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A Note On RH - Open Sets In GTS With Hereditary Classes

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ABSTRACT

In this paper, we investigate the properties of RH-open sets, and characterize, $\mu-$codense hereditary classes by these sets.

Keywords: *H-open sets, RH-open sets, δ_H- open set, δ_H-open interior, $\mu-$codense hereditary classes.*

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1. INTRODUCTION

In 2007, Császár [3] defined a nonempty class of subsets of a nonempty set, called hereditary class and studied modification of generalized topology via hereditary classes. Also it is studied in [11]. The aim of the paper is to extend the study of the properties of the generalized topologies via hereditary classes. A subfamily μ of $\wp(X)$ is called a generalized topology (GT) [2] if $\emptyset \in \mu$ and μ is closed under arbitrary union. The pair (X, μ) is called a generalized topological space (GTS). Members of μ are called μ - open sets and its comple- ment is called a μ - closed set. The largest μ - open set contained in a subset A of X is denoted by $i_\mu(A)$ [1] and is called the μ - interior of A. The smallest μ - closed set containing A is called the μ - closure of A and is denoted by $c_\mu(A)$ [1]. A generalized topology μ is said to be a quasi-topology if μ is closed under finite intersection. Let X be a nonempty set. A hereditary class H of X is a nonempty collection of subset of X such that $A \subset B$, $B \in H$ implies $A \in H$ [3]. A hereditary class of X is an ideal [8] if $A \cup B \in H$ whenever $A \in H$ and $B \in H$. An ideal I in a topological space (X, τ) is said to be codense if $\tau \cap I = \{\emptyset\}$. With respect to the generalized topology μ of all μ - open sets and a hereditary class H, for each subset A of X, a subset $A^*(H)$ or simply A^* of X is defined by $A^* = \{x \in X | M \cap A \notin H$ for every $M \in \mu$ such that $x \in M\}$[3]. H is said to be $\mu-$ codense if $\mu \cap H = \{\emptyset\}$ [3]. If $c^*(A) = A \cup A^*$ for every subset A of X, with respect to μ and a hereditary class H of subsets of X, then $\mu^* = \{A \subset X \mid c^*(X - A) = X - A\}$ is a generalized topology[3], and $i_{\mu^*}(A)$ will denote the interior of A in (X, μ^*). The aim of this paper is to study the properties of RH$-$ open sets [7] and characterize $\mu-$ codense hereditary class in terms of these sets.

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2. PRELIMINARIES

By a space, we always mean a GTS (X,μ) with a hereditary class H with no separation properties assumed. A subset A of a GTS (X,μ) with a hereditary class H is called regularopen if $A= i_\mu(c_\mu(A))$. The family of all regularopen sets is denoted by $RO(X)$. $RO(X)$ is a base for a quasi topological space (X,μ) with a hereditary class H, denoted by μ_S, coarser than μ. The GTS (X,μ) with a hereditary class H is said to be semiregular if $\mu = \mu_S$. A subset A of a GTS (X,μ) with a hereditary class H is said to be H− open [2] if $A \subset i_\mu(A^*)$. The family of all H− open sets is denoted by $HO(X,\mu)$ or simply $HO(X)$. $N(\mu)$ or simply N is the hereditary class of all nowhere dense subsets in (X,μ). The following lemma will be useful in sequel.

Lemma: 2.1 [6, Lemma 1.2] If (X,μ) is a GTS with a hereditary class H. If

$A \subset A^*$, then $A^* = c_\mu(A^*) = c(A) = c_{\mu^*}(A)$

3. RH−OPEN SETS

A subset A of a GTS (X,μ) with a hereditary class H is said to be an RH− open set[7] if $i_\mu(c^*(A)) = A$. We will denote the family of all RH− open sets is denoted by $RHO(X)$. In [7], it was established that $RHO(X)$. is a base for a quasi topological space, denoted by μ_H and $\mu_S \subset \mu_H \subset \mu$. [7, Theorem 2.1]. Elements of μ_H are called δ_H -open sets and complement of δ_H -open sets are called δ_H -closed sets. The interior of a subset A in (X,μ_H) with a hereditary class H will be denoted by δ_H int(A). Clearly, if the GTS (X,μ) with a hereditary class H is semiregular, $\mu_S = \mu_H = \mu$. The following Example 3.1 shows the existence of hereditary class on X such that μ_S, μ_H and μ are all distinct.

Example: 3.1 Let $X=\{a,b,c\}$, $\mu=\{\emptyset,\{a\},\{b\},\{a,b\}\}$ and $H=\{\emptyset,\{b\}\}$. $RO(X) = \{\emptyset,\{a,b\}\}$ and $RHO(X) = \{\emptyset,\{a\},\{a,b\}\}$. Therefore $\mu_S=\{\emptyset,\{a,b\}\}$ and $\mu_H=\{\emptyset,\{a\},\{a,b\}\}$. Therefore μ_S, μ_H and μ are all distinct. Moreover, $\{a\} \in RHO(X)$ but $\{a\} \notin RO(X)$

Every regular open set is an RH− open set. For, if A is a regular open set, then $A = i_\mu c_{\mu^*}(A)$.

Now $i_\mu c_{\mu^*}(A) \subset i_\mu c_{\mu^*}(A) = A$, but $i_\mu(A) \subset A \subset i_\mu c_{\mu^*}(A)$.

Therefore, $i_\mu c_{\mu^*}(A) = A$. Hence A is a RH−open set. The following Theorem 3.2 discuss the relation between these two kinds of sets.

Theorem: 3.2 Let (X,μ) be a quasi topology with a hereditary class H and A be a subset of X.

(a) If $A \in RHO(X)$ and $A \subset A^*$, then $A \in RO(X)$.

(b) If H is μ− codense, then $RHO(X)=RO(X)$ and so $\mu_S = \mu_H$.

Proof:

(a) $A \in RHO(X)$ implies that $A= i_\mu c_{\mu^*}(A) = i_\mu(A \cup A^*) = i_\mu(A^*)$, since $A \subset A^*$. By Lemma 2.1, it follows that $A = i_\mu(c_\mu(A))$ and so $A \in RO(X)$.

(b) Since every regular open set is an RH− open set, $RO(X) \subset RHO(X)$. Conversely, suppose $A \in RHO(X)$. Since H is μ− codense and A is μ− open, by [6, Lemma 1.2], $A \subset A^*$ and so by (a), $A \in RO(X)$. Therefore, $RHO(X) \subset RO(X)$. This completes the proof.

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Corollary: 3.3 Let (X,μ) be a GTS with a hereditary class H. If $H = \{\emptyset\}$ or N, then $\mu_S = \mu_H$.

Proof: Since $\{\emptyset\}$ and N are $\mu-$codense, the proof follows from Theorem 3.2(b).

Example: 3.4 Let $X = \{a,b,c\}, \mu = \{\emptyset,\{a\},\{a,b\}\}$ and $H = \{\emptyset,\{a\}\}$. Then $\mu_S = \mu_H$ but H is not $\mu-$ condense and $\mu_H = \mu$ but $H \neq \mathcal{P}(X)$.

Theorem: 3.5 Let (X,μ) be a GTS with a hereditary class H and A be a subset of X. If A is $\mu-$open and $A \subset A^*$, then $A \in RHO(X)$.

Proof: $A \subset A^*$ implies that $c_\mu{}^*(A) = A$ [6, Lemma 1.2] so $A = i_\mu(A) = i_\mu(c_\mu{}^*(A))$ which implies that $A \in RHO(X)$.

Corollary: 3.6 Let (X,μ) be a GTS with a hereditary class H and A be a subset of X. If A is $\mu-$ open and $A = A^*$, then $A \in RO(X)$.

Proof: Follows from Theorem 3.5 and Theorem 3.2(a).

The following Theorem 3.7 characterizes $\mu-$ codense hereditary class in terms of RH$-$ open sets and δ_H- open sets.

Theorem: 3.7 Let (X,μ) be a quasi topological space with a hereditary class H. Then the following are equivalent.

(a) H is $\mu-$ codense.

(b) $H \cap \mu_H = \{\emptyset\}$.

(c) $H \cap RHO(X) = \{\emptyset\}$.

(d) $A \subset A^*$ for every $A \in \mu_H$

(e) $A \subset A^*$ for every $A \in RHO(X)$.

(f) $[A]_{\delta-H} = A^*$ for every $A \in \mu_H$, where $[A]_{\delta-H}$ is the closure of A in (X,μ_H).

(g) $[A]_{\delta-H} = A^*$ for every $A \in RHO(X)$.

Proof:

(a) $\Rightarrow$ (b): follows from the fact that $\mu_H \subset \mu$.

(b) $\Rightarrow$ (c): follows from the fact that $RHO(X) \subset \mu_H$.

(c) $\Rightarrow$ (a): Suppose $A \in H \cap \mu$. Since $A \in H, A^* = \emptyset$ and $c_\mu{}^*(A) = A$. Since A is $\mu-$ open, $i_\mu(c_\mu(A)) = i_\mu(A) = A$ and so $A \in RHO(X)$. By (c), $A = \emptyset$ and the proof follows.

(b) $\Rightarrow$ (d): If $A \in \mu_H$, then $A \in \mu$ and so by [6, Lemma 1.5], $A \subset A^*$.

(d) $\Rightarrow$ (e): follows from the fact that $RHO(X) \subset \mu_H$.

(e) $\Rightarrow$ (f): If $A \in \mu_H$, then $A = \cup\{A_\alpha / A_\alpha \in RHO(X)$ for every $\alpha\}$. By (e) $A_\alpha \subset A_\alpha{}^*$ for every α and $A = \cup A_\alpha \subset \cup A^* \subset (\cup A_\alpha)^* = A^*$. By Lemma 2.1, $A^* = c_\mu(A)$ and so A^* is regularclosed and hence is δ_H-closed. Therefore, $[A]_{\delta-H} \subset [A^*]_{\delta-H} = A^*$. Clearly

$$A^* = c_\mu(A) \subset [A]_{\delta-H}. \text{ Hence } [A]_{\delta-H} = A^*$$

which proves (f).

(f) $\Rightarrow$ (g): follows from the fact that $RHO(X) \subset \mu_H$.

(g) ⇒ (c): Suppose $A \in H \cap RHO(X)$. $A \in H$ implies that $A^* = \emptyset$ and by (g), $A \in RHO(X)$ implies that $[A]_{\delta-H} \subset A^* = \emptyset$. Therefore $[A]_{\delta-H} = \emptyset$ which implies that $A = \emptyset$. This completes the proof.

The following Theorem 3.8 gives characterizations of $\mu-$ codense hereditary classes.

Theorem: 3.8 Let (X, μ) be a quasi topological space with a hereditary class H. Then the following are equivalent.

(a) H is $\mu-$ codense.

(b) $\delta_H i_\mu(A) \subset \delta_H i_\mu(A^*)$ for every subset A of X.

(c) $\delta_H i_\mu(I) = \emptyset$ for every element I of H.

Proof: (a) ⇒ (b): Since $\delta_H i_\mu(A) \subset A$ for every subset A of X, $(\delta\ i\ (A))^* \subset A^*$ By Theorem 3.7(d), $\delta_H i_\mu(A) \subset (\delta_H i_\mu (A))^*$ and so $\delta\ i\ (A) \subset \delta\ i\ (A^*)$ which completes the proof.

(b) ⇒ (c): If $I \in H$, then $A^* = \emptyset$ and so the proof follows from (b).

(c) ⇒ (a): Suppose $A \in H \cap RHO(X)$. Since $A \in H$, by (c) $\delta_H\ i_\mu\ (A) = \emptyset$. Since $A \in RHO(X)$, $A = \emptyset$. By Theorem3.7(c), H is $\mu-$ codense.

The following Theorem 3.9 discuss the relation between H−open sets and RH− open sets.

Theorem: 3.9 Let (X, μ) be a quasi topological space with a hereditary class H. If A is an $H-$open subset of X, then there is an $RH-$open set G such that $A \subset G$ and $A^* = G^*$. The converse holds, if H is $\mu-$codense.

Proof: Suppose A is an $H-$ open subset of X. Then $A \subset i_\mu(A^*)$. If $G = i_\mu(A^*)$, then G is an $\mu-$ open set such that $A \subset G$. Now $A \subset i_\mu(A^*)$ implies that $A \subset i_\mu(A^*) \subset A^*$ and so $A \subset (i_\mu(A^*))^* \subset (A^*)^*$, by Corollary 2.6 of [3]. Since $(A^*)^* \subset A$ by [3], we have $A^* = (i_\mu(A^*))^*$ and so $A^* = G^*$. Next, we prove that G is $RH-$ open.

Now, $i_\mu(c_\mu^*(G)) = i_\mu(c_\mu^*(i_\mu(A^*))) \subset i_\mu(c_\mu^* A^*)) = i_\mu(c_\mu^*(G))) = G$ and so $i_\mu(c_\mu^*(G))) \subset G$, by Lemma 2.1. On the other hand, $G = i_\mu(A^*)$ implies that $G = i_\mu(c_\mu^*(G))$, since $A \subset A^*$. Since A is $H-$ open, it follows that $G \subset i_\mu(c_\mu^*(G))$ which implies that G is $RH-$open.

Conversely, suppose A is a subset of X, G is an RH- open set such that $A \subset G$ and $A^* = G^*$ and H is μ-codense. By Theorem 3.6(c), $G \subset G^*$ and so by Lemma 2.1, $G^* = (c_\mu^*(G))$.

Now $i_\mu(A^*) = i_\mu(G^*) = i_\mu(c_\mu^*(G)) = G \supset A$ and so A is H-open.

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