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# ANALYSIS OF SINGLE SERVER BATCH SERVICE QUEUEING SYSTEM UNDER MULTIPLE VACATIONS WITH SECOND OPTIONAL SERVICE

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***ABSTRACT***

*Consider a single server fixed batch service queueing system under multiple vacations with second optional service in which the arrival rate follows a Poisson process with parameter $\lambda$. The concept of Second optional service is introduced in this paper and server provides two types of services namely Essential Service and Second Optional Service. The essential service will be given first to all batches and the second optional service will be extended only if the batch demands optional service with probability p. Both essential and optional services follow an exponential distribution with parameters $\mu_1$ and $\mu_2$. Further assume that the length of the multiple vacations follows an exponential distribution with parameter $\alpha$. Assume that the system initially contains k customers when the server enters in to the system and starts the essential service immediately in a batch of size k. After completion of an essential service, if the batch demands optional service with probability p then the server starts optional service and after completion of this optional service if he finds less than k customers in the queue, then the server goes for a multiple vacation of a length $\alpha$. This model is completely solved by constructing the generating function and Rouche's theorem is applied and we have derived the closed form solutions for probability of number of customers in the queue during the server busy and in vacation. Further we are providing the analytical solution for mean number of customers and variance of the system. Numerical studies have been done for analysis of mean and variance for various values of $\lambda$, $\mu_1$, $\mu_2$, p, $\alpha$ and k and also various particular cases of this model have been discussed.*

***Keywords:*** *Single Server, Batch Service, Second optional service, Multiple vacations, Steady state probability distribution.*

## 1. INTRODUCTION

Second optional service plays a vital role in retrial queueing systems. Kailash C. Madan [8] has studied an M/G/1 queue with second optional service using supplementary variable technique. The work of Madan was generalized by Medhi [9] who studied a single server Poisson input queue with a second optional channel. Gaudham Choudhury [5] studied some aspects of M/G/1 queueing system with second optional service and derived the steady state queue size distribution at the stationary point of time for general second optional service. Wang [10] studied M/G/1 queue with second optional service and server breakdown. Wang and Jianghua LI [11] extended the previous work of Wang [10] and studied a single server retrial queue with general retrial times and two phase service in which they discussed second multi optional services allowing balking and reneging of customers in the retrial queue. Ayyappan and *et al*., [1, 2] have studied the second optional services for retrial queueing system with priority services and Erlang type services.

For batch service queues with vacations, there have been a few related works. Dhas [3] considered Markovian batch service systems and obtained the queue length distributions by matrix-geometric methods. Lee *et al*. [6, 7] obtained various performance measures for $M/G^B/1$ queue with single vacation. Dshalalow and Yellen [4] considered a non-exhaustive batch service system with multiple vacations in which the server starts a multiple vacation whenever the queue drops below a level r and resumes service at the end of a vacation segment when the queue accumulates to at least r.

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In this paper we are analyzing a special batch service queue called the fixed size batch service queue under multiple vacations with second optional service. The model is described in Section 2. In Section 3, we have derived the system steady state equations and using these equations, the probability generating functions for number of customers in the queue when the server is busy (essential and optional) or in vacation are derived and also obtained steady state probability distributions. Section 4 deals with stability condition of the system. Closed form solutions of System performance measures are obtained 6. A numerical study is carried out in Section 7 to test the effectiveness of the system. We are providing the analytical solution for mean number of customers and variance of the system. Numerical studies have been done for analysis of mean and variance for various values of $\lambda$, $\mu_1$, $\mu_2$, p, $\alpha$ and k and also various particular cases of this model have been discussed.

## 2. DESCRIPTION OF THE MODEL

Consider a single server fixed batch service queueing system under multiple vacations with optional service in which the arrival rate $\lambda$ follows a Poisson process. In this paper server provides two types of services namely Essential Service and Second Optional Service. Both Essential and Optional services follow an exponential distribution with parameters $\mu_1$ and $\mu_2$. Further assume that the length of the multiple vacations follows an exponential distribution with parameter $\alpha$.The essential service will be given first to all batches and the second optional service will be extended only if the batch demands an optional service with probability p. Assume that the system initially contains k customers when the server enters in to the system and starts the essential service immediately in a batch of size k. After completion of an essential service, if the batch demands optional service with probability p then the server starts optional service and after completion of this optional service if he finds less than k customers in the queue, then the server goes for a multiple vacation of a length $\alpha$. If there are more than k customers in the queue then the first k customers will be selected from the queue and essential service will be given as a batch. If there are less than k customers in the queue upon his return from the vacation, he immediately leaves for another vacation and so on until he finally finds k or more customers in the queue. Let < N(t), C(t) > be a random process where N(t) be the random variable which represents the number of customers in queue at time t and C(t) be the random variable which represents the server status (busy with essential/ busy with optional / vacation) at time t.

We define
$P_{n,1}(t)$ - Probability that the server is busy with essential service if there are n customers in the queue at time t.
$P_{n,2}(t)$ - Probability that the server is in vacation if there are n customers in the queue at time t
$P_{n,3}(t)$ - Probability that the server is busy with optional service if there are n customers in the queue at time t.

The Chapman-Kolmogorov equations are

$$P_{0,1}^{'}(t) = -(\lambda+\mu_1)P_{0,1}(t) + \mu_1(1-p)P_{k,1}(t) + \alpha P_{k,2}(t) + \mu_2 P_{k,3}(t) \tag{1}$$

$$P_{n,1}^{'}(t) = -(\lambda+\mu_1)P_{n,1}(t) + \lambda P_{n-1,1}(t) + \alpha P_{n+k,2}(t) + \mu_1 P_{n+k,1}(t) + \mu_2 P_{n+k,3}(t) \text{ for n = 1,2,3,....} \tag{2}$$

$$P_{0,2}^{'}(t) = -\lambda P_{0,2}(t) + \mu_1(1-p)P_{0,1}(t) + \mu_2 P_{0,3}(t) \tag{3}$$

$$P_{n,2}^{'}(t) = -\lambda P_{n,2}(t) + \lambda P_{n-1,2}(t) + \mu_1(1-p)P_{n,1}(t) + \mu_2 P_{n,3}(t) \text{ for n = 1,2,3,... ,k-1} \tag{4}$$

$$P_{n,2}^{'}(t) = -(\lambda+\alpha)P_{n,2}(t) + \lambda P_{n-1,2}(t) \text{ for n } \geq \text{k} \tag{5}$$

$$P_{0,3}^{'}(t) = -(\lambda+\mu_2)P_{0,3}(t) + \mu_1 \, pP_{0,1}(t) \tag{6}$$

$$P_{n,3}^{'}(t) = -(\lambda+\mu_2)P_{n,3}(t) + \lambda P_{n-1,3}(t) + \mu_1 pP_{n,1}(t) \text{ for n = 1,2,3,. . .} \tag{7}$$

## 3. EVALUATION OF STEADY STATE PROBABILITIES

In this section we are finding the closed form solutions for number of customers in the queue when the server busy with essential and optional service and also in vacation using Generating functions.

When steady state prevails, the equations (1) to (7) becomes

$$(\lambda+\mu_1)P_{0,1} = \mu_1(1-p)P_{k,1} + \alpha P_{k,2} + \mu_2 P_{k,3} \tag{8}$$

$$(\lambda+\mu_1)P_{n,1} = \lambda P_{n-1,1} + \alpha P_{n+k,2} + \mu_1 P_{n+k,1} + \mu_2 P_{n+k,3} \text{ for n = 1,2,3,....} \tag{9}$$

$$\lambda P_{0,2} = \mu_1(1-p)P_{0,1} + \mu_2 P_{0,3} \tag{10}$$

$$\lambda P_{n,2} = \lambda P_{n-1,2} + \mu_1(1-p)P_{n,1} + \mu_2 P_{n,3} \text{ for n = 1,2,3,...,k-1} \tag{11}$$

$$(\lambda+\alpha)P_{n,2} = \lambda P_{n-1,2} \text{ for n } \geq \text{k} \tag{12}$$

$$(\lambda+\mu_2)P_{0,3} = \mu_1 p P_{0,1} \tag{13}$$

$$(\lambda+\mu_2)P_{n,3} = \lambda P_{n-1,3} + \mu_1 p P_{n,1} \text{ for n = 1,2,3,. . .} \tag{14}$$

Generating functions for the number of customers in the queue when the server is busy with essential service, the server is in vacation and the server is busy with second optional service are defined as

$$G(z) = \sum_{n=0}^{\infty} P_{n,1} z^n \ , \ H(z) = \sum_{n=0}^{\infty} P_{n,2} z^n \text{ and } I(z) = \sum_{n=0}^{\infty} P_{n,3} z^n$$

Multiply the equation (8) with 1 and (9) with $z^n$ on both sides and summing over n = 0 to ∞, we get

$$G(z)[\lambda z^{k+1} - (\lambda+\mu_1)z^k + \mu_1(1-p)] + \alpha H(z) + \mu_2 I(z)$$

$$= \mu_1(1-p)\sum_{n=0}^{k-1} P_{n,1} z^n + \alpha \sum_{n=0}^{k-1} P_{n,2} z^n + \mu_2 \sum_{n=0}^{k-1} P_{n3} z^n \tag{15}$$

Adding equation (10), (11) and (12) after multiply with 1, $z^n$ and $z^n$ (n = 1, 2, 3, . . .) respectively, we get

$$H(z)[\alpha + \lambda(1-z)] = \mu_1(1-p)\sum_{n=0}^{k-1} P_{n,1} z^n + \alpha \sum_{n=0}^{k-1} P_{n,2} z^n + \mu_2 \sum_{n=0}^{k-1} P_{n,3} z^n \tag{16}$$

Multiply the equation (13) with 1 and (14) with $z^n$ on both sides and summing over n= 0 to ∞, we get

$$I(z)\left[\mu_2 + \lambda - \lambda z\right] = \mu_1 p G(z) \tag{17}$$

Substitute equation (16) and (17) in (15) we get

$$G(z)\left[\frac{\lambda^2 z^{k+1} - \lambda(\lambda+\mu_1+\mu_2)z^k + \mu_1\mu_2(1+z+...+z^{k-1}) + \mu_1(1-p)\lambda}{\mu_2 + \lambda - \lambda z}\right] = \lambda H(z) \tag{18}$$

From equation (16), we get

$$H(z) = \frac{\mu_1(1-p)\sum_{n=0}^{k-1} P_{n,1} z^n + \alpha \sum_{n=0}^{k-1} P_{n,2} z^n + \mu_2 \sum_{n=0}^{k-1} P_{n,3} z^n}{\alpha + \lambda(1-z)} \tag{19}$$

Equation (19) represents the probability generating function for number of customers in the queue when the server is in vacation.

From (18) and (19), we get

$$G(z) = \frac{\lambda\left(\mu_1(1-p)\sum_{n=0}^{k-1} P_{n,1} z^n + \alpha \sum_{n=0}^{k-1} P_{n,2} z^n + \mu_2 \sum_{n=0}^{k-1} P_{n,3} z^n\right)(\mu_2 + \lambda - \lambda z)}{\left(\alpha + \lambda(1-z)\right)\left(\lambda^2 z^{k+1} - \lambda(\lambda+\mu_1+\mu_2)z^k + \mu_1\mu_2(1+z+...+z^{k-1}) + \mu_1(1-p)\lambda\right)} \tag{20}$$

Equation (20) represents the probability generating function for number of customers in the queue when the server is busy with essential service.

Put z = 1 in equations (17) and (18), we get

$$I(1) = \left(\frac{\mu_1 p}{\mu_2}\right) G(1) \tag{21}$$

$$G(1) = \frac{\frac{\lambda}{k\mu_1} H(1)}{1 - \frac{\lambda}{k\mu_1}\left(1 + \frac{\mu_1 p}{\mu_2}\right)} \tag{22}$$

The normalized condition is $G(1)+H(1)+I(1)=1$ (23)

Substituting (21) and (22) in (23), we get

$$H(1) = 1 - \frac{\lambda}{k\mu_1}\left(1 + \frac{\mu_1 p}{\mu_2}\right) \tag{24}$$

From the equations (21), (22) and (24), we get

Steady state probability that the server is busy with essential service $= G(1) = \frac{\lambda}{k\mu_1}$

Steady state probability that the server is in vacation $\Rightarrow H(1) = 1 - \frac{\lambda}{k\mu_1}\left(1 + \frac{\mu_1 p}{\mu_2}\right)$

Steady state probability that the server is busy with second optional service $= I(1) = \left(\frac{\mu_1 p}{\mu_2}\right)\frac{\lambda}{k\mu_1}$

The generating function G(z) has the property that it must converge inside the unit circle $|z| < 1$.

We notice that the expression in the denominator of G(z) , $\lambda^2 z^{k+1} - \lambda(\lambda + \mu_1 + \mu_2) z^k + \mu_1\mu_2(1 + z + \ldots + z^{k-1}) + \mu_1(1-p)\lambda$ has k+1 zeros. By Rouche's theorem, we notice that k-1 zeros of this expression lies inside the circle $|z| < 1$ and must coincide with k-1 zeros of numerator of G(z) and two zero lies outside the circle $|z| < 1$. Let $z_0$ and $z_1$ be two zeros lie outside the circle $|z| < 1$.

As G(z) converges, k-1 zeros of numerator and denominator will be cancelled, we get

$$G(z) = \frac{A(\mu_2 + \lambda - \lambda z)}{(\alpha + \lambda - \lambda z)\lambda^2 (z - z_0)(z - z_1)} \tag{25}$$

As z = 1, from the equation (25), we get

$$G(1) = \frac{A\mu_2}{\alpha\lambda^2 (1 - z_0)(1 - z_1)} \tag{26}$$

Using (22) and (24) in (26), we obtain

$$A = \left(\frac{\lambda^3 \alpha}{k\mu_1\mu_2}\right)(1 - z_0)(1 - z_1) \tag{27}$$

From the equations (25) and (27), we get

$$G(z)=\frac{\lambda\alpha}{k\mu_1\mu_2}(1-z_0)(1-z_1)\frac{\mu_2+\lambda-\lambda z}{(\alpha+\lambda-\lambda z)(z-z_0)(z-z_1)} \tag{28}$$

By applying partial fractions, we get

$$G(z)=\frac{\lambda\alpha}{k\mu_1\mu_2}(1-z_0)(1-z_1)\left(\frac{A_1}{(\alpha+\lambda-\lambda z)}+\frac{A_2}{(z-z_0)}+\frac{A_3}{(z-z_1)}\right)$$

$$G(z)=\frac{\lambda\alpha}{k\mu_1\mu_2}\left(\frac{(r_1-1)(r_2-1)}{r_1r_2}\right)\left[\left(\frac{A_1}{\alpha+\lambda}\right)\sum_{n=0}^{\infty}s^nz^n-A_2\sum_{n=0}^{\infty}r_1^{n+1}z^n-A_3\sum_{n=0}^{\infty}r_2^{n+1}z^n\right] \tag{29}$$

where

$$r_1=\frac{1}{z_0},r_2=\frac{1}{z_1},\ \mathrm{s}=\frac{\lambda}{\lambda+\alpha},A_1=\frac{r_1r_2s^2(\mu_2-\alpha)}{(r_1-s)(r_2-s)},A_2=\frac{\left[r_1(\mu_2+\lambda)-\lambda\right]r_1r_2}{\left[r_1(\alpha+\lambda)-\lambda\right](r_2-r_1)}$$

$$\text{and } A_3=\frac{\left[r_2(\mu_2+\lambda)-\lambda\right]r_1r_2}{\left[r_2(\alpha+\lambda)-\lambda\right](r_1-r_2)}$$

Comparing the coefficient of $z^n$ (n = 0, 1, 2,. . . ) on both sides of the equation (22), we get

$$p_{n,1}=\frac{\lambda\alpha}{k\mu_1\mu_2}(r_1-1)(r_2-1)\left[\frac{(\mu_2-\alpha)s^{n+2}}{(r_1-s)(r_2-s)(\alpha+\lambda)}-\frac{\left[r_1(\mu_2+\lambda)-\lambda\right]}{\left[r_1(\alpha+\lambda)-\lambda\right](r_2-r_1)}r_1^{n+1}-\frac{\left[r_2(\mu_2+\lambda)-\lambda\right]}{\left[r_2(\alpha+\lambda)-\lambda\right](r_1-r_2)}r_2^{n+1}\right] \tag{30}$$

Equation (30) represents that the steady state probabilities that there are n customers in the queue when the server is busy with essential service.

From equation (13), we get

$$P_{0,3}=\left(\frac{\mu_1 p}{\lambda+\mu_2}\right)P_{0,1} \tag{31}$$

Using equation (24) in (14) and apply recursive for n = 1, 2, 3, . . . , we get

$$P_{n,3}=\frac{\mu_1 p}{\lambda+\mu_2}\sum_{t=0}^{n}P_{n-t,1}\left(\frac{\lambda}{\lambda+\mu_2}\right)^t \quad \text{for } n\geq 0 \tag{32}$$

Equations (31) and (32) represent the steady state probabilities that there are n customers in the queue when the server is busy with optional service.

From equation (10), we get

$$P_{0,2}=\frac{\mu_1(1-p)}{\lambda}P_{0,1}+\frac{\mu_2}{\lambda}P_{0,3} \tag{33}$$

Using equation (26) in (11) and apply recursive for n = 1, 2, 3, . . . , k-1 , we get

$$P_{n,2}=\frac{\mu_1(1-p)}{\lambda}\sum_{t=0}^{n}P_{t,1}+\frac{\mu_2}{\lambda}\sum_{t=0}^{n}P_{t,3} \quad \text{for n = 0,1,2, . . . , k-1} \tag{34}$$

From (12), we get

$$P_{n,2}=\left(\frac{\lambda}{\lambda+\alpha}\right)^{n-k+1}P_{k-1,2} \quad \text{for } n\geq k \tag{35}$$

Equations (33), (34) and (35) represent the steady state probabilities that there are n customers in the queue when the server is in vacation.

## 4. STABILITY CONDITION

The necessary and sufficient condition for the system to be stable is $\frac{\lambda}{k\mu_1}\left(1+\frac{\mu_1 p}{\mu_2}\right) < 1$.

## 5. PARTICULAR CASES

As p → 0, this model becomes a single server batch service queueing system with multiple vacations.

## 6. SYSTEM PERFORMANCE MEASURES

In this section, we will list some important performance measures along with their formulas. These measures are used to bring out the qualitative behavior of the queueing model under study. Numerical study has been dealt in very large scale to study the following measures.

1. $$p_{n,1} = \frac{\lambda\alpha}{k\mu_1\mu_2}(r_1-1)(r_2-1)\left[\frac{(\mu_2-\alpha)s^{n+2}}{(r_1-s)(r_2-s)(\alpha+\lambda)} - \frac{\left[r_1(\mu_2+\lambda)-\lambda\right]}{\left[r_1(\alpha+\lambda)-\lambda\right](r_2-r_1)}r_1^{n+1} - \frac{\left[r_2(\mu_2+\lambda)-\lambda\right]}{\left[r_2(\alpha+\lambda)-\lambda\right](r_1-r_2)}r_2^{n+1}\right]$$
   for n = 0,1,2,. . .
2. $$P_{n,2} = \frac{\mu_1(1-p)}{\lambda}\sum_{t=0}^{n}P_{t,1} + \frac{\mu_2}{\lambda}\sum_{t=0}^{n}P_{t,3}$$ for n = 0,1,2, . . . ,k-1
3. $$P_{n,2} = \left(\frac{\lambda}{\lambda+\alpha}\right)^{n-k+1}P_{k-1,2}$$ for n ≥ k
4. $$P_{n,3} = \frac{\mu_1 p}{\lambda+\mu_2}\sum_{t=0}^{n}P_{n-t,1}\left(\frac{\lambda}{\lambda+\mu_2}\right)^{t}$$ for n ≥ 0
5. $$L_q = \sum_{n=0}^{\infty}n(P_{n,1}+P_{n,2}) + \sum_{n=0}^{\infty}nP_{n,3}$$
6. $$V(x) = \left(\sum_{n=0}^{\infty}n^2P_{n,1} + \sum_{n=0}^{\infty}n^2P_{n,2} + \sum_{n=0}^{\infty}n^2P_{n,3}\right) - (L_q)^2$$

## 7. NUMERICAL STUDY

The values of parameters $\lambda$, $\mu_1$, $\mu_2$, $\alpha$, p and k are chosen so that they satisfy the stability condition discussed in section 4.

The system performance measures of this model have been done and expressed in the form of tables for various values $\lambda$, $\mu_1$, $\mu_2$, $\alpha$, p and k.

Tables 1, 5, 9, 13 and 17 show the steady probabilities distribution when server is busy with essential service for various values of $\lambda$ and different batch size k.

Tables 2, 6, 10, 14 and 18 show the steady state probabilities distribution when the server is in vacation for various values of $\lambda$ and different batch size k.

Tables 3, 7, 11, 15 and 19 show the steady state probabilities distribution when the server is busy with second optional service for various values of $\lambda$ and different batch size k.

Tables 4, 8, 12, 16 and 20 show the impact of arrival rate $\lambda$ and k over Mean number of customers in the queue. Further we infer the following

- Mean number of customers in the queue increases as arrival rate $\lambda$ increases.
- $P_1$, $P_2$ and $P_3$ are probabilities of server busy with essential service and in vacation and busy with second optional service.

Tables 21, 22, 23, 24 and 25 show the impact of probability p and k over Mean number of customers in the queue. Further we infer the following

- Mean number of customers in the queue decreases as p decreases.
- $P_1$ , $P_2$ and $P_3$ are probabilities of server busy with essential service and in vacation and busy with second optional service.
- As $p \to 0$, this model coincides with single server batch service queueing system with multiple vacations.

**Table 1:** Steady state Probabilities distribution for various values of $\lambda$, $\mu_1 = 10$, $\mu_2 = 20$, $p = 0.2$ and $\alpha = 15$ when the server is busy with an essential service and k = 2

| λ | P01 | P11 | P21 | P31 | P41 | P51 | P61 | P71 | P81 | P91 | P101 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.0426 | 0.0066 | 0.0008 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 1.5 | 0.0591 | 0.0132 | 0.0022 | 0.0003 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2 | 0.0731 | 0.0211 | 0.0046 | 0.0009 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2.5 | 0.0848 | 0.0298 | 0.0079 | 0.0019 | 0.0004 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3 | 0.0946 | 0.0388 | 0.0121 | 0.0034 | 0.0009 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3.5 | 0.1025 | 0.0478 | 0.0169 | 0.0054 | 0.0016 | 0.0005 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 4 | 0.1090 | 0.0567 | 0.0224 | 0.0080 | 0.0027 | 0.0009 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
| 4.5 | 0.1140 | 0.0652 | 0.0283 | 0.0111 | 0.0041 | 0.0015 | 0.0005 | 0.0002 | 0.0001 | 0.0000 | 0.0000 |
| 5 | 0.1178 | 0.0732 | 0.0346 | 0.0147 | 0.0059 | 0.0023 | 0.0009 | 0.0003 | 0.0001 | 0.0000 | 0.0000 |
| 5.5 | 0.1204 | 0.0806 | 0.0411 | 0.0188 | 0.0082 | 0.0035 | 0.0014 | 0.0006 | 0.0002 | 0.0001 | 0.0000 |
| 6 | 0.1220 | 0.0873 | 0.0476 | 0.0234 | 0.0109 | 0.0049 | 0.0022 | 0.0010 | 0.0004 | 0.0002 | 0.0001 |
| 6.5 | 0.1227 | 0.0933 | 0.0540 | 0.0282 | 0.0140 | 0.0067 | 0.0032 | 0.0015 | 0.0007 | 0.0003 | 0.0001 |
| 7 | 0.1225 | 0.0985 | 0.0603 | 0.0333 | 0.0175 | 0.0089 | 0.0045 | 0.0022 | 0.0011 | 0.0005 | 0.0003 |
| 7.5 | 0.1216 | 0.1029 | 0.0663 | 0.0386 | 0.0214 | 0.0115 | 0.0061 | 0.0032 | 0.0017 | 0.0009 | 0.0004 |
| 8 | 0.1200 | 0.1064 | 0.0720 | 0.0439 | 0.0255 | 0.0144 | 0.0080 | 0.0044 | 0.0024 | 0.0013 | 0.0007 |
| 8.5 | 0.1178 | 0.1091 | 0.0771 | 0.0492 | 0.0299 | 0.0177 | 0.0103 | 0.0059 | 0.0034 | 0.0019 | 0.0011 |
| 9 | 0.1149 | 0.1109 | 0.0817 | 0.0544 | 0.0345 | 0.0213 | 0.0129 | 0.0078 | 0.0047 | 0.0028 | 0.0017 |
| 9.5 | 0.1115 | 0.1118 | 0.0856 | 0.0593 | 0.0391 | 0.0252 | 0.0159 | 0.0100 | 0.0062 | 0.0039 | 0.0024 |

**Table 2:** Steady state Probabilities distribution for various values of $\lambda$, $\mu_1 = 10$, $\mu_2 = 20$, $p = 0.2$, $\alpha = 15$ when the server is in vacation and K = 2

| λ | P02 | P12 | P22 | P32 | P42 | P52 | P62 | P72 | P82 | P92 | P102 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.4217 | 0.4906 | 0.0307 | 0.0019 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 1.5 | 0.3887 | 0.4807 | 0.0437 | 0.0040 | 0.0004 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2 | 0.3588 | 0.4687 | 0.0551 | 0.0065 | 0.0008 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2.5 | 0.3317 | 0.4549 | 0.0650 | 0.0093 | 0.0013 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3 | 0.3070 | 0.4400 | 0.0733 | 0.0122 | 0.0020 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3.5 | 0.2842 | 0.4243 | 0.0803 | 0.0152 | 0.0029 | 0.0005 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 4 | 0.2633 | 0.4079 | 0.0859 | 0.0181 | 0.0038 | 0.0008 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 4.5 | 0.2440 | 0.3912 | 0.0903 | 0.0208 | 0.0048 | 0.0011 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
| 5 | 0.2261 | 0.3742 | 0.0935 | 0.0234 | 0.0058 | 0.0015 | 0.0004 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
| 5.5 | 0.2094 | 0.3571 | 0.0958 | 0.0257 | 0.0069 | 0.0019 | 0.0005 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
| 6 | 0.1940 | 0.3400 | 0.0972 | 0.0278 | 0.0079 | 0.0023 | 0.0006 | 0.0002 | 0.0001 | 0.0000 | 0.0000 |
| 6.5 | 0.1795 | 0.3230 | 0.0977 | 0.0295 | 0.0089 | 0.0027 | 0.0008 | 0.0002 | 0.0001 | 0.0000 | 0.0000 |
| 7 | 0.1660 | 0.3061 | 0.0974 | 0.0310 | 0.0099 | 0.0031 | 0.0010 | 0.0003 | 0.0001 | 0.0000 | 0.0000 |
| 7.5 | 0.1533 | 0.2895 | 0.0965 | 0.0322 | 0.0107 | 0.0036 | 0.0012 | 0.0004 | 0.0001 | 0.0000 | 0.0000 |
| 8 | 0.1414 | 0.2730 | 0.0949 | 0.0330 | 0.0115 | 0.0040 | 0.0014 | 0.0005 | 0.0002 | 0.0001 | 0.0000 |
| 8.5 | 0.1303 | 0.2567 | 0.0929 | 0.0336 | 0.0121 | 0.0044 | 0.0016 | 0.0006 | 0.0002 | 0.0001 | 0.0000 |
| 9 | 0.1198 | 0.2408 | 0.0903 | 0.0339 | 0.0127 | 0.0048 | 0.0018 | 0.0007 | 0.0003 | 0.0001 | 0.0000 |
| 9.5 | 0.1098 | 0.2251 | 0.0873 | 0.0338 | 0.0131 | 0.0051 | 0.0020 | 0.0008 | 0.0003 | 0.0001 | 0.0000 |

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**Table 3:** Steady state Probabilities distribution for various values of $\lambda$, $\mu_1 = 10$, $\mu_2 = 20$, $p = 0.2$ and $\alpha = 15$ when the server busy with an optional service and K=2

| λ | P03 | P13 | P23 | P33 | P43 | P53 | P63 | P73 | P83 | P93 | P103 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.0041 | 0.0008 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 1.5 | 0.0055 | 0.0016 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2 | 0.0066 | 0.0025 | 0.0007 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2.5 | 0.0075 | 0.0035 | 0.0011 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3 | 0.0082 | 0.0044 | 0.0016 | 0.0005 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3.5 | 0.0087 | 0.0054 | 0.0022 | 0.0008 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 4 | 0.0091 | 0.0062 | 0.0029 | 0.0011 | 0.0004 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 4.5 | 0.0093 | 0.0070 | 0.0036 | 0.0016 | 0.0006 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 5 | 0.0094 | 0.0077 | 0.0043 | 0.0020 | 0.0009 | 0.0004 | 0.0001 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
| 5.5 | 0.0094 | 0.0084 | 0.0050 | 0.0026 | 0.0012 | 0.0005 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
| 6 | 0.0094 | 0.0089 | 0.0057 | 0.0031 | 0.0016 | 0.0007 | 0.0003 | 0.0002 | 0.0001 | 0.0000 | 0.0000 |
| 6.5 | 0.0093 | 0.0093 | 0.0064 | 0.0037 | 0.0020 | 0.0010 | 0.0005 | 0.0002 | 0.0001 | 0.0001 | 0.0000 |
| 7 | 0.0091 | 0.0097 | 0.0070 | 0.0043 | 0.0024 | 0.0013 | 0.0007 | 0.0003 | 0.0002 | 0.0001 | 0.0000 |
| 7.5 | 0.0088 | 0.0099 | 0.0075 | 0.0049 | 0.0029 | 0.0016 | 0.0009 | 0.0005 | 0.0002 | 0.0001 | 0.0001 |
| 8 | 0.0086 | 0.0101 | 0.0080 | 0.0054 | 0.0034 | 0.0020 | 0.0011 | 0.0006 | 0.0004 | 0.0002 | 0.0001 |
| 8.5 | 0.0083 | 0.0101 | 0.0084 | 0.0060 | 0.0039 | 0.0024 | 0.0014 | 0.0008 | 0.0005 | 0.0003 | 0.0002 |
| 9 | 0.0079 | 0.0101 | 0.0088 | 0.0065 | 0.0044 | 0.0028 | 0.0018 | 0.0011 | 0.0007 | 0.0004 | 0.0002 |
| 9.5 | 0.0076 | 0.0100 | 0.0090 | 0.0069 | 0.0049 | 0.0033 | 0.0021 | 0.0014 | 0.0009 | 0.0005 | 0.0003 |

**Table 4:** Average number of customers in the queue and Variance for various values of $\lambda$, $\mu_1 = 10$, $\mu_2 = 20$, $p = 0.2$, $\alpha = 15$ and $K = 2$

| λ | P1 | P2 | P3 | MEAN | VARIANCE |
|---|---|---|---|---|---|
| 1 | 0.0500 | 0.9450 | 0.0050 | 0.5677 | 2.4576 |
| 1.5 | 0.0750 | 0.9175 | 0.0075 | 0.6032 | 2.5704 |
| 2 | 0.1000 | 0.8900 | 0.0100 | 0.6404 | 2.6919 |
| 2.5 | 0.1250 | 0.8625 | 0.0125 | 0.6799 | 2.8243 |
| 3 | 0.1500 | 0.8350 | 0.0150 | 0.7220 | 2.9699 |
| 3.5 | 0.1750 | 0.8075 | 0.0175 | 0.7672 | 3.1313 |
| 4 | 0.2000 | 0.7800 | 0.0200 | 0.8160 | 3.3114 |
| 4.5 | 0.2250 | 0.7525 | 0.0225 | 0.8688 | 3.5137 |
| 5 | 0.2500 | 0.7250 | 0.0250 | 0.9262 | 3.7423 |
| 5.5 | 0.2750 | 0.6975 | 0.0275 | 0.9888 | 4.0020 |
| 6 | 0.3000 | 0.6700 | 0.0300 | 1.0574 | 4.2986 |
| 6.5 | 0.3250 | 0.6425 | 0.0325 | 1.1328 | 4.6390 |
| 7 | 0.3500 | 0.6150 | 0.0350 | 1.2159 | 5.0319 |
| 7.5 | 0.3750 | 0.5875 | 0.0375 | 1.3080 | 5.4878 |
| 8 | 0.4000 | 0.5600 | 0.0400 | 1.4104 | 6.0199 |
| 8.5 | 0.4250 | 0.5325 | 0.0425 | 1.5246 | 6.6449 |
| 9 | 0.4500 | 0.5050 | 0.0450 | 1.6528 | 7.3839 |
| 9.5 | 0.4750 | 0.4775 | 0.0475 | 1.7973 | 8.2645 |

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**Table 5:** Steady state Probabilities distribution for various values of $\lambda$, $\mu_1 = 10$, $\mu_2 = 20$, $p = 0.2$, $\alpha = 15$ when the server is busy with an essential service and K=3

| $\lambda$ | P01 | P11 | P21 | P31 | P41 | P51 | P61 | P71 | P81 | P91 | P101 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.0284 | 0.0044 | 0.0005 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 1.5 | 0.0395 | 0.0088 | 0.0015 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2 | 0.0490 | 0.0140 | 0.0030 | 0.0006 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2.5 | 0.0570 | 0.0196 | 0.0051 | 0.0012 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3 | 0.0639 | 0.0256 | 0.0077 | 0.0021 | 0.0005 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3.5 | 0.0697 | 0.0315 | 0.0108 | 0.0033 | 0.0010 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 4 | 0.0746 | 0.0374 | 0.0142 | 0.0048 | 0.0016 | 0.0005 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 4.5 | 0.0786 | 0.0432 | 0.0179 | 0.0067 | 0.0024 | 0.0008 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
| 5 | 0.0820 | 0.0487 | 0.0219 | 0.0088 | 0.0034 | 0.0012 | 0.0004 | 0.0002 | 0.0001 | 0.0000 | 0.0000 |
| 5.5 | 0.0847 | 0.0539 | 0.0260 | 0.0112 | 0.0046 | 0.0018 | 0.0007 | 0.0003 | 0.0001 | 0.0000 | 0.0000 |
| 6 | 0.0868 | 0.0589 | 0.0302 | 0.0139 | 0.0060 | 0.0025 | 0.0010 | 0.0004 | 0.0002 | 0.0001 | 0.0000 |
| 6.5 | 0.0885 | 0.0634 | 0.0344 | 0.0167 | 0.0077 | 0.0034 | 0.0015 | 0.0006 | 0.0003 | 0.0001 | 0.0000 |
| 7 | 0.0897 | 0.0677 | 0.0386 | 0.0197 | 0.0095 | 0.0045 | 0.0020 | 0.0009 | 0.0004 | 0.0002 | 0.0001 |
| 7.5 | 0.0905 | 0.0715 | 0.0427 | 0.0229 | 0.0116 | 0.0057 | 0.0027 | 0.0013 | 0.0006 | 0.0003 | 0.0001 |
| 8 | 0.0909 | 0.0750 | 0.0468 | 0.0262 | 0.0138 | 0.0071 | 0.0035 | 0.0017 | 0.0009 | 0.0004 | 0.0002 |
| 8.5 | 0.0910 | 0.0781 | 0.0507 | 0.0295 | 0.0162 | 0.0086 | 0.0045 | 0.0023 | 0.0012 | 0.0006 | 0.0003 |
| 9 | 0.0908 | 0.0808 | 0.0545 | 0.0329 | 0.0188 | 0.0104 | 0.0056 | 0.0030 | 0.0016 | 0.0008 | 0.0004 |
| 9.5 | 0.0903 | 0.0832 | 0.0580 | 0.0363 | 0.0214 | 0.0122 | 0.0069 | 0.0038 | 0.0021 | 0.0011 | 0.0006 |

**Table 6:** Steady state Probabilities distribution for various values of $\lambda$, $\mu_1 = 10$, $\mu_2 = 20$, $p = 0.2$, $\alpha = 15$ when the server is in vacation and K = 3

| $\Lambda$ | P02 | P12 | P22 | P32 | P42 | P52 | P62 | P72 | P82 | P92 | P102 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.2814 | 0.3271 | 0.3327 | 0.0208 | 0.0013 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 1.5 | 0.2597 | 0.3207 | 0.3314 | 0.0301 | 0.0027 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2 | 0.2404 | 0.3130 | 0.3293 | 0.0387 | 0.0046 | 0.0005 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2.5 | 0.2231 | 0.3044 | 0.3265 | 0.0466 | 0.0067 | 0.0010 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3 | 0.2074 | 0.2952 | 0.3229 | 0.0538 | 0.0090 | 0.0015 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3.5 | 0.1931 | 0.2856 | 0.3186 | 0.0603 | 0.0114 | 0.0022 | 0.0004 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
| 4 | 0.1802 | 0.2758 | 0.3137 | 0.0660 | 0.0139 | 0.0029 | 0.0006 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
| 4.5 | 0.1683 | 0.2660 | 0.3082 | 0.0711 | 0.0164 | 0.0038 | 0.0009 | 0.0002 | 0.0000 | 0.0000 | 0.0000 |
| 5 | 0.1574 | 0.2562 | 0.3023 | 0.0756 | 0.0189 | 0.0047 | 0.0012 | 0.0003 | 0.0001 | 0.0000 | 0.0000 |
| 5.5 | 0.1473 | 0.2464 | 0.2960 | 0.0794 | 0.0213 | 0.0057 | 0.0015 | 0.0004 | 0.0001 | 0.0000 | 0.0000 |
| 6 | 0.1381 | 0.2368 | 0.2894 | 0.0827 | 0.0236 | 0.0067 | 0.0019 | 0.0006 | 0.0002 | 0.0000 | 0.0000 |
| 6.5 | 0.1295 | 0.2273 | 0.2825 | 0.0854 | 0.0258 | 0.0078 | 0.0024 | 0.0007 | 0.0002 | 0.0001 | 0.0000 |
| 7 | 0.1215 | 0.2181 | 0.2753 | 0.0876 | 0.0279 | 0.0089 | 0.0028 | 0.0009 | 0.0003 | 0.0001 | 0.0000 |
| 7.5 | 0.1141 | 0.2090 | 0.2680 | 0.0893 | 0.0298 | 0.0099 | 0.0033 | 0.0011 | 0.0004 | 0.0001 | 0.0000 |
| 8 | 0.1071 | 0.2002 | 0.2605 | 0.0906 | 0.0315 | 0.0110 | 0.0038 | 0.0013 | 0.0005 | 0.0002 | 0.0001 |
| 8.5 | 0.1007 | 0.1916 | 0.2528 | 0.0914 | 0.0331 | 0.0120 | 0.0043 | 0.0016 | 0.0006 | 0.0002 | 0.0001 |
| 9 | 0.0946 | 0.1832 | 0.2451 | 0.0919 | 0.0345 | 0.0129 | 0.0048 | 0.0018 | 0.0007 | 0.0003 | 0.0001 |
| 9.5 | 0.0889 | 0.1750 | 0.2374 | 0.0920 | 0.0357 | 0.0138 | 0.0054 | 0.0021 | 0.0008 | 0.0003 | 0.0001 |

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**Table 7:** Steady state Probabilities distribution for various values of $\lambda$, $\mu_1 = 10$, $\mu_2 = 20$, $p = 0.2$, $\alpha = 15$ when the server is busy with an optional service and K = 3

| λ | P03 | P13 | P23 | P33 | P43 | P53 | P63 | P73 | P83 | P93 | P103 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.0027 | 0.0005 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 1.5 | 0.0037 | 0.0011 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2 | 0.0045 | 0.0017 | 0.0004 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2.5 | 0.0051 | 0.0023 | 0.0007 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3 | 0.0056 | 0.0029 | 0.0011 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3.5 | 0.0059 | 0.0036 | 0.0015 | 0.0005 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 4 | 0.0062 | 0.0042 | 0.0019 | 0.0007 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 4.5 | 0.0064 | 0.0047 | 0.0023 | 0.0010 | 0.0004 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 5 | 0.0066 | 0.0052 | 0.0028 | 0.0013 | 0.0005 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 5.5 | 0.0066 | 0.0057 | 0.0033 | 0.0016 | 0.0007 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 6 | 0.0067 | 0.0061 | 0.0037 | 0.0019 | 0.0009 | 0.0004 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
| 6.5 | 0.0067 | 0.0064 | 0.0042 | 0.0023 | 0.0011 | 0.0005 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
| 7 | 0.0066 | 0.0067 | 0.0046 | 0.0027 | 0.0014 | 0.0007 | 0.0003 | 0.0002 | 0.0001 | 0.0000 | 0.0000 |
| 7.5 | 0.0066 | 0.0070 | 0.0050 | 0.0030 | 0.0017 | 0.0009 | 0.0004 | 0.0002 | 0.0001 | 0.0000 | 0.0000 |
| 8 | 0.0065 | 0.0072 | 0.0054 | 0.0034 | 0.0020 | 0.0011 | 0.0006 | 0.0003 | 0.0001 | 0.0001 | 0.0000 |
| 8.5 | 0.0064 | 0.0074 | 0.0058 | 0.0038 | 0.0023 | 0.0013 | 0.0007 | 0.0004 | 0.0002 | 0.0001 | 0.0001 |
| 9 | 0.0063 | 0.0075 | 0.0061 | 0.0042 | 0.0026 | 0.0015 | 0.0009 | 0.0005 | 0.0003 | 0.0001 | 0.0001 |
| 9.5 | 0.0061 | 0.0076 | 0.0064 | 0.0045 | 0.0029 | 0.0018 | 0.0010 | 0.0006 | 0.0003 | 0.0002 | 0.0001 |

**Table 8:** Average number of customers in the queue and Variance for various values of $\lambda$, $\mu_1 = 10$, $\mu_2 = 20$, $p = 0.2$, $\alpha = 15$ and K = 3

| λ | P1 | P2 | P3 | MEAN | VARIANCE |
|---|---|---|---|---|---|
| 1 | 0.0333 | 0.9633 | 0.0033 | 1.0668 | 3.8714 |
| 1.5 | 0.0500 | 0.9450 | 0.0050 | 1.1004 | 3.9780 |
| 2 | 0.0667 | 0.9267 | 0.0067 | 1.1344 | 4.0882 |
| 2.5 | 0.0833 | 0.9083 | 0.0083 | 1.1691 | 4.2030 |
| 3 | 0.1000 | 0.8900 | 0.0100 | 1.2046 | 4.3231 |
| 3.5 | 0.1167 | 0.8717 | 0.0117 | 1.2410 | 4.4496 |
| 4 | 0.1333 | 0.8533 | 0.0133 | 1.2787 | 4.5836 |
| 4.5 | 0.1500 | 0.8350 | 0.0150 | 1.3177 | 4.7261 |
| 5 | 0.1667 | 0.8167 | 0.0167 | 1.3583 | 4.8785 |
| 5.5 | 0.1833 | 0.7983 | 0.0183 | 1.4007 | 5.0421 |
| 6 | 0.2000 | 0.7800 | 0.0200 | 1.4451 | 5.2185 |
| 6.5 | 0.2167 | 0.7617 | 0.0217 | 1.4916 | 5.4093 |
| 7 | 0.2333 | 0.7433 | 0.0233 | 1.5407 | 5.6165 |
| 7.5 | 0.2500 | 0.7250 | 0.0250 | 1.5924 | 5.8422 |
| 8 | 0.2667 | 0.7067 | 0.0267 | 1.6470 | 6.0887 |
| 8.5 | 0.2833 | 0.6883 | 0.0283 | 1.7049 | 6.3588 |
| 9 | 0.3000 | 0.6700 | 0.0300 | 1.7663 | 6.6555 |
| 9.5 | 0.3167 | 0.6517 | 0.0317 | 1.8316 | 6.9824 |

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**Table 9:** Steady state Probabilities distribution for various values of $\lambda$, $\mu_1 = 10$, $\mu_2 = 20$, $p = 0.2$ and $\alpha = 15$ when the server is busy with an essential service and K = 4

| $\lambda$ | P01 | P11 | P21 | P31 | P41 | P51 | P61 | P71 | P81 | P91 | P101 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.0213 | 0.0033 | 0.0004 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 1.5 | 0.0296 | 0.0066 | 0.0011 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2 | 0.0368 | 0.0105 | 0.0023 | 0.0004 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2.5 | 0.0428 | 0.0147 | 0.0038 | 0.0009 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3 | 0.0480 | 0.0191 | 0.0058 | 0.0016 | 0.0004 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3.5 | 0.0525 | 0.0236 | 0.0080 | 0.0024 | 0.0007 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 4 | 0.0563 | 0.0280 | 0.0105 | 0.0036 | 0.0011 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 4.5 | 0.0595 | 0.0323 | 0.0133 | 0.0049 | 0.0017 | 0.0006 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
| 5 | 0.0622 | 0.0365 | 0.0162 | 0.0064 | 0.0024 | 0.0009 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
| 5.5 | 0.0645 | 0.0405 | 0.0192 | 0.0081 | 0.0033 | 0.0013 | 0.0005 | 0.0002 | 0.0001 | 0.0000 | 0.0000 |
| 6 | 0.0663 | 0.0442 | 0.0223 | 0.0100 | 0.0043 | 0.0017 | 0.0007 | 0.0003 | 0.0001 | 0.0000 | 0.0000 |
| 6.5 | 0.0679 | 0.0478 | 0.0254 | 0.0121 | 0.0054 | 0.0023 | 0.0010 | 0.0004 | 0.0002 | 0.0001 | 0.0000 |
| 7 | 0.0691 | 0.0511 | 0.0285 | 0.0142 | 0.0067 | 0.0030 | 0.0014 | 0.0006 | 0.0003 | 0.0001 | 0.0000 |
| 7.5 | 0.0701 | 0.0541 | 0.0316 | 0.0165 | 0.0081 | 0.0039 | 0.0018 | 0.0008 | 0.0004 | 0.0002 | 0.0001 |
| 8 | 0.0708 | 0.0570 | 0.0346 | 0.0188 | 0.0096 | 0.0048 | 0.0023 | 0.0011 | 0.0005 | 0.0002 | 0.0001 |
| 8.5 | 0.0713 | 0.0596 | 0.0376 | 0.0212 | 0.0113 | 0.0058 | 0.0029 | 0.0014 | 0.0007 | 0.0003 | 0.0002 |
| 9 | 0.0716 | 0.0620 | 0.0405 | 0.0237 | 0.0130 | 0.0069 | 0.0036 | 0.0018 | 0.0009 | 0.0005 | 0.0002 |
| 9.5 | 0.0717 | 0.0641 | 0.0433 | 0.0261 | 0.0149 | 0.0082 | 0.0044 | 0.0023 | 0.0012 | 0.0006 | 0.0003 |

**Table 10:** Steady state Probabilities distribution for various values of $\lambda$, $\mu_1 = 10$, $\mu_2 = 20$, $p = 0.2$, $\alpha = 15$ when the server is in vacation and K = 4

| | P02 | P12 | P22 | P32 | P42 | P52 | P62 | P72 | P82 | P92 | P102 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.2110 | 0.2453 | 0.2495 | 0.2499 | 0.0156 | 0.0010 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 1.5 | 0.1949 | 0.2406 | 0.2486 | 0.2498 | 0.0227 | 0.0021 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2 | 0.1805 | 0.2348 | 0.2470 | 0.2494 | 0.0293 | 0.0035 | 0.0004 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2.5 | 0.1676 | 0.2284 | 0.2449 | 0.2489 | 0.0356 | 0.0051 | 0.0007 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
| 3 | 0.1560 | 0.2217 | 0.2423 | 0.2480 | 0.0413 | 0.0069 | 0.0011 | 0.0002 | 0.0000 | 0.0000 | 0.0000 |
| 3.5 | 0.1455 | 0.2147 | 0.2392 | 0.2469 | 0.0467 | 0.0088 | 0.0017 | 0.0003 | 0.0001 | 0.0000 | 0.0000 |
| 4 | 0.1360 | 0.2076 | 0.2356 | 0.2454 | 0.0517 | 0.0109 | 0.0023 | 0.0005 | 0.0001 | 0.0000 | 0.0000 |
| 4.5 | 0.1273 | 0.2005 | 0.2318 | 0.2436 | 0.0562 | 0.0130 | 0.0030 | 0.0007 | 0.0002 | 0.0000 | 0.0000 |
| 5 | 0.1194 | 0.1934 | 0.2276 | 0.2416 | 0.0604 | 0.0151 | 0.0038 | 0.0009 | 0.0002 | 0.0001 | 0.0000 |
| 5.5 | 0.1121 | 0.1865 | 0.2232 | 0.2392 | 0.0642 | 0.0172 | 0.0046 | 0.0012 | 0.0003 | 0.0001 | 0.0000 |
| 6 | 0.1055 | 0.1797 | 0.2186 | 0.2366 | 0.0676 | 0.0193 | 0.0055 | 0.0016 | 0.0005 | 0.0001 | 0.0000 |
| 6.5 | 0.0993 | 0.1730 | 0.2138 | 0.2338 | 0.0707 | 0.0214 | 0.0065 | 0.0020 | 0.0006 | 0.0002 | 0.0001 |
| 7 | 0.0936 | 0.1666 | 0.2089 | 0.2307 | 0.0734 | 0.0234 | 0.0074 | 0.0024 | 0.0008 | 0.0002 | 0.0001 |
| 7.5 | 0.0883 | 0.1603 | 0.2040 | 0.2275 | 0.0758 | 0.0253 | 0.0084 | 0.0028 | 0.0009 | 0.0003 | 0.0001 |
| 8 | 0.0834 | 0.1542 | 0.1989 | 0.2240 | 0.0779 | 0.0271 | 0.0094 | 0.0033 | 0.0011 | 0.0004 | 0.0001 |
| 8.5 | 0.0788 | 0.1483 | 0.1939 | 0.2204 | 0.0797 | 0.0288 | 0.0104 | 0.0038 | 0.0014 | 0.0005 | 0.0002 |
| 9 | 0.0746 | 0.1426 | 0.1888 | 0.2166 | 0.0812 | 0.0305 | 0.0114 | 0.0043 | 0.0016 | 0.0006 | 0.0002 |
| 9.5 | 0.0706 | 0.1370 | 0.1837 | 0.2127 | 0.0825 | 0.0320 | 0.0124 | 0.0048 | 0.0019 | 0.0007 | 0.0003 |

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**Table 11:** Steady state Probabilities distribution for various values of $\lambda$, $\mu_1 = 10$, $\mu_2 = 20$, $p = 0.2$, $\alpha = 15$ when the server is busy with an optional service and K = 4

| $\lambda$ | P03 | P13 | P23 | P33 | P43 | P53 | P63 | P73 | P83 | P93 | P103 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.0020 | 0.0004 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 1.5 | 0.0028 | 0.0008 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2 | 0.0033 | 0.0013 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2.5 | 0.0038 | 0.0017 | 0.0005 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3 | 0.0042 | 0.0022 | 0.0008 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3.5 | 0.0045 | 0.0027 | 0.0011 | 0.0004 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 4 | 0.0047 | 0.0031 | 0.0014 | 0.0005 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 4.5 | 0.0049 | 0.0035 | 0.0017 | 0.0007 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 5 | 0.0050 | 0.0039 | 0.0021 | 0.0009 | 0.0004 | 0.0001 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 5.5 | 0.0051 | 0.0043 | 0.0024 | 0.0012 | 0.0005 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 6 | 0.0051 | 0.0046 | 0.0028 | 0.0014 | 0.0007 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 6.5 | 0.0051 | 0.0049 | 0.0031 | 0.0017 | 0.0008 | 0.0004 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
| 7 | 0.0051 | 0.0051 | 0.0034 | 0.0019 | 0.0010 | 0.0005 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
| 7.5 | 0.0051 | 0.0053 | 0.0037 | 0.0022 | 0.0012 | 0.0006 | 0.0003 | 0.0001 | 0.0001 | 0.0000 | 0.0000 |
| 8 | 0.0051 | 0.0055 | 0.0040 | 0.0025 | 0.0014 | 0.0007 | 0.0004 | 0.0002 | 0.0001 | 0.0000 | 0.0000 |
| 8.5 | 0.0050 | 0.0057 | 0.0043 | 0.0028 | 0.0016 | 0.0009 | 0.0005 | 0.0002 | 0.0001 | 0.0001 | 0.0000 |
| 9 | 0.0049 | 0.0058 | 0.0046 | 0.0031 | 0.0018 | 0.0011 | 0.0006 | 0.0003 | 0.0002 | 0.0001 | 0.0000 |
| 9.5 | 0.0049 | 0.0059 | 0.0048 | 0.0033 | 0.0021 | 0.0012 | 0.0007 | 0.0004 | 0.0002 | 0.0001 | 0.0001 |

**Table 12:** Average number of customers in the queue and Variance for various values of $\lambda$, $\mu_1 = 10$, $\mu_2 = 20$, $p = 0.2$, $\alpha = 15$ and K = 4

| $\Lambda$ | P1 | P2 | P3 | MEAN | VARIANCE |
|---|---|---|---|---|---|
| 1 | 0.0250 | 0.9725 | 0.0025 | 1.5667 | 5.4545 |
| 1.5 | 0.0375 | 0.9588 | 0.0038 | 1.6001 | 5.5602 |
| 2 | 0.0500 | 0.9450 | 0.0050 | 1.6335 | 5.6684 |
| 2.5 | 0.0625 | 0.9312 | 0.0062 | 1.6671 | 5.7795 |
| 3 | 0.0750 | 0.9175 | 0.0075 | 1.7010 | 5.8937 |
| 3.5 | 0.0875 | 0.9038 | 0.0088 | 1.7353 | 6.0116 |
| 4 | 0.1000 | 0.8900 | 0.0100 | 1.7699 | 6.1335 |
| 4.5 | 0.1125 | 0.8763 | 0.0113 | 1.8052 | 6.2602 |
| 5 | 0.1250 | 0.8625 | 0.0125 | 1.8411 | 6.3921 |
| 5.5 | 0.1375 | 0.8488 | 0.0138 | 1.8777 | 6.5299 |
| 6 | 0.1500 | 0.8350 | 0.0150 | 1.9153 | 6.6744 |
| 6.5 | 0.1625 | 0.8213 | 0.0163 | 1.9538 | 6.8263 |
| 7 | 0.1750 | 0.8075 | 0.0175 | 1.9934 | 6.9865 |
| 7.5 | 0.1875 | 0.7938 | 0.0188 | 2.0343 | 7.1558 |
| 8 | 0.2000 | 0.7800 | 0.0200 | 2.0765 | 7.3352 |
| 8.5 | 0.2125 | 0.7663 | 0.0213 | 2.1201 | 7.5258 |
| 9 | 0.2250 | 0.7525 | 0.0225 | 2.1653 | 7.7287 |
| 9.5 | 0.2375 | 0.7388 | 0.0237 | 2.2123 | 7.9452 |

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**Table 13:** Steady state Probabilities distribution for various values of $\lambda$, $\mu_1 = 10$, $\mu_2 = 20$, $p = 0.2$, $\alpha = 15$ when the server is busy with an essential service and K = 5

| λ | P01 | P11 | P21 | P31 | P41 | P51 | P61 | P71 | P81 | P91 | P101 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.0170 | 0.0026 | 0.0003 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 1.5 | 0.0237 | 0.0052 | 0.0009 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2 | 0.0294 | 0.0084 | 0.0018 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2.5 | 0.0343 | 0.0118 | 0.0031 | 0.0007 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3 | 0.0385 | 0.0153 | 0.0046 | 0.0012 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3.5 | 0.0420 | 0.0189 | 0.0064 | 0.0019 | 0.0006 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 4 | 0.0451 | 0.0224 | 0.0084 | 0.0028 | 0.0009 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 4.5 | 0.0477 | 0.0258 | 0.0106 | 0.0039 | 0.0013 | 0.0004 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 5 | 0.0499 | 0.0292 | 0.0129 | 0.0051 | 0.0019 | 0.0007 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
| 5.5 | 0.0518 | 0.0324 | 0.0153 | 0.0064 | 0.0026 | 0.0010 | 0.0004 | 0.0001 | 0.0001 | 0.0000 | 0.0000 |
| 6 | 0.0534 | 0.0354 | 0.0177 | 0.0079 | 0.0033 | 0.0014 | 0.0005 | 0.0002 | 0.0001 | 0.0000 | 0.0000 |
| 6.5 | 0.0547 | 0.0382 | 0.0202 | 0.0095 | 0.0042 | 0.0018 | 0.0008 | 0.0003 | 0.0001 | 0.0001 | 0.0000 |
| 7 | 0.0558 | 0.0409 | 0.0227 | 0.0112 | 0.0052 | 0.0024 | 0.0010 | 0.0004 | 0.0002 | 0.0001 | 0.0000 |
| 7.5 | 0.0567 | 0.0434 | 0.0251 | 0.0130 | 0.0063 | 0.0030 | 0.0014 | 0.0006 | 0.0003 | 0.0001 | 0.0001 |
| 8 | 0.0574 | 0.0458 | 0.0275 | 0.0148 | 0.0075 | 0.0037 | 0.0018 | 0.0008 | 0.0004 | 0.0002 | 0.0001 |
| 8.5 | 0.0579 | 0.0480 | 0.0299 | 0.0167 | 0.0088 | 0.0044 | 0.0022 | 0.0011 | 0.0005 | 0.0002 | 0.0001 |
| 9 | 0.0583 | 0.0500 | 0.0323 | 0.0186 | 0.0101 | 0.0053 | 0.0027 | 0.0014 | 0.0007 | 0.0003 | 0.0002 |
| 9.5 | 0.0586 | 0.0518 | 0.0345 | 0.0205 | 0.0115 | 0.0062 | 0.0033 | 0.0017 | 0.0009 | 0.0004 | 0.0002 |

**Table 14:** Steady state Probabilities distribution for various values of $\lambda$, $\mu_1 = 10$, $\mu_2 = 20$, $p = 0.2$ and $\alpha = 15$ when the server is in vacation and K = 5

| λ | P02 | P12 | P22 | P32 | P42 | P52 | P62 | P72 | P82 | P92 | P102 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.1688 | 0.1963 | 0.1996 | 0.2000 | 0.2000 | 0.0125 | 0.0008 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 1.5 | 0.1559 | 0.1925 | 0.1988 | 0.1998 | 0.2000 | 0.0182 | 0.0017 | 0.0002 | 0.0000 | 0.0000 | 0.0000 |
| 2 | 0.1444 | 0.1879 | 0.1976 | 0.1996 | 0.1999 | 0.0235 | 0.0028 | 0.0003 | 0.0000 | 0.0000 | 0.0000 |
| 2.5 | 0.1341 | 0.1828 | 0.1959 | 0.1991 | 0.1998 | 0.0285 | 0.0041 | 0.0006 | 0.0001 | 0.0000 | 0.0000 |
| 3 | 0.1248 | 0.1774 | 0.1938 | 0.1984 | 0.1996 | 0.0333 | 0.0055 | 0.0009 | 0.0002 | 0.0000 | 0.0000 |
| 3.5 | 0.1165 | 0.1718 | 0.1914 | 0.1975 | 0.1993 | 0.0377 | 0.0071 | 0.0013 | 0.0003 | 0.0000 | 0.0000 |
| 4 | 0.1090 | 0.1662 | 0.1886 | 0.1964 | 0.1989 | 0.0419 | 0.0088 | 0.0019 | 0.0004 | 0.0001 | 0.0000 |
| 4.5 | 0.1021 | 0.1606 | 0.1855 | 0.1950 | 0.1983 | 0.0458 | 0.0106 | 0.0024 | 0.0006 | 0.0001 | 0.0000 |
| 5 | 0.0958 | 0.1550 | 0.1823 | 0.1934 | 0.1976 | 0.0494 | 0.0123 | 0.0031 | 0.0008 | 0.0002 | 0.0000 |
| 5.5 | 0.0901 | 0.1496 | 0.1788 | 0.1916 | 0.1967 | 0.0528 | 0.0142 | 0.0038 | 0.0010 | 0.0003 | 0.0001 |
| 6 | 0.0849 | 0.1443 | 0.1753 | 0.1896 | 0.1957 | 0.0559 | 0.0160 | 0.0046 | 0.0013 | 0.0004 | 0.0001 |
| 6.5 | 0.0800 | 0.1391 | 0.1716 | 0.1874 | 0.1946 | 0.0588 | 0.0178 | 0.0054 | 0.0016 | 0.0005 | 0.0001 |
| 7 | 0.0756 | 0.1341 | 0.1678 | 0.1850 | 0.1933 | 0.0615 | 0.0196 | 0.0062 | 0.0020 | 0.0006 | 0.0002 |
| 7.5 | 0.0715 | 0.1292 | 0.1640 | 0.1826 | 0.1918 | 0.0639 | 0.0213 | 0.0071 | 0.0024 | 0.0008 | 0.0003 |
| 8 | 0.0677 | 0.1246 | 0.1602 | 0.1800 | 0.1902 | 0.0662 | 0.0230 | 0.0080 | 0.0028 | 0.0010 | 0.0003 |
| 8.5 | 0.0641 | 0.1200 | 0.1563 | 0.1772 | 0.1885 | 0.0682 | 0.0247 | 0.0089 | 0.0032 | 0.0012 | 0.0004 |
| 9 | 0.0608 | 0.1156 | 0.1525 | 0.1744 | 0.1866 | 0.0700 | 0.0262 | 0.0098 | 0.0037 | 0.0014 | 0.0005 |
| 9.5 | 0.0577 | 0.1114 | 0.1487 | 0.1715 | 0.1847 | 0.0716 | 0.0278 | 0.0108 | 0.0042 | 0.0016 | 0.0006 |

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**Table 15:** Steady state Probabilities distribution for various values of $\lambda$ and $\mu_1 = 10$, $\mu_2 = 20$, $p = 0.2$ and $\alpha = 15$ when the server is busy with an optional service and K=5

| $\lambda$ | P03 | P13 | P23 | P33 | P43 | P53 | P63 | P73 | P83 | P93 | P103 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.0016 | 0.0003 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 1.5 | 0.0022 | 0.0006 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2 | 0.0027 | 0.0010 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2.5 | 0.0030 | 0.0014 | 0.0004 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3 | 0.0033 | 0.0018 | 0.0006 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3.5 | 0.0036 | 0.0021 | 0.0009 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 4 | 0.0038 | 0.0025 | 0.0011 | 0.0004 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 4.5 | 0.0039 | 0.0028 | 0.0014 | 0.0006 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 5 | 0.0040 | 0.0031 | 0.0017 | 0.0007 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 5.5 | 0.0041 | 0.0034 | 0.0019 | 0.0009 | 0.0004 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 6 | 0.0041 | 0.0037 | 0.0022 | 0.0011 | 0.0005 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 6.5 | 0.0041 | 0.0039 | 0.0025 | 0.0013 | 0.0006 | 0.0003 | 0.0001 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
| 7 | 0.0041 | 0.0041 | 0.0027 | 0.0015 | 0.0008 | 0.0004 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
| 7.5 | 0.0041 | 0.0043 | 0.0030 | 0.0018 | 0.0009 | 0.0005 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
| 8 | 0.0041 | 0.0044 | 0.0032 | 0.0020 | 0.0011 | 0.0006 | 0.0003 | 0.0001 | 0.0001 | 0.0000 | 0.0000 |
| 8.5 | 0.0041 | 0.0046 | 0.0035 | 0.0022 | 0.0013 | 0.0007 | 0.0004 | 0.0002 | 0.0001 | 0.0000 | 0.0000 |
| 9 | 0.0040 | 0.0047 | 0.0037 | 0.0024 | 0.0015 | 0.0008 | 0.0004 | 0.0002 | 0.0001 | 0.0001 | 0.0000 |
| 9.5 | 0.0040 | 0.0048 | 0.0039 | 0.0026 | 0.0016 | 0.0009 | 0.0005 | 0.0003 | 0.0002 | 0.0001 | 0.0000 |

**Table 16:** Average number of customers in the queue and Variance for various values of $\lambda$ , $\mu_1 = 10$, $\mu_2 = 20$, $p = 0.2$, $\alpha = 15$ and $K = 5$

| $\Lambda$ | P1 | P2 | P3 | MEAN | VARIANCE |
|---|---|---|---|---|---|
| 1 | 0.0200 | 0.9780 | 0.0020 | 2.0667 | 7.2044 |
| 1.5 | 0.0300 | 0.9670 | 0.0030 | 2.1000 | 7.3100 |
| 2 | 0.0400 | 0.9560 | 0.0040 | 2.1334 | 7.4179 |
| 2.5 | 0.0500 | 0.9450 | 0.0050 | 2.1668 | 7.5281 |
| 3 | 0.0600 | 0.9340 | 0.0060 | 2.2002 | 7.6408 |
| 3.5 | 0.0700 | 0.9230 | 0.0070 | 2.2338 | 7.7563 |
| 4 | 0.0800 | 0.9120 | 0.0080 | 2.2676 | 7.8746 |
| 4.5 | 0.0900 | 0.9010 | 0.0090 | 2.3016 | 7.9961 |
| 5 | 0.1000 | 0.8900 | 0.0100 | 2.3358 | 8.1211 |
| 5.5 | 0.1100 | 0.8790 | 0.0110 | 2.3705 | 8.2500 |
| 6 | 0.1200 | 0.8680 | 0.0120 | 2.4055 | 8.3831 |
| 6.5 | 0.1300 | 0.8570 | 0.0130 | 2.4410 | 8.5209 |
| 7 | 0.1400 | 0.8460 | 0.0140 | 2.4771 | 8.6637 |
| 7.5 | 0.1500 | 0.8350 | 0.0150 | 2.5138 | 8.8122 |
| 8 | 0.1600 | 0.8240 | 0.0160 | 2.5512 | 8.9669 |
| 8.5 | 0.1700 | 0.8130 | 0.0170 | 2.5893 | 9.1282 |
| 9 | 0.1800 | 0.8020 | 0.0180 | 2.6283 | 9.2968 |
| 9.5 | 0.1900 | 0.7910 | 0.0190 | 2.6682 | 9.4734 |

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**Table 17:** Steady state Probabilities distribution for various values of $\lambda$, $\mu_1 = 10$, $\mu_2 = 20$, $p = 0.2$, $\alpha = 15$ when the server is busy with an essential service and K= 6

| λ | P01 | P11 | P21 | P31 | P41 | P51 | P61 | P71 | P81 | P91 | P101 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.0142 | 0.0022 | 0.0003 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 1.5 | 0.0198 | 0.0044 | 0.0007 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2 | 0.0245 | 0.0070 | 0.0015 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2.5 | 0.0286 | 0.0098 | 0.0025 | 0.0006 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3 | 0.0320 | 0.0127 | 0.0038 | 0.0010 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3.5 | 0.0350 | 0.0157 | 0.0053 | 0.0016 | 0.0005 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 4 | 0.0376 | 0.0187 | 0.0070 | 0.0024 | 0.0007 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 4.5 | 0.0398 | 0.0215 | 0.0088 | 0.0032 | 0.0011 | 0.0004 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 5 | 0.0416 | 0.0243 | 0.0107 | 0.0042 | 0.0016 | 0.0006 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
| 5.5 | 0.0432 | 0.0270 | 0.0127 | 0.0053 | 0.0021 | 0.0008 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
| 6 | 0.0446 | 0.0295 | 0.0147 | 0.0066 | 0.0028 | 0.0011 | 0.0004 | 0.0002 | 0.0001 | 0.0000 | 0.0000 |
| 6.5 | 0.0457 | 0.0319 | 0.0168 | 0.0079 | 0.0035 | 0.0015 | 0.0006 | 0.0003 | 0.0001 | 0.0000 | 0.0000 |
| 7 | 0.0467 | 0.0341 | 0.0188 | 0.0093 | 0.0043 | 0.0019 | 0.0008 | 0.0004 | 0.0002 | 0.0001 | 0.0000 |
| 7.5 | 0.0475 | 0.0363 | 0.0209 | 0.0107 | 0.0052 | 0.0024 | 0.0011 | 0.0005 | 0.0002 | 0.0001 | 0.0000 |
| 8 | 0.0481 | 0.0382 | 0.0229 | 0.0123 | 0.0062 | 0.0030 | 0.0014 | 0.0007 | 0.0003 | 0.0001 | 0.0001 |
| 8.5 | 0.0486 | 0.0401 | 0.0249 | 0.0138 | 0.0072 | 0.0036 | 0.0018 | 0.0009 | 0.0004 | 0.0002 | 0.0001 |
| 9 | 0.0490 | 0.0418 | 0.0268 | 0.0154 | 0.0083 | 0.0043 | 0.0022 | 0.0011 | 0.0005 | 0.0003 | 0.0001 |
| 9.5 | 0.0493 | 0.0433 | 0.0287 | 0.0170 | 0.0095 | 0.0051 | 0.0027 | 0.0014 | 0.0007 | 0.0004 | 0.0002 |

**Table 18:** Steady state Probabilities distribution for various values of $\lambda$, $\mu_1 = 10$, $\mu_2 = 20$, $p = 0.2$, $\alpha = 15$ when the server is in vacation and K = 6

| λ | P02 | P12 | P22 | P32 | P42 | P52 | P62 | P72 | P82 | P92 | P102 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.1407 | 0.1636 | 0.1663 | 0.1666 | 0.1667 | 0.1667 | 0.0104 | 0.0007 | 0.0000 | 0.0000 | 0.0000 |
| 1.5 | 0.1299 | 0.1604 | 0.1657 | 0.1665 | 0.1666 | 0.1667 | 0.0152 | 0.0014 | 0.0001 | 0.0000 | 0.0000 |
| 2 | 0.1203 | 0.1566 | 0.1647 | 0.1663 | 0.1666 | 0.1667 | 0.0196 | 0.0023 | 0.0003 | 0.0000 | 0.0000 |
| 2.5 | 0.1117 | 0.1523 | 0.1633 | 0.1659 | 0.1665 | 0.1666 | 0.0238 | 0.0034 | 0.0005 | 0.0001 | 0.0000 |
| 3 | 0.1040 | 0.1478 | 0.1615 | 0.1653 | 0.1663 | 0.1666 | 0.0278 | 0.0046 | 0.0008 | 0.0001 | 0.0000 |
| 3.5 | 0.0971 | 0.1432 | 0.1595 | 0.1646 | 0.1661 | 0.1665 | 0.0315 | 0.0060 | 0.0011 | 0.0002 | 0.0000 |
| 4 | 0.0908 | 0.1385 | 0.1572 | 0.1636 | 0.1657 | 0.1664 | 0.0350 | 0.0074 | 0.0016 | 0.0003 | 0.0001 |
| 4.5 | 0.0851 | 0.1339 | 0.1547 | 0.1625 | 0.1653 | 0.1662 | 0.0384 | 0.0089 | 0.0020 | 0.0005 | 0.0001 |
| 5 | 0.0800 | 0.1293 | 0.1520 | 0.1612 | 0.1647 | 0.1660 | 0.0415 | 0.0104 | 0.0026 | 0.0006 | 0.0002 |
| 5.5 | 0.0752 | 0.1248 | 0.1491 | 0.1597 | 0.1640 | 0.1657 | 0.0444 | 0.0119 | 0.0032 | 0.0009 | 0.0002 |
| 6 | 0.0709 | 0.1204 | 0.1462 | 0.1580 | 0.1631 | 0.1653 | 0.0472 | 0.0135 | 0.0039 | 0.0011 | 0.0003 |
| 6.5 | 0.0669 | 0.1161 | 0.1431 | 0.1562 | 0.1622 | 0.1648 | 0.0498 | 0.0151 | 0.0046 | 0.0014 | 0.0004 |
| 7 | 0.0632 | 0.1120 | 0.1401 | 0.1543 | 0.1611 | 0.1642 | 0.0523 | 0.0166 | 0.0053 | 0.0017 | 0.0005 |
| 7.5 | 0.0598 | 0.1080 | 0.1370 | 0.1523 | 0.1600 | 0.1636 | 0.0545 | 0.0182 | 0.0061 | 0.0020 | 0.0007 |
| 8 | 0.0567 | 0.1042 | 0.1338 | 0.1502 | 0.1587 | 0.1629 | 0.0566 | 0.0197 | 0.0069 | 0.0024 | 0.0008 |
| 8.5 | 0.0538 | 0.1005 | 0.1307 | 0.1480 | 0.1573 | 0.1620 | 0.0586 | 0.0212 | 0.0077 | 0.0028 | 0.0010 |
| 9 | 0.0511 | 0.0969 | 0.1276 | 0.1458 | 0.1558 | 0.1611 | 0.0604 | 0.0227 | 0.0085 | 0.0032 | 0.0012 |
| 9.5 | 0.0486 | 0.0935 | 0.1245 | 0.1434 | 0.1542 | 0.1602 | 0.0621 | 0.0241 | 0.0093 | 0.0036 | 0.0014 |

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**Table 19:** Steady state Probabilities distribution for various values of $\lambda$, $\mu_1 = 10$, $\mu_2 = 20$, $p = 0.2$ and $\alpha = 15$ when the server is busy with an optional service and K=6

| $\lambda$ | P03 | P13 | P23 | P33 | P43 | P53 | P63 | P73 | P83 | P93 | P103 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.0014 | 0.0003 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 1.5 | 0.0018 | 0.0005 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2 | 0.0022 | 0.0008 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2.5 | 0.0025 | 0.0012 | 0.0004 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3 | 0.0028 | 0.0015 | 0.0005 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3.5 | 0.0030 | 0.0018 | 0.0007 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 4 | 0.0031 | 0.0021 | 0.0009 | 0.0004 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 4.5 | 0.0032 | 0.0024 | 0.0012 | 0.0005 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 5 | 0.0033 | 0.0026 | 0.0014 | 0.0006 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 5.5 | 0.0034 | 0.0028 | 0.0016 | 0.0008 | 0.0003 | 0.0001 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 6 | 0.0034 | 0.0031 | 0.0018 | 0.0009 | 0.0004 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 6.5 | 0.0035 | 0.0033 | 0.0021 | 0.0011 | 0.0005 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 7 | 0.0035 | 0.0034 | 0.0023 | 0.0013 | 0.0007 | 0.0003 | 0.0001 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
| 7.5 | 0.0035 | 0.0036 | 0.0025 | 0.0015 | 0.0008 | 0.0004 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
| 8 | 0.0034 | 0.0037 | 0.0027 | 0.0016 | 0.0009 | 0.0005 | 0.0002 | 0.0001 | 0.0001 | 0.0000 | 0.0000 |
| 8.5 | 0.0034 | 0.0038 | 0.0029 | 0.0018 | 0.0011 | 0.0006 | 0.0003 | 0.0001 | 0.0001 | 0.0000 | 0.0000 |
| 9 | 0.0034 | 0.0039 | 0.0031 | 0.0020 | 0.0012 | 0.0007 | 0.0004 | 0.0002 | 0.0001 | 0.0000 | 0.0000 |
| 9.5 | 0.0033 | 0.0040 | 0.0032 | 0.0022 | 0.0013 | 0.0008 | 0.0004 | 0.0002 | 0.0001 | 0.0001 | 0.0000 |

**Table 20:** Average number of customers in the queue and Variance for various values of $\lambda$, $\mu_1 = 10$, $\mu_2 = 20$, $p = 0.2$. $\alpha = 15$ and $K = 6$

| $\lambda$ | P1 | P2 | P3 | MEAN | VARIANCE |
|---|---|---|---|---|---|
| 1 | 0.0167 | 0.9817 | 0.0017 | 2.5667 | 9.1211 |
| 1.5 | 0.0250 | 0.9725 | 0.0025 | 2.6000 | 9.2267 |
| 2 | 0.0333 | 0.9633 | 0.0033 | 2.6333 | 9.3345 |
| 2.5 | 0.0417 | 0.9542 | 0.0042 | 2.6667 | 9.4445 |
| 3 | 0.0500 | 0.9450 | 0.0050 | 2.7001 | 9.5569 |
| 3.5 | 0.0583 | 0.9358 | 0.0058 | 2.7335 | 9.6716 |
| 4 | 0.0667 | 0.9267 | 0.0067 | 2.7669 | 9.7888 |
| 4.5 | 0.0750 | 0.9175 | 0.0075 | 2.8005 | 9.9086 |
| 5 | 0.0833 | 0.9083 | 0.0083 | 2.8342 | 10.0311 |
| 5.5 | 0.0917 | 0.8992 | 0.0092 | 2.8680 | 10.1566 |
| 6 | 0.1000 | 0.8900 | 0.0100 | 2.9020 | 10.2852 |
| 6.5 | 0.1083 | 0.8808 | 0.0108 | 2.9363 | 10.4172 |
| 7 | 0.1167 | 0.8717 | 0.0117 | 2.9708 | 10.5529 |
| 7.5 | 0.1250 | 0.8625 | 0.0125 | 3.0057 | 10.6925 |
| 8 | 0.1333 | 0.8533 | 0.0133 | 3.0410 | 10.8364 |
| 8.5 | 0.1417 | 0.8442 | 0.0142 | 3.0767 | 10.9849 |
| 9 | 0.1500 | 0.8350 | 0.0150 | 3.1128 | 11.1383 |
| 9.5 | 0.1583 | 0.8258 | 0.0158 | 3.1495 | 11.2972 |

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**Table 21:** Average number of customers in the queue and Variance for various values of p, $\lambda = 5$, $\mu_1 = 10$, $\mu_2 = 20$, $\alpha = 15$ and K = 2

| p | P1 | P2 | P3 | Mean | Variance |
|---|---|---|---|---|---|
| 1.0000 | 0.2500 | 0.6250 | 0.1250 | 0.9983 | 4.0553 |
| 0.5000 | 0.2500 | 0.6875 | 0.0625 | 0.9511 | 3.8496 |
| 0.2500 | 0.2500 | 0.7188 | 0.0313 | 0.9302 | 3.7595 |
| 0.1250 | 0.2500 | 0.7344 | 0.0156 | 0.9203 | 3.7172 |
| 0.0625 | 0.2500 | 0.7422 | 0.0078 | 0.9154 | 3.6966 |
| 0.0313 | 0.2500 | 0.7461 | 0.0039 | 0.9130 | 3.6865 |
| 0.0156 | 0.2500 | 0.7480 | 0.0020 | 0.9119 | 3.6815 |
| 0.0078 | 0.2500 | 0.7490 | 0.0010 | 0.9113 | 3.6790 |
| 0.0039 | 0.2500 | 0.7495 | 0.0005 | 0.9110 | 3.6777 |
| 0.0020 | 0.2500 | 0.7498 | 0.0002 | 0.9108 | 3.6771 |
| 0.0010 | 0.2500 | 0.7499 | 0.0001 | 0.9108 | 3.6768 |
| 0.0005 | 0.2500 | 0.7499 | 0.0001 | 0.9107 | 3.6767 |
| 0.0002 | 0.2500 | 0.7500 | 0.0000 | 0.9107 | 3.6766 |
| 0.0001 | 0.2500 | 0.7500 | 0.0000 | 0.9107 | 3.6765 |

**Table 22:** Average number of customers in the queue and Variance for various values of p, $\lambda = 5$, $\mu_1 = 10$, $\mu_2 = 20$, $\alpha = 15$ and K = 3

| p | P1 | P2 | P3 | Mean | Variance |
|---|---|---|---|---|---|
| 1.0000 | 0.1667 | 0.7500 | 0.0833 | 1.3754 | 4.9491 |
| 0.5000 | 0.1667 | 0.7917 | 0.0417 | 1.3645 | 4.9041 |
| 0.2500 | 0.1667 | 0.8125 | 0.0208 | 1.3593 | 4.8827 |
| 0.1250 | 0.1667 | 0.8229 | 0.0104 | 1.3568 | 4.8722 |
| 0.0625 | 0.1667 | 0.8281 | 0.0052 | 1.3555 | 4.8671 |
| 0.0313 | 0.1667 | 0.8307 | 0.0026 | 1.3549 | 4.8645 |
| 0.0156 | 0.1667 | 0.8320 | 0.0013 | 1.3546 | 4.8632 |
| 0.0078 | 0.1667 | 0.8327 | 0.0007 | 1.3544 | 4.8626 |
| 0.0039 | 0.1667 | 0.8330 | 0.0003 | 1.3543 | 4.8623 |
| 0.0020 | 0.1667 | 0.8332 | 0.0002 | 1.3543 | 4.8621 |
| 0.0010 | 0.1667 | 0.8333 | 0.0001 | 1.3543 | 4.8620 |
| 0.0005 | 0.1667 | 0.8333 | 0.0000 | 1.3543 | 4.8620 |
| 0.0002 | 0.1667 | 0.8333 | 0.0000 | 1.3543 | 4.8620 |
| 0.0001 | 0.1667 | 0.8333 | 0.0000 | 1.3543 | 4.8620 |

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**Table 23:** Average number of customers in the queue and Variance for various values of p, $\lambda = 5$, $\mu_1 = 10$, $\mu_2 = 20$, $\alpha = 15$ and K = 4

| p | P1 | P2 | P3 | Mean | Variance |
|---|---|---|---|---|---|
| 1.0000 | 0.1250 | 0.8125 | 0.0625 | 1.8462 | 6.4129 |
| 0.5000 | 0.1250 | 0.8437 | 0.0313 | 1.8430 | 6.3998 |
| 0.2500 | 0.1250 | 0.8594 | 0.0156 | 1.8414 | 6.3934 |
| 0.1250 | 0.1250 | 0.8672 | 0.0078 | 1.8406 | 6.3902 |
| 0.0625 | 0.1250 | 0.8711 | 0.0039 | 1.8402 | 6.3886 |
| 0.0313 | 0.1250 | 0.8730 | 0.0020 | 1.8400 | 6.3878 |
| 0.0156 | 0.1250 | 0.8740 | 0.0010 | 1.8399 | 6.3874 |
| 0.0078 | 0.1250 | 0.8745 | 0.0005 | 1.8399 | 6.3872 |
| 0.0039 | 0.1250 | 0.8748 | 0.0002 | 1.8398 | 6.3871 |
| 0.0020 | 0.1250 | 0.8749 | 0.0001 | 1.8398 | 6.3871 |
| 0.0010 | 0.1250 | 0.8749 | 0.0001 | 1.8398 | 6.3871 |
| 0.0005 | 0.1250 | 0.8750 | 0.0000 | 1.8398 | 6.3870 |
| 0.0002 | 0.1250 | 0.8750 | 0.0000 | 1.8398 | 6.3870 |
| 0.0001 | 0.1250 | 0.8750 | 0.0000 | 1.8398 | 6.3870 |

**Table 24:** Average number of customers in the queue and Variance for various values of p, $\lambda = 5$, $\mu_1 = 10$, $\mu_2 = 20$, $\alpha = 15$ and K = 5

| p | P1 | P2 | P3 | Mean | Variance |
|---|---|---|---|---|---|
| 1.0000 | 0.1000 | 0.8500 | 0.0500 | 2.3375 | 8.1278 |
| 0.5000 | 0.1000 | 0.8750 | 0.0250 | 2.3365 | 8.1236 |
| 0.2500 | 0.1000 | 0.8875 | 0.0125 | 2.3359 | 8.1216 |
| 0.1250 | 0.1000 | 0.8938 | 0.0063 | 2.3357 | 8.1205 |
| 0.0625 | 0.1000 | 0.8969 | 0.0031 | 2.3356 | 8.1200 |
| 0.0313 | 0.1000 | 0.8984 | 0.0016 | 2.3355 | 8.1198 |
| 0.0156 | 0.1000 | 0.8992 | 0.0008 | 2.3355 | 8.1196 |
| 0.0078 | 0.1000 | 0.8996 | 0.0004 | 2.3354 | 8.1196 |
| 0.0039 | 0.1000 | 0.8998 | 0.0002 | 2.3354 | 8.1195 |
| 0.0020 | 0.1000 | 0.8999 | 0.0001 | 2.3354 | 8.1195 |
| 0.0010 | 0.1000 | 0.9000 | 0.0000 | 2.3354 | 8.1195 |
| 0.0005 | 0.1000 | 0.9000 | 0.0000 | 2.3354 | 8.1195 |
| 0.0002 | 0.1000 | 0.9000 | 0.0000 | 2.3354 | 8.1195 |
| 0.0001 | 0.1000 | 0.9000 | 0.0000 | 2.3354 | 8.1195 |

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**Table 25:** Average number of customers in the queue and Variance for various values of p, $\lambda = 5$, $\mu_1 = 10$, $\mu_2 = 20$, $\alpha = 15$ and $K = 6$

| p | P1 | P2 | P3 | Mean | Variance |
|---|---|---|---|---|---|
| 1.0000 | 0.0833 | 0.8750 | 0.0417 | 2.8347 | 10.0333 |
| 0.5000 | 0.0833 | 0.8958 | 0.0208 | 2.8344 | 10.0319 |
| 0.2500 | 0.0833 | 0.9063 | 0.0104 | 2.8342 | 10.0312 |
| 0.1250 | 0.0833 | 0.9115 | 0.0052 | 2.8341 | 10.0309 |
| 0.0625 | 0.0833 | 0.9141 | 0.0026 | 2.8341 | 10.0307 |
| 0.0313 | 0.0833 | 0.9154 | 0.0013 | 2.8340 | 10.0306 |
| 0.0156 | 0.0833 | 0.9160 | 0.0007 | 2.8340 | 10.0306 |
| 0.0078 | 0.0833 | 0.9163 | 0.0003 | 2.8340 | 10.0306 |
| 0.0039 | 0.0833 | 0.9165 | 0.0002 | 2.8340 | 10.0306 |
| 0.0020 | 0.0833 | 0.9166 | 0.0001 | 2.8340 | 10.0305 |
| 0.0010 | 0.0833 | 0.9166 | 0.0000 | 2.8340 | 10.0305 |
| 0.0005 | 0.0833 | 0.9166 | 0.0000 | 2.8340 | 10.0305 |
| 0.0002 | 0.0833 | 0.9167 | 0.0000 | 2.8340 | 10.0305 |
| 0.0001 | 0.0833 | 0.9167 | 0.0000 | 2.8340 | 10.0305 |

## 8. CONCLUSION

The Numerical studies show the changes in the system due to impact of arrival rate, batch size and optional service. The mean number of customers in the system increases as arrival rate increases for various batch sizes. This research work can be extended further by introducing various concepts like Bernoulli vacation, vacation interruption etc.,

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