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MASS TRANSFER EFFECTS ON FLOW PAST A PARABOLIC STARTED VERTICAL PLATE WITH VARIABLE TEMPERATURE AND MASS DIFFUSION

[1]R. MUTHUCUMARASWAMY*
[1]Department of Applied Mathematics, Sri Venkateswara College of Engineering, Irungattukottai - 602 117, Sriperumbudur Taluk, India.

[2]A. NEEL ARMSTRONG
[2]Department of Mathematics, SKR Engineering College, Nazarthpet, Poonamallee, Chennai - 600 123, India.

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ABSTRACT

The hypothetical solution of flow past a parabolic starting motion of the infinite vertical plate with variable temperature and variable mass diffusion has been studied. The plate temperature and the concentration level near the plate are raised linearly with time. The dimensionless governing equations are solved by using the Laplace-transform technique. The effect of velocity profiles are studied for different physical parameters like thermal Grashof number, mass Grashof number, Schmidt number, and time. It is observed that the velocity increases as the value of the thermal Grashof number or mass Grashof number increase. The trend is just reversed with respect to the Schmidt number.

***Keywords**: parabolic, variable temperature, vertical plate, variable, heat and mass transfer.*

1. INTRODUCTION

The real applications are related on building thermal insulation, petroleum reservoir operations, food processing, contaminant transport in groundwater, casting and welding in manufacturing processes, drying processes, cooling of nuclear reactors and stability of snow and the other applications are purification of crude oil, molten plastics, pulps, paper industry, textile industry; the cooling of threads or sheets is of importance in the process industries. After the pioneering work by Agrawal [1], a great deal of papers researching several aspects of these convection processes taking into the account the applicability of the boundary layer approximation have been published.

A fluid flowing through the receiver takes the heat away towards the thermal power cycle, where e.g. high pressure, high temperature steam is generated to drive a turbine. Air, water, oil and molten salt can be used as heat transfer fluids. Natural convection on flow past a linearly accelerated vertical plate in the presence of viscous dissipative heat using perturbation method by Gupta *et al* [3]. Kafousias and Raptis [5] extended this problem to include mass transfer effects subjected to variable suction or injection.

Soundalgekar [8] studied the mass transfer effects on flow past a uniformly accelerated vertical plate. Mass transfer effects on flow past an accelerated vertical plate with uniform heat flux was analyzed by Singh and Singh [6]. Free convection effects on flow past an exponentially accelerated vertical plate was studied by Singh and Naveen Kumar [7]. The skin friction for accelerated vertical plate has been studied analytically by Hossain and Shayo [4]. Mass transfer effects on exponentially accelerated infinite vertical plate with constant heat flux and uniform mass diffusion was studied by Basant Kumar Jha *et al* [2].

The effect of a transverse magnetic field on unsteady free convective flow and mass diffusion of an electrically conducting elasto-viscous fluid past a parabolic starting motion of the infinite vertical plate was analyzed by Agrawal *et al* [1]. MHD effects on impulsively started infinite vertical plate with variable temperature in the presence of the transverse magnetic field were studied by Soundalgekar *et al*. [8]. The governing equations are tackled using Laplace transform technique.

Hence, it is proposed to study the effects on flow past a parabolic started an infinite vertical plate with variable mass diffusion. The dimensionless governing equations are solved by using the Laplace-Transform Technique. The solutions are in terms of exponential and complementary error function.

Corresponding author: [1]R. MUTHUCUMARASWAMY*
E-mail: msamy@svce.ac.in

2. GOVERNING EQUATIONS

Consider the unsteady flow of a viscous incompressible fluid past an infinite vertical plate with variable temperature and variable mass diffusion has been considered. The x' -axis is taken along the plate in the vertically upward direction and the y -axis is taken normal to the plate. At time $t' \le 0$, the plate and fluid are at the same temperature T_∞ and concentration C'_∞. At time $t' > 0$, the plate is started with a velocity $u = u_0.t'^2$ in its own plane against gravitational field. The plate temperature is raised uniformly and the mass is diffused from the plate to the fluid is made to raise linearly with time t. Since the plate is infinite in length, all the terms in the governing equations will be independent of x' and there is no flow along y-direction. Then under usual Boussinesq's approximation for unsteady parabolic starting motion is governed by the following equations:

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu \frac{\partial^2 u}{\partial y^2} \tag{1}$$

$$\rho C_P \frac{\partial T}{\partial t'} = k\, \frac{\partial^2 T}{\partial y^2} \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y^2} \tag{3}$$

With the following initial and boundary conditions:

$$\begin{aligned}
&u = 0, \quad T = T_\infty, \quad C' = C'_\infty \quad \text{for all } y, t' \le 0\\
&t' > 0: \quad u = u_0.t'^2, \quad T' = T'_\infty + (T'_w - T'_\infty)At', \quad C' = C'_\infty + (C'_w - C'_\infty)At' \quad \text{at} \quad y = 0\\
&u \to 0 \quad T \to T_\infty, \quad C' \to C'_\infty \quad \text{as} \quad y \to \infty
\end{aligned} \tag{4}$$

where $A = \frac{u_0^{\,2}}{\nu}$.

On introducing the following non-dimensional quantities:

$$U = u\left(\frac{u_0}{\nu^2}\right)^{1/3}, \quad t = \left(\frac{u_0^{\,2}}{\nu}\right)^{1/3} t', \quad Y = y\left(\frac{u_0}{\nu^2}\right)^{1/3}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \; Gr = \frac{g\beta(T - T_\infty)}{(\nu.u_0)^{1/3}}$$

$$C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \; Gc = \frac{g\beta(C' - C'_\infty)}{(\nu.u_0)^{1/3}}, \quad \Pr = \frac{\mu C_p}{k}, \quad Sc = \frac{\nu}{D} \tag{5}$$

Substitute in equations (1) to (4), leads to

$$\frac{\partial U}{\partial t} = Gr\theta + GcC + \frac{\partial^2 U}{\partial Y^2} \tag{6}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial Y^2} \tag{7}$$

$$\frac{\partial C}{\partial t} = \frac{1}{\mathrm{Sc}}\frac{\partial^2 C}{\partial Y^2} \tag{8}$$

The initial and boundary conditions in non-dimensional quantities are

$$\begin{aligned}
&U = 0, \quad \theta = 0, \quad C = 0 \quad \text{for all } Y, t \le 0\\
&\mathrm{t} > 0: \quad U = \mathrm{t}^2, \quad \theta = \mathrm{t}, \quad \mathrm{C} = \mathrm{t} \quad \text{at } Y = 0\\
&U \to 0, \quad \theta \to 0, \quad C \to 0 \quad \text{as} \quad Y \to \infty
\end{aligned} \tag{9}$$

3. METHOD OF SOLUTION

The dimensionless governing equations (6) to (8), subject to the corresponding initial and boundary conditions (9) are tackled by using Laplace transform technique and the solutions are derived as follows:

$$\theta = t\left(\left(1+2\eta^2 \Pr\right)\operatorname{erfc}(\eta\sqrt{\Pr}) - 2c\exp\left(-\eta^2 \Pr\right)\right) \tag{10}$$

$$C = t\left(\left(1+2\eta^2 Sc\right)\operatorname{erfc}(\eta\sqrt{Sc}) - 2d\exp\left(-\eta^2 Sc\right)\right) \tag{11}$$

$$\begin{aligned} u = \frac{t^2}{3} & \left((3+12\eta^2+4\eta^4)\operatorname{erfc}(\eta) - e\left(10+4\eta^2\right)\exp\left(-\eta^2\right)\right) \\ & - a\left((3+12\eta^2+4\eta^4)\operatorname{erfc}(\eta) - e\left(10+4\eta^2\right)\exp\left(-\eta^2\right)\right) \\ & - b\left((3+12\eta^2+4\eta^4)\operatorname{erfc}(\eta) - e\left(10+4\eta^2\right)\exp\left(-\eta^2\right)\right) \\ & + a\left((3+12\eta^2\Pr+4\eta^4(\Pr)^2)\left(\operatorname{erfc}(\eta\sqrt{\Pr})\right) - c(10+4\eta^2\Pr)\exp\left(-\eta^2\Pr\right)\right) \\ & + b\left((3+12\eta^2 Sc+4\eta^4(Sc)^2)\left(\operatorname{erfc}(\eta\sqrt{Sc})\right) - d(10+4\eta^2 Sc)\exp\left(-\eta^2 Sc\right)\right) \end{aligned} \tag{12}$$

where $a = \dfrac{Gr\,t^2}{6(1-\Pr)}$; $b = \dfrac{Gc\,t^2}{6(1-Sc)}$; $c = \dfrac{\eta\sqrt{\Pr}}{\sqrt{\pi}}$; $d = \dfrac{\eta\sqrt{Sc}}{\sqrt{\pi}}$; $e = \dfrac{\eta}{\sqrt{\pi}}$; $\eta = \dfrac{Y}{2\sqrt{t}}$

RESULTS AND DISCUSSION

For physical understanding of the problem, numerical computations are carried out for different physical parameters depending upon the nature of the flow and transport. The value of the Schmidt number Sc is taken to be 0.6 which corresponds to water-vapor. Also, the value of Prandtl number (Pr = 0.71) is chosen such that they represent air. The numerical values of the velocity, temperature and concentration are computed for the different physical parameters like Prandtl number, thermal Grashof number, mass Grashof number, Schmidt number and time.

Figure 1 illustrates the velocity profile for different values of Schmidt number (Sc=0.16, 0.3, 0.6, 2.01), Gr = Gc = 5 and t = 0.4. As the graph represents the velocity increases as the Schmidth number decreases. The relative variation with regard to the velocity can be noticed.

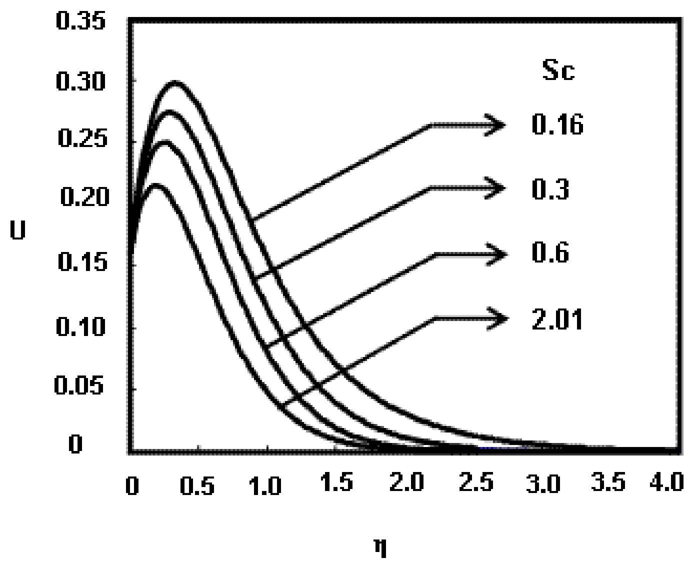

Figure 1: Velocity profile for different values of Sc

In this graph the velocity profile for different value of thermal Grashof number (Gr=2.5) and mass Grashof Number (Gc=5,10) are observed. It is clear that as the velocity increases with the thermal Grashof number or mass Grashof number. The effect of thermal Grashof number is very dominant, since the temperature of the plate is assumed to be uniform.

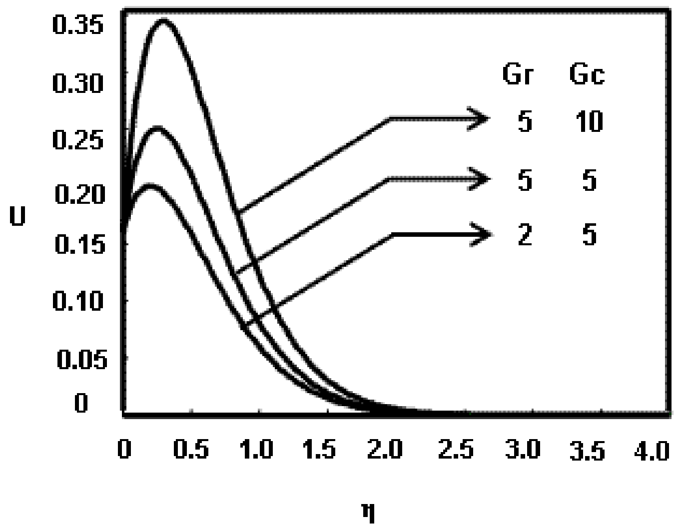

Figure 2: Velocity Profile for different values of Gr,Gc

Figure 3 shows that the velocity profile for different value of time (t = 0.2.0.4,0.6,0.8) is observed. Here the velocity increases slowly as the time increases and Gr = Gc = 5.

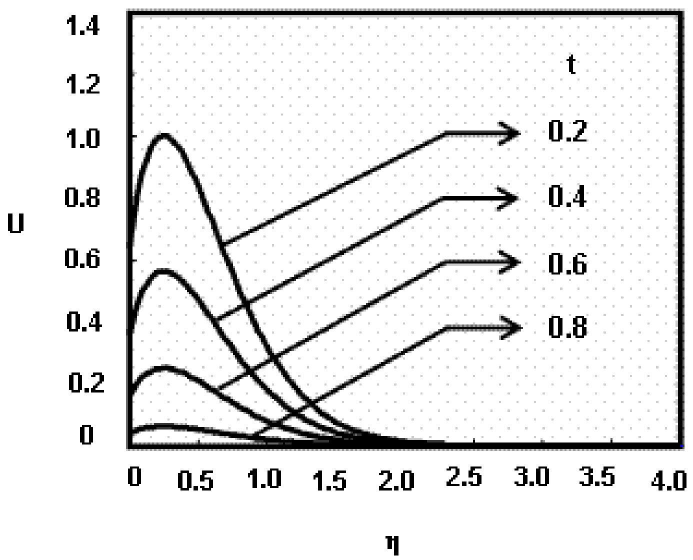

Figure 3: Velocity Profile for different values of t

Figure 4 represents the value of the temperatre gradually decreases time (t=0.2, 0.4, 0.6, 0.8) and Sc=0.6. The trend of the temperature reaches the parameter η after a certain period of time.

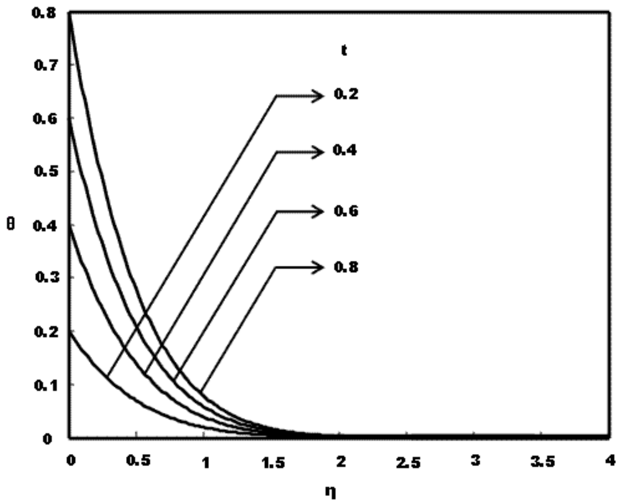

Figure 4: Temperature Profile for different t

Figure 5 illustrate the concentration profile for different Schmidth number can be illustrated. The effect of concentration here decreases in a monotone style from their respective values to a zero value. The concentration increases with decreasing value of Schmidth number (Sc = 0.6, 0.78, 1, 2.01)

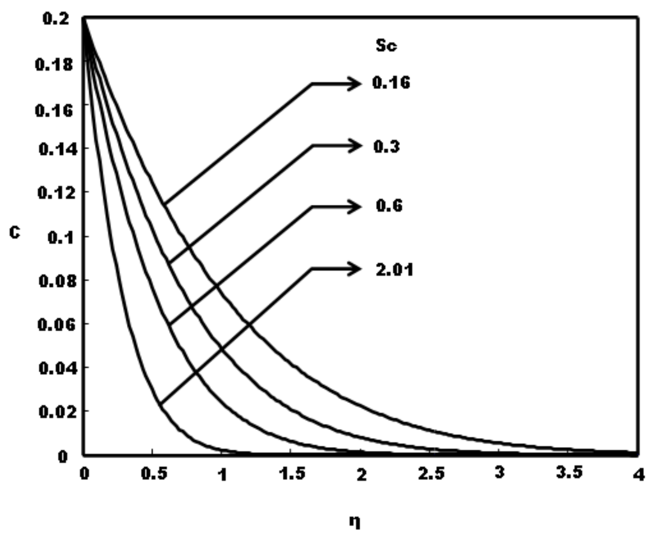

Figure 5: Concentration Profile for different Sc

4. CONCLUSION

The hypothetical solution of flow past a parabolic starting motion of the infinite vertical plate in the presence of variable mass diffusion has been studied. The dimensionless governing equations are solved by the usual Laplace transform technique. The effect of different physical parameters like thermal Grashof number, mass Grashof number, Schmidt number and Time are studied graphically. The conclusions of the study are as follows:

(i) The velocity increases with increasing thermal Grashof number or mass Grashof number.
(ii) The velocity increases with increasing values of the time t, but the trend is just reversed with respect to the Schmidt number.

NOMENCLATURE

A Constants
C' species concentration in the fluid $kg\, m^{-3}$
C dimensionless concentration
C_p specific heat at constant pressure $J.kg^{-1}.k$
D mass diffusion coefficient $m^2.s^{-1}$
Gc mass Grashof number
Gr thermal Grashof number
g acceleration due to gravity $m.s^{-2}$
k thermal conductivity $W.m^{-1}.K^{-1}$
Pr Prandtl number
Sc Schmidt number
T temperature of the fluid near the plate K
t' time s
u velocity of the fluid in the x'-direction $m.s^{-1}$
u_0 velocity of the plate $m.s^{-1}$
u dimensionless velocity
y coordinate axis normal to the plate m
Y dimensionless coordinate axis normal to the plate

GREEK SYMBOLS

β volumetric coefficient of thermal expansion K^{-1}

β^{*} volumetric coefficient of expansion with concentration K^{-1}

μ coefficient of viscosity $Ra.s$

ν kinematic viscosity $m^2.s^{-1}$

ρ density of the fluid $kg.m^{-3}$

τ dimensionless skin-friction $kg.m^{-1}.s^2$

θ dimensionless temperature

η similarity parameter

$erfc$ complementary error function

SUBSCRIPTS

w conditions at the wall

∞ free stream conditions

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