temperature depreciates with increasing chemical reactions  $k$  or  $\alpha$  (fig.11). From fig (12), it can be observed that the actual temperature enhances with increasing  $Q_1$ . Also it depreciates with  $\gamma \leq 0.9$  an enhances with higher  $\gamma \geq 1.2$

The non-dimensional concentration (C) is shown in figs (13)-(16) for different parametric values. The variation of C with N shows that the actual concentration depreciates with increased  $|N|$  (fig.13). From fig (14), it can be seen that higher the concentration of the channel walls smaller the concentration (fig.14). The variation of C with  $k$  shows that the actual concentration enhances with increase  $k \leq 1.5$  & depreciates with higher  $k \geq 2.5$ . Also the actual concentration depreciates with increase  $\alpha$  (fig15). An increasing  $Q_1$  or  $\gamma$  leads to an enhancement with actual concentration (fig.16).

The rate of heat transfer (Nu) at the walls  $\eta = \pm 1$  is exhibited in tables 1-8 for different values of G, K,  $\alpha$ , N,  $\beta$ ,  $k$ ,  $\gamma$ ,  $Q_1$ . It is found that the rate of heat transfer depreciates  $\eta=1$  and enhances  $\eta=-1$  with increase  $G>0$ , while for  $G<0$ ,  $|N_u|$  enhances at  $\eta=1$  and depreciates at  $\eta=-1$ . It enhances at  $\eta=1$  with increase in  $k$  for all G while at  $\eta=-1$ , it reduces with  $k \leq 1.5$  and enhances with higher  $k \geq 2.5$  in the heating case and in the cooling case it reduces at  $\eta=-1$  for all G. An increase in the strength of heat generating source ( $\alpha>0$ ) reduces  $|N_u|$  and at  $\eta=-1$ , it depreciates with  $\alpha$  and for higher  $\alpha \geq 6$ , it depreciates in the heating cases and enhances cooling cases. With respect to the buoyancy ratio N it can

be seen that  $|N_u|$  depreciates at  $\eta=1$  and enhances at  $\eta=-1$  irrespective of the directions of the buoyancy force (tables 1 & 3). The variation of Nu with  $\beta$  shows that higher the constriction of the channel walls  $|\beta|=0.3$  larger  $|N_u|$  at  $\eta=1$  and smaller  $\eta=-1$  and for further higher  $|\beta|=0.5$  larger  $|N_u|$  at both the walls and for further higher  $|\beta|=0.9$ ,  $|N_u|$  depreciates at  $\eta=\pm 1$  for  $G > 0$  and for  $G < 0$ , it depreciates at  $\eta=+1$  and enhances  $\eta=-1$ . An increasing  $\gamma$  reduces  $|N_u|$  at both the walls. An increase the radiation absorption  $Q_1$  results in an enhancement in  $|N_u|$  at  $\eta=1$  and depreciates  $\eta=-1$  (tables.2&4)

The rate of mass transfer (Sh) at  $\eta=\pm 1$  is shown in tables 5-8 for different parametric values. It is found that the rate of mass transfer enhances with increase  $G > 0$  at  $\eta=\pm 1$  and for  $G < 0$ , |Sh| enhances at  $\eta=1$  and reduces at  $\eta=-1$ . An increase in the strength of heat source ( $\alpha$ ) reduces |Sh| at both the walls. When the molecular buoyancy force dominates over the thermal buoyancy force |Sh| reduces at  $\eta=1$  and enhances  $\eta=-1$ . An increase in  $k \leq 1.5$  enhances |Sh| and reduces with higher  $k > 2.5$  at  $\eta=1$  while at  $\eta=-1$ , it enhances with  $k$  for all  $G$ . (tables 5 & 7). Higher the constriction of the channel walls larger |Sh| at  $\eta=1$  and smaller  $\eta=-1$ . An increasing the Womersly number  $\gamma$  leads to an enhancement in |Sh| at both the walls. It is found that the rate of mass transfer enhances with increasing  $Q_1$  (tables.6&8).

**Table - 1**  
Nusselt Number (Nu) at  $\eta = -1$

G	I	II	III	IV	V	VI	VII	VIII
100	-3.0956	-3.1256	-3.0656	-3.2956	1.62613	-3.4075	-0.6015	0.60799
300	-8.9231	-8.9632	-8.8232	-8.9932	2.25503	4.751	-1.2249	0.18186
-100	-1.1486	-1.1587	-1.1287	-1.1467	0.58132	0.65312	-0.0707	0.99502
-300	-0.7743	-0.1757	-1.1643	-1.1843	-0.4958	1.98386	0.38688	1.34809
N	1	2	-0.50	-0.8	1	1	1	1
K	0.5	0.5	0.5	0.5	1.5	2.5	0.5	0.5
$\alpha$	2	2	2	2	2	2	4	6

**Table - 2**  
Nusselt Number (Nu) at  $\eta = -1$

G	I	II	III	IV	V	VI	VII	II	III
100	-0.93212	-3.0956	-13.071	5.53849	-3.0666	-3.0656	-3.0056	-6.2153	1.11974
300	-1.60651	-8.9231	7.28034	1.72347	-8.9031	-8.9632	-8.9032	3.66535	0.24093
-100	-0.40413	-1.1486	-1.6761	-0.49421	-1.1087	-1.1387	-1.1287	-1.0836	-0.6738
-300	0.02047	-0.7743	-0.2674	-1.94409	-0.1442	-0.1943	-0.1847	-0.3827	-0.3035
$\beta$	-0.3	-0.5	-0.7	-0.9	-0.5	-0.5	-0.5	-0.5	-0.5
$\gamma$	0.5	0.5	0.5	0.5	1.5	2.5	3.5	0.5	0.5
$Q_1$	1	1	1	1	1	1	1	2	4

**Table - 3**  
Nusselt Number (Nu) at  $\eta = +1$

G	I	II	III	IV	V	VI	VII	VIII
100	3.93981	3.86981	3.92981	3.90986	5.79446	-21.6.15	1.68197	0.69067
300	3.5501	3.5002	3.5601	3.52016	8.41267	-10.423	2.08373	0.90559
-100	5.31257	5.30256	5.32257	5.30256	0.86171	-5.1139	1.19833	0.46399
-300	-6.5693	-6.5083	-6.5793	-6.5593	-11.886	-68.535	0.60496	0.22454
N	1	2	-0.5	-0.8	1	1	1	1
K	0.5	0.5	0.5	0.5	1.5	2.5	0.5	0.5
$\alpha$	2	2	2	2	2	2	4	6

**Table - 4**  
Nusselt Number (Nu) at  $\eta = +1$

G	I	II	III	IV	V	VI	VII	II	III
100	2.11207	3.93981	4.1624	3.06388	3.92981	3.90986	3.86982	2.38819	1.31219
300	2.38928	3.5501	3.6002	1.98344	3.5501	3.50016	3.4506	1.79319	1.04232
-100	1.71256	5.31257	-6.1852	-3.44084	5.31257	5.30256	5.26256	4.6577	0.33386
-300	1.0869	-6.5693	-7.2755	0.17944	-6.5693	-6.5294	-6.5093	6.86688	0.58859
$\beta$	-0.3	-0.5	-0.7	-0.9	-0.5	-0.5	-0.5	-0.5	-0.5
$\gamma$	0.5	0.5	0.5	0.5	1.5	2.5	3.5	0.5	0.5
$Q_1$	1	1	1	1	1	1	1	2	4

**Table - 5**  
Sherwood Number (Sh) at  $\eta = +1$

G	I	II	III	IV	V	VI	VII	VIII
100	3.97909	3.96969	3.97806	3.96906	-4.3119	-2.246	-0.2635	-0.2314
300	12.4526	12.4466	12.4427	12.4329	-13.751	-6.0896	-0.3433	-0.2467
-100	-4.3581	-4.346	-4.348	-4.3386	4.58405	2.18497	-0.1838	-0.216
-300	-12.562	-12.54	-12.522	-12.519	15.4119	7.34903	-0.1043	-0.2007
N	1	2	-0.5	-0.8	1	1	1	1
K	0.5	0.5	0.5	0.5	1.5	2.5	0.5	0.5
$\alpha$	2	2	2	2	2	2	4	6

**Table - 6**  
Sherwood Number (Sh) at  $\eta = +1$

G	I	II	III	IV	V	VI	VII	II	III
100	-0.55801	3.97909	46.0525	324.526	3.98969	3.99929	4.01969	38.6477	109.048
300	-1.26623	12.4526	142.977	1095.74	12.4666	12.4767	12.4827	118.014	336.174
-100	0.14792	-4.3581	-48.09	-292.481	-4.3686	-4.3786	-4.3886	-38.55	-106.81
-300	0.85157	-12.562	-130.95	-797.326	-12.57	-12.586	-12.599	-113.67	-312.23
$\beta$	-0.3	-0.5	-0.7	-0.9	-0.5	-0.5	-0.5	-0.5	-0.5
$\gamma$	0.5	0.5	0.5	0.5	1.5	2.5	3.5	0.5	0.5
Q1	1	1	1	1	1	1	1	2	4

**Table - 7**  
Sherwood Number (Sh) at  $\eta = -1$

G	I	II	III	IV	V	VI	VII	VIII
100	-0.8468	-0.8668	-0.8568	-0.8868	-1.1126	-2.2942	-0.7997	-0.7954
300	-0.8776	-0.8866	-0.8876	-0.8976	-0.9171	-1.843	-0.7976	-0.7908
-100	-0.817	-0.829	-0.827	-0.847	-1.4201	-3.0025	-0.8018	-0.8001
-300	-0.7883	-0.7987	-0.7983	-0.8083	-1.9738	-4.275	-0.8039	-0.8047
N	1	2	-0.5	-0.8	1	1	1	1
K	0.5	0.5	0.5	0.5	1.5	2.5	0.5	0.5
$\alpha$	2	2	2	2	2	2	4	6

**Table - 8**  
Sherwood Number (Sh) at  $\eta = -1$

G	I	II	III	IV	V	VI	VII	VIII	IX
100	-0.91765	-0.8468	-0.8236	-0.89825	-0.8568	-0.8668	-0.8728	-0.8674	-0.9095
300	-0.921	-0.8776	-0.8405	-1.37275	-0.8876	-0.8976	-0.9076	-0.8947	-0.931
-100	-0.91432	-0.817	-0.7213	-0.61198	-0.827	-0.829	-0.8369	-0.8418	-0.8904
-300	-0.91102	-0.7883	-0.631	-0.42048	-0.7983	-0.7981	-0.8087	-0.8176	-0.8732
$\beta$	-0.3	-0.5	-0.7	-0.9	-0.5	-0.5	-0.5	-0.5	-0.5
$\gamma$	0.5	0.5	0.5	0.5	1.5	2.5	3.5	0.5	0.5
Q1	1	1	1	1	1	1	1	2	4

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