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ANTI Q-FUZZY NORMAL SUBSEMIRING OF A SEMIRING

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ABSTRACT

In this paper, we made an attempt to study the algebraic nature of anti Q-fuzzy normal subsemiring of a semiring.

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Key Words: Q-*fuzzy set, Q-fuzzy subsemiring, anti Q-fuzzy subsemiring, anti Q-fuzzy normal subsemiring, Q-lower level subset.*

INTRODUCTION

There are many concepts of universal algebras generalizing an associative ring (R*;* +;.). Some of them in particular, nearrings and several kinds of semirings have been proven very useful. An algebra (R ; +, .) is said to be a semiring if (R ; +) and (R ; .) are semigroups satisfying a. (b+c) = a. b+a. c and (b+c) .a = b. a+c. a for all a, b and c in R. A semiring R is said to be additively commutative if a+b = b+a for all a, b in R. A semiring R may have an identity 1, defined by 1. a = a = a. 1 and a zero 0, defined by 0+a = a = a+0 and a.0 = 0 = 0.a for all a in R. After the introduction of fuzzy sets by L.A.Zadeh [11], several researchers explored on the generalization of the concept of fuzzy sets. Vanathi.K, Subramanian.V.S.A, Arjunan.K [9] defined as anti Q-fuzzy subsemiring of a semiring. In this paper, we introduce the some theorems in anti Q-fuzzy normal subsemiring of a semiring.

1. PRELIMINARIES

Definition: 1.1 Let X be a non empty set and Q be a non empty set. A Q-**fuzzy subset A** of X is a function A: X×Q → [0, 1].

Definition: 1.2 Let R be a semiring and Q be a non empty set. A Q-fuzzy subset A of R is said to be a Q-fuzzy subsemiring (QFSSR) of R if it satisfies the following conditions:
(i) $\mu_A(x+y, q) \geq \min\{\mu_A(x, q), \mu_A(y, q)\}$,
(ii) $\mu_A(xy, q) \geq \min\{\mu_A(x, q), \mu_A(y, q)\}$, for all x and y in R and q in Q.

Definition: 1.3 Let R be a semiring and Q be a non empty set. A Q-fuzzy subset A of R is said to be an anti Q-fuzzy subsemiring (AQFSSR) of R if it satisfies the following conditions:
(i) $\mu_A(x+y, q) \leq \max\{ \mu_A(x, q), \mu_A(y, q)\}$,
(ii) $\mu_A(xy, q) \leq \max\{\mu_A(x, q), \mu_A(y, q)\}$, for all x and y in R and q in Q.

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Definition: 1.4 Let R be a semiring and Q be a non empty set. An anti Q-fuzzy subsemiring A of R is said to be an anti Q-fuzzy normal subsemiring (AQFNSSR) of R if it satisfies the following conditions:
(i) $\mu_A(x+y, q) = \mu_A(y+x, q)$,
(ii) $\mu_A(xy, q) = \mu_A(yx, q)$, for all x and y in R and q in Q.

Definition: 1.5 Let A and B be Q-fuzzy subsets of sets G and H, respectively. The anti-product of A and B, denoted by A×B, is defined as $A\times B = \{\langle((x, y), q), \mu_{A\times B}((x,y), q)\rangle$ for all x in G and y in H and q in Q$\}$,
where $\mu_{A\times B}((x, y), q) = \max\{\mu_A(x, q), \mu_B(y, q)\}$.

Definition: 1.6 Let A be a Q-fuzzy subset in a set S, the anti-strongest Q-fuzzy relation on S, that is a Q-fuzzy relation on A is V given by $\mu_V((x,y), q)= \max\{\mu_A(x, q), \mu_A(y, q)\}$, for all x and y in S and q in Q.

Definition: 1.7 Let $(R, +, \cdot)$ and $(R^{I}, +, \cdot)$ be any two semirings. Let $f: R \to R^{I}$ be any function and A be an anti Q-fuzzy subsemiring in R, V be an anti Q-fuzzy subsemiring in $f(R) = R^{I}$, defined by $\mu_V(y, q) = \inf_{x\in f^{-1}(y)} \mu_A(x, q)$, for all x in R and y in R^{I} and q in Q. Then A is called a preimage of V under f and is denoted by $f^{-1}(V)$.

Definition: 1.8 Let A be a Q-fuzzy subset of X. For α in [0, 1], the Q-lower level subset of A is the set
$A_\alpha = \{x\in X: \mu_A(x, q) \le \alpha\}$.

2. PROPERTIES

Theorem]: 2.1 Let (R, +,.) be a semiring and Q be a non empty set. If A and B are two anti Q-fuzzy normal subsemirings of R, then their union A∪B is an anti Q-fuzzy normal subsemiring of R.

Proof: Let x and y in R and q in Q. Let $A = \{\langle(x, q), \mu_A(x, q)\rangle / x\in R$ and $q\in Q\}$ and $B = \{\langle(x, q), \mu_B(x, q)\rangle / x\in R$ and $q\in Q\}$ be anti Q-fuzzy normal subsemirings of a semiring R. Let C = A∪B and $C = \{\langle (x, q), \mu_C(x, q)\rangle / x\in R$ and $q\in Q\}$. Then, clearly C is an anti Q-fuzzy subsemiring of a semiring R, since A and B are two anti Q-fuzzy subsemirings of a semiring R. And,
(i) $\mu_C(x+y, q) = \max\{\mu_A(x+y, q), \mu_B(x+y, q)\}, = \max\{\mu_A(y+x, q), \mu_B(y+x, q)\}= \mu_C(y+x, q)$, for all x and y in R and q in Q. Therefore, $\mu_C(x+y, q) = \mu_C(y+x, q)$, for all x and y in R and q in Q.
(ii) $\mu_C(xy, q) = \max\{\mu_A(xy, q), \mu_B(xy, q)\}= \max\{\mu_A(yx, q), \mu_B(yx, q)\} = \mu_C(yx, q)$, for all x and y in R and q in Q. Therefore, $\mu_C(xy, q) = \mu_C(yx, q)$, for all x and y in R and q in Q. Hence A∪B is an anti Q-fuzzy normal subsemiring of a semiring R.

Theorem: 2.2 Let R be a semiring and Q be a non empty set. The union of a family of anti Q-fuzzy normal subsemirings of R is an anti Q-fuzzy normal subsemiring of R.

Proof: Let $\{A_i\}_{i\in I}$ be a family of anti Q-fuzzy normal subsemirings of a semiring R and let $A = \bigcup_{i\in I} A_i$. Then for x and y in R and q in Q. Clearly the union of a family of anti Q-fuzzy subsemirings of a semiring R is an anti Q-fuzzy subsemiring of a semiring R.
(i) $\mu_A(x+y, q) = \sup_{i\in I} \mu_{A_i}(x+y,q) = \sup_{i\in I} \mu_{A_i}(y+x,q) = \mu_A(y+x, q)$, for all x and y in R and q in Q. Therefore, $\mu_A(x+y, q) = \mu_A(y+x, q)$, for all x and y in R and q in Q.

(ii) $\mu_A(xy, q) = \sup_{i\in I} \mu_{A_i}(xy,q) = \sup_{i\in I} \mu_{A_i}(yx,q) = \mu_A(yx, q)$, for all x and y in R and q in Q.
Therefore, $\mu_A(xy, q) = \mu_A(yx, q)$, for all x and y in R and q in Q. Hence the union of a family of anti Q-fuzzy normal subsemirings of a semiring R is an anti Q-fuzzy normal subsemiring of a semiring R.

Theorem: 2.3 Let A and B be anti Q-fuzzy subsemiring of the semirings G and H, respectively. If A and B are anti Q-fuzzy normal subsemirings, then A×B is an anti Q-fuzzy normal subsemiring of G×H.

Proof: Let A and B be anti Q-fuzzy normal subsemirings of the semirings G and H respectively. Clearly A×B is an anti Q-fuzzy subsemiring of G×H. Let x_1 and x_2 be in G, y_1 and y_2 be in H. Then (x_1, y_1) and (x_2, y_2) are in G×H and q in Q.

Now, $\mu_{A\times B}[(x_1, y_1) + (x_2, y_2), q] = \mu_{A\times B}((x_1+ x_2, y_1+y_2), q) = \max\{\mu_A(x_1+x_2, q), \mu_B(y_1+y_2, q)\}= \max\{\mu_A(x_2+x_1, q), \mu_B(y_2 +y_1, q)\}= \mu_{A\times B}((x_2+x_1, y_2+y_1), q) = \mu_{A\times B}[(x_2, y_2) + (x_1, y_1), q]$.

Therefore, $\mu_{A\times B}[(x_1, y_1) + (x_2, y_2), q] = \mu_{A\times B}[(x_2, y_2)+(x_1, y_1), q]$. And, $\mu_{A\times B}[(x_1, y_1)(x_2, y_2), q] = \mu_{A\times B}((x_1x_2, y_1y_2), q) = \max\{\mu_A(x_1x_2, q), \mu_B(y_1y_2, q)\} = \max\{\mu_A(x_2x_1, q), \mu_B(y_2y_1, q)\} = \mu_{A\times B}((x_2x_1, y_2y_1), q) = \mu_{A\times B}[(x_2, y_2)(x_1, y_1), q]$.

Therefore, $\mu_{A\times B}[(x_1, y_1)(x_2, y_2), q] = \mu_{A\times B}[(x_2, y_2)(x_1, y_1), q]$. Hence A×B is an anti Q-fuzzy normal subsemiring of G×H.

Theorem: 2.4 Let A be a Q-fuzzy subset in a semiring R and V be the strongest anti Q-fuzzy relation on R. Then A is an anti Q-fuzzy normal subsemiring of R if and only if V is an anti Q-fuzzy normal subsemiring of R×R.

Proof: Suppose that A is an anti Q-fuzzy normal subsemiring of R. Then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in R×R and q in Q. Clearly V is a anti Q-fuzzy subsemiring of R×R. We have, $\mu_V(x+y, q) = \mu_V[(x_1, x_2)+(y_1, y_2), q] = \mu_V((x_1+y_1, x_2+y_2), q) = \mu_A((x_1+y_1), q) \wedge \mu_A((x_2+y_2), q) = \mu_A((y_1+x_1), q) \wedge \mu_A((y_2+x_2), q) = \mu_V((y_1+x_1, y_2+x_2), q) = \mu_V[(y_1, y_2)+(x_1, x_2), q] = \mu_V(y+x, q)$ Therefore, $\mu_V(x+y, q) = \mu_V(y+x, q)$, for all x and y in R×R and q in Q. We have, $\mu_V(xy, q) = \mu_V[(x_1, x_2)(y_1, y_2), q] = \mu_V((x_1y_1, x_2y_2), q) = \mu_A((x_1y_1), q) \wedge \mu_A((x_2y_2), q) = \mu_A((y_1x_1), q) \wedge \mu_A((y_2x_2), q) = \mu_V((y_1x_1, y_2x_2), q) = \mu_V[(y_1, y_2)(x_1, x_2), q] = \mu_V(yx, q)$. Therefore, $\mu_V(xy, q) = \mu_V(yx, q)$, for all x and y in R×R and q in Q. This proves that V is a anti Q-fuzzy normal subsemiring of R×R. Conversely, assume that V is a anti Q-fuzzy normal subsemiring of R×R, then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in R×R, we have $\mu_A(x_1+y_1, q) \wedge \mu_A(x_2+y_2, q) = \mu_V((x_1+y_1, x_2+y_2), q) = \mu_V[(x_1, x_2)+(y_1, y_2), q] = \mu_V(x+y, q) = \mu_V(y+x, q) = \mu_V[(y_1, y_2)+(x_1, x_2), q] = \mu_V((y_1+x_1, y_2+x_2), q) = \mu_A(y_1+x_1, q) \wedge \mu_A(y_2+x_2, q)$. We get, $\mu_A(x_1+y_1, q) = \mu_A(y_1+x_1, q)$, for all x_1 and y_1 in R and q in Q. And $\mu_A(x_1y_1, q) \wedge \mu_A(x_2y_2, q) = \mu_V((x_1y_1, x_2y_2), q) = \mu_V[(x_1, x_2)(y_1, y_2), q] = \mu_V(xy, q) = \mu_V(yx, q) = \mu_V[(y_1, y_2)(x_1, x_2), q] = \mu_V((y_1x_1, y_2x_2), q) = \mu_A(y_1x_1, q) \wedge \mu_A(y_2x_2, q)$. We get, $\mu_A(x_1y_1, q) = \mu_A(y_1x_1, q)$, for all x_1 and y_1 in R and q in Q. Hence A is an anti Q-fuzzy normal subsemiring of R.

Theorem: 2.5 Let (R, +, .) and (R^I, +, .) be any two semirings and Q be a non empty set. The homomorphic image of an anti Q-fuzzy normal subsemiring of R is an anti Q-fuzzy normal subsemiring of R^I.

Proof: Let (R, +, .) and (R^I, +, .) be any two semirings and $f: R \to R^I$ be a homomorphism. Then, f(x+y) = f(x) + f(y) and f(xy) = f(x) f(y), for all x and y in R. Let V = f(A), where A is an anti Q-fuzzy normal subsemiring of a semiring R. We have to prove that V is an anti Q-fuzzy normal subsemiring of a semiring R^I. Now, for f(x), f(y) in R^I, clearly V is an anti Q-fuzzy subsemiring of a semiring R^I, since A is an anti Q-fuzzy subsemiring of a semiring R. Now, $\mu_v(f(x)+f(y), q) = \mu_v(f(x+y), q) \leq \mu_A(x+y, q) = \mu_A(y+x, q) \geq \mu_v(f(y+x), q) = \mu_v(f(y)+f(x), q)$, which implies that $\mu_v(f(x)+f(y), q) = \mu_v(f(y)+f(x), q)$, for all f(x) and f(y) in R^I and q in Q.

Again, $\mu_v(f(x)f(y), q) = \mu_v(f(xy), q) \leq \mu_A(xy, q) = \mu_A(yx, q) \geq \mu_v(f(yx), q) = \mu_v(f(y)f(x), q)$, which implies that $\mu_v(f(x)f(y), q) = \mu_v(f(y)f(x), q)$, for all f(x) and f(y) in R^I and q in Q. Hence V is an anti Q-fuzzy normal subsemiring of a semiring R^I.

Theorem: 2.6 Let (R, +, .) and (R^I, +, .) be any two semirings and Q be a non empty set. The homomorphic preimage of an anti Q-fuzzy normal subsemiring of R^I is an anti Q-fuzzy normal subsemiring of R.

Proof: Let (R, +, .) and (R^I, +, .) be any two semirings and $f : R \to R^I$ be a homomorphism. Then, f(x+y) = f(x) + f(y) and f(xy) = f(x) f(y), for all x and y in R. Let V = f(A), where V is an anti Q-fuzzy normal subsemiring of a semiring R^I. We have to prove that A is an anti Q-fuzzy normal subsemirring of a semiring R. Let x and y in R and q in Q. Then, clearly A is an anti Q-fuzzy subsemiring of a semiring R, since V is an anti Q-fuzzy subsemiring of a semiring R^I. Now, $\mu_A(x+y, q)=\mu_v(f(x+y), q)= \mu_v(f(x)+f(y), q) = \mu_v(f(y)+f(x), q) = \mu_v(f(y+x), q) = \mu_A(y+x, q)$, which implies that $\mu_A(x+y, q) = \mu_A(y+x, q)$, for all x and y in R and q in Q. Again, $\mu_A(xy, q) = \mu_v(f(xy), q) = \mu_v(f(x)f(y), q) = \mu_v(f(y)f(x), q) = \mu_v(f(yx), q) = \mu_A(yx, q)$, which implies that $\mu_A(xy, q) = \mu_A(yx, q)$, for all x and y in R and q in Q. Hence A is an anti Q-fuzzy normal subsemiring of a semiring R.

Theorem: 2.7 Let (R, +, .) and (R^I, +, .) be any two semirings and Q be a non empty set. The anti-homomorphic image of an anti Q-fuzzy normal subsemiring of R is an anti Q-fuzzy normal subsemiring of R^I.

Proof: Let (R, +, .) and (R^I, +, .) be any two semirings and $f : R \to R^I$ be an anti-homomorphism. Then, f(x+y) = f(y)+f(x) and f(xy) = f(y) f(x), for all x and y in R. Let V = f(A), where A is an anti Q-fuzzy normal subsemiring of a semiring R. We have to prove that V is an anti Q-fuzzy normal subsemiring of a semiring R^I. Now, for f(x) and f(y) in R^I, clearly V is an anti Q-fuzzy subsemiring of a semiring R^I, since A is an anti Q-fuzzy subsemiring of a semiring R. Now, $\mu_v(f(x)+f(y), q) = \mu_v(f(y+x), q) \leq \mu_A(y+x, q) = \mu_A(x+y, q) \geq \mu_v(f(x+y), q) = \mu_v(f(y)+f(x), q)$, which implies that $\mu_v(f(x)+f(y), q) = \mu_v(f(y) + f(x), q)$, for all f(x) and f(y) in R^I and q in Q.

Again, $\mu_v(f(x)f(y), q) = \mu_v(f(yx), q) \leq \mu_A(yx, q) = \mu_A(xy, q) \geq \mu_v(f(xy), q) = \mu_v(f(y)f(x), q)$, which implies that $\mu_v(f(x)f(y), q) = \mu_v(f(y)f(x), q)$, for all f(x) and f(y) in R^I and q in Q. Hence V is an anti Q-fuzzy normal subsemiring of a semiring R^I.

Theorem: 2.8 Let (R, +, .) and (R^l, +, .) be any two semirings and Q be a non empty set. The anti-homomorphic preimage of an anti Q-fuzzy normal subsemiring of R^l is an anti Q-fuzzy normal subsemiring of R.

Proof: Let (R, +, .) and (R^l, +, .) be any two semirings and f: R $\rightarrow$ R^l be an anti-homomorphism. Then, f(x+y) = f(y)+f(x) and f(xy) = f(y) f(x), for all x and y in R. Let V = f(A), where V is an anti Q-fuzzy normal subsemiring of a semiring R^l. We have to prove that A is an anti Q-fuzzy normal subsemiring of a semiring R. Let x and y in R and q in Q, then, clearly A is an anti Q-fuzzy subsemiring of a semiring R, since V is an anti Q-fuzzy subsemiring of a semiring R^l. Now, $\mu_A(x + y, q) = \mu_v(f(x + y), q) = \mu_v(f(y)+f(x), q) = \mu_v(f(x)+f(y), q) = \mu_v(f(y+x), q) = \mu_A(y+x, q)$, which implies that $\mu_A(x+y, q) = \mu_A(y+x, q)$, for all x and y in R and q in Q. Again, $\mu_A(xy, q) = \mu_v(f(xy), q) = \mu_v(f(y)f(x), q) = \mu_v(f(x)f(y), q) = \mu_v(f(yx), q) = \mu_A(yx, q)$, which implies that $\mu_A(xy, q) = \mu_A(yx, q)$, for all x and y in R. Hence A is an anti Q-fuzzy normal subsemiring of a semiring R.

Theorem: 2.9 Let A be an anti Q-fuzzy subsemiring of a semiring H and f is an isomorphism from a semiring R onto H. If A is an anti Q-fuzzy normal subsemiring of the semiring H, then A$\circ$f is an anti Q-fuzzy normal subsemiring of the semiring R.

Proof: Let x and y in R, q in Q and A be an anti Q-fuzzy normal subsemiring of a semiring H. Then we have, clearly A$\circ$f is an anti Q-fuzzy subsemiring of a semiring R.

Now, $(\mu_A\circ f)(x+y, q) = \mu_A(f(x+y), q) = \mu_A(f(x)+f(y), q)= \mu_A(f(y)+f(x), q) = \mu_A(f(y+x), q) = (\mu_A\circ f)(y+x, q)$, which implies that $(\mu_A\circ f)(x+y, q) = (\mu_A\circ f)(y+x, q)$, for all x and y in R and q in Q. And, $(\mu_A\circ f)(xy, q) = \mu_A(f(xy), q) = \mu_A(f(x)f(y), q) = \mu_A(f(y)f(x), q) = \mu_A(f(yx), q) = (\mu_A\circ f)(yx, q)$, which implies that $(\mu_A\circ f)(xy, q) = (\mu_A\circ f)(yx, q)$, for all x and y in R and q in Q. Hence A$\circ$f is an anti Q-fuzzy normal subsemiring of a semiring R.

Theorem: 2.10 Let A be an anti Q-fuzzy subsemiring of a semiring H and f is an anti-isomorphism from a semiring R onto H. If A is an anti Q-fuzzy normal subsemiring of the semiring H, then A$\circ$f is an anti Q-fuzzy normal subsemiring of the semiring R.

Proof: Let x and y in R, q in Q and A be an anti Q-fuzzy normal subsemiring of a semiring H. Then we have, clearly A$\circ$f is an anti Q-fuzzy subsemiring of a semiring R.

Now, $(\mu_A\circ f)(x+y, q) = \mu_A(f(x+y), q) = \mu_A(f(y)+f(x), q) = \mu_A(f(x)+f(y), q) = \mu_A(f(y+x), q) = (\mu_A\circ f)(y+x, q)$, which implies that $(\mu_A\circ f)(x+y, q) = (\mu_A\circ f)(y+x, q)$, for all x and y in R and q in Q. And, $(\mu_A\circ f)(xy, q) = \mu_A(f(xy), q) = \mu_A(f(y)f(x), q) = \mu_A(f(x)f(y), q) = \mu_A(f(yx), q) = (\mu_A\circ f)(yx, q)$, which implies that $(\mu_A\circ f)(xy, q) = (\mu_A\circ f)(yx, q)$, for all x and y in R and q in Q. Hence A$\circ$f is an anti Q-fuzzy normal subsemiring of a semiring R.

Theorem: 2.11 Let A be an anti Q-fuzzy normal subsemiring of a semiring R. Then for α in [0, 1], A_α is a Q-lower level subsemiring of R.

Proof: It is trivial.

Theorem: 2.12 Let A be an anti Q-fuzzy normal subsemiring of a semiring R. Then two Q-lower level subsemiring $A_{\alpha1}$, $A_{\alpha2}$ and α_1, α_2 are in [0,1] with $\alpha_1 < \alpha_2$ of A are equal if and only if there is no x in R such that $\alpha_2 > \mu_A(x, q) > \alpha_1$.

Proof: It is trivial.

Theorem: 2.13 Let A be an anti Q-fuzzy normal subsemiring of a semiring R. If any two Q-lower level subsemirings of A belongs to R, then their intersection is also Q-lower level subsemiring of A in R.

Proof: It is trivial.

Theorem: 2.14 Let A be an anti Q-fuzzy normal subsemiring of a semiring R. If $\alpha_i\in[0,1]$, and $A_{\alpha i}$, $i\in I$ is a collection of Q-lower level subsemirings of A, then their intersection is also a Q-lower level subsemiring of A.

Proof: It is trivial.

Theorem: 2.15 Let A be an anti Q-fuzzy normal subsemiring of a semiring R. If any two Q-lower level subsemirings of A belongs to R, then their union is also a Q-lower level subsemiring of A in R.

Proof: It is trivial.

Theorem: 2.16 Let A be an anti Q-fuzzy normal subsemiring of a semiring R. If $\alpha_i \in [0,1]$, and $A_{\alpha i}$, $i \in I$ is a collection of Q-lower level subsemirings of A, then their union is also a Q-lower level subsemiring of A.

Proof: It is trivial.

Theorem: 2.17 The homomorphic image of a Q-lower level subsemiring of an anti Q-fuzzy normal subsemiring of a semiring R is a Q-lower level subsemiring of an anti Q-fuzzy normal subsemiring of a semiring R^l.

Proof: It is trivial.

Theorem: 2.18 The homomorphic pre-image of a Q-lower level subsemiring of an anti Q-fuzzy normal subsemiring of a semiring R^l is a Q-lower level subsemiring of an anti Q-fuzzy normal subsemiring of a semiring R.

Proof: It is trivial.

Theorem: 2.19 The anti-homomorphic image of a Q-lower level subsemiring of an anti Q-fuzzy normal subsemiring of a semiring R is a Q-lower level subsemiring of an anti Q-fuzzy normal subsemiring of a semiring R^l.

Proof: It is trivial.

Theorem: 2.20 The anti-homomorphic pre-image of a Q-lower level subsemiring of an anti Q-fuzzy normal subsemiring of a semiring R^l is a Q-lower level subsemiring of an anti Q-fuzzy normal subsemiring of a semiring R.

Proof: It is trivial.

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