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COMMON FIXED POINT THEOREM FOR INTEGRAL TYPE MAPPING IN FUZZY METRIC SPACE

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ABSTRACT

In this paper we prove a common fixed point theorem in fuzzy metric space using integral type mapping by relaxing compatibility to compatible maps of type (α) or (β).

Key words: *Common fixed point, fuzzy metric space, −norm, compatible maps of type (α) or (β).*

1.1 INTRODUCTION

In real world, the complexity generally arises from uncertainly in the form of ambiguity. The probability theory has been age old and effective tool to handle uncertainly, but it can be applied only to the situations whose characteristics are based on random processes, i.e., process in which the occurrence of events is strictly determined by chance. Uncertainly may arise due to partial information about the problem, or due to information which is not fully reliable, or due to inherent imprecision in the language with which the problem is defined or due to receipt of information from more than one source. Fuzzy set theory is an excellent mathematical tool to handle the uncertainly arising due to vagueness. In 1965, Lotfi A- Zadeh [15] propounded the fuzzy set theory in his paper.

The concept of fuzzy set theory is the new way to represent vagueness in everyday life. Subsequently, it was developed by many authors and used in various field. To used the concept of topology and analysis, several researchers has been defined fuzzy metric spaces defined by Kramosil and Michalek [12] and modified by George and Veeramani [6]. Recently, Grebiec [7] has proved fixed point results for fuzzy metric space. In the sequel, Singh and Chauhan [14] introduced the concept of compatible mappings of fuzzy metric space and proved the common fixed point theorem. Jungck *et.al.* [10] introduced the concept of compatible map of type (A) in metric space and proved fixed point theorems. Cho [4, 5] introduced the concept of compatible map of type (α) and compatible map of type β in fuzzy metric space. Using the concept of compatible maps of type (A). Jain *et.al.* [9] proved a fixed point theorem for six self maps in fuzzy metric space, using the concept of compatible map of type (β). Our aim of this paper is to find some more results for compatible map of type (β) in fuzzy metric space.

For the sake of completeness, we recall some definition and known results in Fuzzy metric space, which are used in this chapter.

Definition: 1.1.1 Let X be any set. A fuzzy set in X is a function with domain X and values in [0, 1].

Definition: 1.1.2 A binary operation $\star : [0, 1] \times [0, 1] \rightarrow [0, 1]$ is continuous $t -$ norm if $\star$ is satisfying the following conditions:
1.1.2. (a)$\star$ is commutative and associativ
1.1.2 (b)$\star$ is continuous,
1.1.2 (c)$\star 1 = a$ $for\ all$ $a \in [0,1]$
1.1.2 (d) $a \star b \leq c \star d$ whenever $a \leq c$ and $b \leq d$, for all $a,b,c,d \in [0,1]$ Examples of $t{-}norm$ are $a \star b = min\{a, b\}$ and $a \star b = ab$.

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Definition: 1.1.3 A triplet $(X,M,\star)$ is a fuzzy metric space whenever X is an arbitrary set, $\star$ is continuous $t-$norm and M is fuzzy set on $X \times X \times (0,\infty_+)$ satisfying, for every x, y, $z \in X$ and s, $t > 0$, the following condition:
1.1.3 (a) $M(x,y,t) > 0$
1.1.3 (b) $M(x,y,0) = 0$
1.1.3 (c) $(x,y,t) = 1$ iff $x=y$
1.1.3 (d) $M(x,y,t) = M(y,x,t)$
1.1.3 (e) $M(x,y,t) \star M(y,z,s) \leq M(x,z,t+s)$
1.1.3 (f) $M(x,y,\cdot) : (0,\infty+) \to [0,1]$ $is\ continuous.$

Example: 1.1.4 Let (X, d) be a metric space. Define $a\star b = min\ \{a, b\}$and $M\ (x, y, t) = \frac{t}{t+d(x,y)}$ for all x, $y \in X$ and all $t > 0$. Then $(X, M, \star)$ is a fuzzy metric space. It is called the fuzzy metric space induced by d.

Definition: 1.1.5 A sequence $\{x_n\}$ in a fuzzy metric space $(X, M, \star)$ is said to be a converges to x iff for each $\varepsilon > 0$ and each $t > 0$, $n_0 \in N$ such that $M\ (x_n, x, t) > 1-\varepsilon\ for\ all\ n \geq n_0$.

Definition: 1.1.6 A sequence$\{x_n\}$in a fuzzy metric space $(X,M,\star)$ is said to be a Cauchy sequence converges to x iff for each $\varepsilon > 0$ and each $t>0$, $n_0 \in N$ such that $M\ (x_m,x_n,t) > 1-\varepsilon\ for\ all\ m,\ n \geq n_0$.

A fuzzy metric space $(X, M, \star)$ is said to be complete if every Cauchy sequence in it converges to a point in it.

Definition: 1.1.7 Self mapping A and S of a fuzzy metric space $(X, M, \star)$ are said to be compatible if and only if $M\ (ASx_{n,n},t) \to 1$ for all $t > 0$, where $\{x_n\}$ is a sequence in X such that Sx_n ,$Ax_n \to p$ for some $p \in X\ as\ n \to \infty$.

Definition: 1.1.8 Self map A and S of a fuzzy metric space $(X, M, \star)$ are said to be compatible of type (β) if and only if $M\ (AAx_{n,n},t) \to 1$ for all $t > 0$, where $\{x_n\}$ is a sequence in X such that Sx_n, $Ax_n \to p$ for some $p \in X\ as\ n \to \infty$.

Definition: 1.1.9 Two maps A and B from a fuzzy metric space $(X, M, \star)$ into itself are said to be weakly compatible if they commute at their coincidence points i.e., $Ax = Bx$ implies $ABx = BAx$ for some $x \in X$.

Remark: 1.1.10 The concept of compatible map of type (β) is more general then the concept of compatible map in fuzzy metric space.

Definition: 1.1.11 Let A and S be two self maps of a fuzzy metric space $(X,M,\star)$ then A and S is said to be a weakly commuting if$M(ASx_n,SAx_n,t) \leq M(Sx_n,Ax_n,t)$ for all x in X.

It can be seen that commuting maps $ASx=SAx\ \forall\ x \in X$ are weakly compatible but converse is not true.

Lemma: 1.1.12 In a fuzzy metric space $(X, M, \star)$ limit of a sequence is unique.

Lemma: 1.1.13 Let $(X, M, \star)$ be a fuzzy metric space. Then for all x, y$\in X$, $(x, y,\ .)$ is a non decreasing function.

Lemma: 1.1.14 Let $(X, M, \star)$ be a fuzzy metric space. If there exists $k \in (0,1)$ such that for all $x,y \in X, M(\ x,y,kt) \geq M\ (x,y,t\ ) \forall t > 0, then\ x = y.$

Lemma: 1.1.15 Let $\{x_n\}$ be a sequence in a fuzzy metric space $(X, M, \star)$. If there exists a number $k \in (0, 1)$ such that $M\ (x_{n+2}, X_{n+1}, kt) \geq M\ (x_{n+1}, x_n\ , t)\ \forall\ t > 0\ and\ n \in N.$

Then$\{x_n\}$is a Cauchy sequence in X.

Lemma: 1.1.16 The only $t-norm$ $\star$ satisfying $r\star r=r$ for all $r \in [0, 1]$ is the minimum $t-norm$ that is $a\star b = min\ \{a, b\}$ for all a, b$\in [0, 1]$.

On the way of generalization of Banach contraction principle [1] one of the most famous generalization was introduced by Branciari [2] in general setting of lebgesgueintegrable function and proved following fixed point theorem in metric spaces.

2.1 Theorem: Let $(X$, d) be a complete metric space, $\alpha \in (0,1)$ and let T: $X \to X$, be a mapping such that for each x, $y \in X$

$$\int_0^{d(Tx,Ty)} \xi(\upsilon) d\upsilon \leq \alpha \int_0^{d(x,y)} \xi(\upsilon)\ d\upsilon$$

where ξ: [0, +∞] → [0, +∞] is a lebgesgueintegrable mapping which is summable on each compact subset of [0, +∞], non negative, and such that, $\forall\varepsilon > 0$, $\int_0^\varepsilon \xi(\upsilon)dv>0$ Then, T has unique fixed point $z\in X$ such that for each $x\in X$, $T^n x\to z$ as $n\to\infty$.

It should be noted that if ξ(ν) = 1 then Banach contraction principle is obtained.

Inspired from the result of Branciari [2] we prove following common fixed point theorems in fuzzy metric spaces.

2.2 Theorem: Let (X, M, ⋆) be a complete fuzzy metric space and let A, B, S, T, P and$Q$be mappings from X into itself such that the following conditions are satisfied:
2.2 (a) P(X)⊂ST(X) and $Q(X)\subset AB(X)$,
2.2 (b) AB=BA, = TS, PB=BP, QT=TQ,
2.2 (c) either P or AB is continuous,
2.2 (d) (P,) is compatible of type (β) and (Q, ST) is weak compatible,
2.2 (e) there exists $k\in$ (0, 1) such that for every $x, y\in X$ and $t > 0$

$$\int_0^{M^2(Px,Qy,kt)} \xi(\upsilon)d\upsilon \geq \int_0^{W(Px,Qy,t)} \xi(\upsilon)d\upsilon$$

W(Px,Qy,t) =$M^2(ABx,STy,t) \star M^2(Px,ABx,t)\star M^2(Qy,STy,t) \star M^2(Px,STy,t)$

where ξ: [0, +∞] → [0, +∞] is a lebgesgueintegrable mapping which is summable on each compact subset of [0, +∞], non negative, and such that, $\forall\varepsilon> 0$, $\int_0^\varepsilon \xi(\upsilon)dv>0$. Then A, B, S, T, P and Q have a unique common fixed point in X.

Proof: Let $x_0\in X$, then from 2.2(a) we have x_1, $x_2\in X$ such that

$Px_0=STx_1$ and $Qx_1=ABx_2$

Inductively, we construct sequences$\{x_n\}$ and $\{y_n\}$ in X such that for $n\in N$,

$Px_{2n-2}=STx_{2n-1}=y_{2n-1}$ nd $Qx_{2n-1}= ABx_{2n}=y_{2n}$

Step - 1: Put $x= x_{2n}andy=x_{2n+1}$ in 2.2 (e) then we have

$\int_0^{M^2(Px_{2n},Qx_{2n+1},kt)} \xi(\upsilon)dv\geq \int_0^{W(Px_{2n},Qx_{2n+1},t)} \xi(\upsilon)dv$

W(Px_{2n}, Qx_{2n+1}, t)= $M^2(ABx_{2n},STx_{2n+1},t) \star M^2(Px_{2n},ABx_{2n},t) \star M^2(Qx_{2n+1}, STx_{2n+1},t) \star M^2(Px_{2n},STx_{2n+1},t)$

$\int_0^{M^2(y_{2n+1},\ y_{2n+2},\ kt)} \xi(\upsilon)dv\geq (v)\int_0^{W(y_{2n+1},y_{2n+2},t)} \xi(\upsilon)dv$ 2.2(i)

$W(y_{2n+1},y_{2n+2},t)=M^2(y_{2n},y_{2n+1},t) \star M^2(y_{2n+1},y_{2n},t)\star M^2(y_{2n+2}, y_{2n+1},t) \star M^2(y_{2n+1},y_{2n+1},t)$

From 1.1.3(c), 1.1.3(b), 1.1.2(c) and lemma 1.1.14 we have

$W(y_{2n+1},y_{2n+2},t)= M^2(y_{2n},y_{2n+1},t)\star M^2(y_{2n+2},y_{2n+1},t)$

Therefore, 2.2. (i) can be written as

$\int_0^{M^2(y_{2n+1},\ y_{2n+2},\ kt)} \xi(v)\ dv \geq \int_0^{M^2(y_{2n},\ y_{2n+1},t)\star M^2(y_{2n+2},\ y_{2n+1},t)} \xi(v)\ dv$

Since $\xi(v)$ is Lesbesgueintegrable function so that

$M^2(y_{2n+1},y_{2n+2},kt) \geq M^2(y_{2n,}y_{2n+1,}t) \star M^2(y_{2n+2},y_{2n+1},t)$

Similarly $M^2(y_{2n+2}, y_{2n+3}, kt) \geq M^2(y_{2n+1}, y_{2n+2}, t) \star M^2(y_{2n+3}, y_{2n+2}, t)$

Therefore, for all n even or odd, we have,

$M^2(y_{n+1}, y_{n+2}, kt) \geq M^2(y_n, y_{n+1}, t) \star M^2(y_{n+2}, y_{n+1}, t)$

Consequently, $M^2(y_{n+1}, y_{n+2}, t) \geq M^2(y_n, y_{n+1}, k^{-1} t) \star M^2(y_{n+2}, y_{n+1}, k^{-1}t)$

By a simple induction, we have

$M^2(y_{n+1}, y_{n+2}, t) \geq M^2(y_n, y_{n+1}, k^{-1} t) \star M^2(y_{n+2}, y_{n+1}, k^{-m} t)$

Since $M^2(y_{n+2}, y_{n+1}, k^{-m}t) \to 1$ as $m \to \infty$,

It follows that

$M^2(y_{n+1}, y_{n+2}, kt) \geq M^2(y_n, y_{n+1}, t)$

$M^2(y_{n+1}, y_{n+2}, t) \geq M^2(y_n, y_{n+1}, k^{-1} t)$

Thus we have $M(y_{n+1}, y_{n+2}, kt) \geq M(y_n, y_{n+1}, t)$

$M(y_{n+1}, y_{n+2}, t) \geq M(y_n, y_{n+1}, \frac{t}{k})$

$M(y_n, y_{n+1}, t) \geq M(y_0, y_1, \frac{t}{k^n}) \to 1$ $as \to \infty$,

and hence $M(y_n, y_{n+1}, t) \to 1$ $as n \to \infty$ $for\ all\ t > 0$.

For each $\epsilon > 0$ and $t > 0$, we can choose $n_0 \in N$ such that

$M(y_n, y_{n+1}, t) > 1-\epsilon$ $for\ all\ n > n_0$.

For any m, n $\in N$ we suppose that $m \geq$ n. Then we have

$M(y_n, y_m, t) \geq M(y_n, y_{n+1}, \frac{t}{m-n}) \star M(y_{n+1}, y_{n+2}, \frac{t}{m-n}) \star \ldots\ldots \star \ldots\ldots \star M(y_{m-1}, y_m, \frac{t}{m-n})$

$M(y_n, y_m, t) \geq (1-\epsilon) \star (1-\epsilon) \star \ldots\ldots \star (1-\epsilon)\ (m-n)\ times$

$M(y_n, y_m, t) \geq (1-\in)$

And hance $\{y_n\}$ is a Cauchy sequence in X.

Since $(X, M, \star)$ is complete, $\{y_n\}$ converges to some point $z \in X$. Also its subsequences converges to the same point $z \in X$. That is

$\{Px_{2n+2}\} \to z$ and $\{STx_{2n+1}\} \to z$ 2.2 (ii)

$\{Qx_{2n+1}\} \to d$ $\{ABx_{2n}\} \to z$ 2.2(iii)

Case 1: Suppose AB is continuous

Since AB is continuous, we have $AB^2x_{2n} \to ABz$ and $ABPx_{2n} \to ABz$

As $(P$, AB) is compatible pair of type, we have

$M(PPx_{2n}, (AB)(AB)x_{2n}, t) = 1$, for $all\ t > 0$

Therefore, $PPx_{2n} \to ABz$ as $n \to \infty$

Step - 2: Put x= Px_{2n} and $y=x_{2n+1}$ in 4.2(e) we have

$\int_0^{M^2(PPx_{2n}, Qx_{2n+1}, kt)} \xi(v)\ dv \geq \int_0^{W(PPx_{2n}, Qx_{2n+1}, t)} \xi(v) dv$

where

$W(PPx_{2n}, Qx_{2n+1}, t) = M^2(ABPx_{2n}, STx_{2n+1}, t) \star M^2(PPx_{2n}, AB\ Px_{2n}, t) \star M^2(Qx_{2n+1}, STx_{2n+1},) \star M^2(PP_{2n}, STx_{2n+1}, t)$

Therefore we have,

$$\int_0^{M^2(PPx_{2n},Qx_{2n+1},kt)} \xi(v)\ dv \geq \int_0^{M^2(ABPx_{2n},STx_{2n+1},t)\star M^2(PPx_{2n},ABPx_{2n},t)\star M^2(Qx_{2n+1},STx_{2n+1},t)\star M^2(PPx_{2n},STx_{2n+1},t)} \xi(v)\, dv$$

$$M^2(PPx_{2n}, Qx_{2n+1},kt) \geq M^2(AB\, Px_{2n},STx_{2n+1},t)\star M^2(PPx_{2n},AB\, Px_{2n}, t)\star M^2(Qx_{2n+1}, STx_{2n+1},t) \star M^2(PP\, x_{2n},STx_{2n+1},t)$$

Taking $n\to\infty$ we get

$$M^2((AB)\, z, z, kt) \geq M^2((AB)\, z,z,t) \star M^2((AB)\, z,(AB)\, z,t)\star M^2(z, z, t)\star M^2((AB)\, z,z,t)$$

$$\geq M^2((AB)z,z,t)$$

Therefore

$$\int_0^{M^2((AB)z,z,kt)} \xi(\upsilon)d\upsilon \geq \int_0^{M^2((AB)z,z,t)} \xi(\upsilon)d\upsilon$$

That is from the property of $\xi(v)$ we have

$M((AB)z,z,kt \geq M((AB)z,z,t)$

Therefore by lemma 1.1.14 we have

$ABz = z$ 2.2(iv)

Step -3: Put $x = z$ and $y = x_{2n+1}$ in 2.2(e) we have

$$\int_0^{M^2(Pz,Qx_{2n+1},kt)} \xi(\upsilon)d\upsilon \geq \int_0^{W(Pz,Qx_{2n+1},t)} \xi(\upsilon)d\upsilon$$

$$W(Pz,Qx_{2n+1},t) = M^2(ABz,STx_{2n+1},t)\star M^2(Pz,ABz,t)\star M^2(Qx_{2n+1},STx_{2n+1},t) \star M^2(Pz,ST\, x_{2n+1},t)$$

Taking $n\to\infty$ and using equation 2.2 (ii) we have

$$\int_0^{M^2(Pz,z,kt)} \xi(\upsilon)d\upsilon \geq \int_0^{W(z,x_{2n+1},t} \xi(\upsilon)\, d\upsilon$$

$$W(Pz, z, t) \geq M^2(ABz,z,t) \star M^2(Pz,ABz,t)\star M^2(z, z, t)\star M^2(Pz,z,t)$$

$$\geq M^2(Pz,z,t)$$

Therefore,

$$\int_0^{M^2(Pz,z,kt)} \xi(\upsilon)d\upsilon \geq \int_0^{M^2(Pz,z,t)} \xi(\upsilon)\, d\upsilon$$

So that $M^2(Pz, z, kt) \geq M^2(Pz,z,t)$. And hence $M(Pz,z, kt) \geq M(Pz,z,t)$

Therefore by using lemma 1.1.14we get $Pz=z$

So we have $ABz=Pz=z$.

Step- 4: Putting $x=Bz$ *nd* $y= x_{2n+1}$ in 2.2(e), we *get*

$$\int_0^{M^2(PBz,Qx_{2n+1},kt)} \xi(\upsilon)d\upsilon \geq \int_0^{W(PBz,Qx_{2n+1},t)} \xi(\upsilon)d\upsilon$$

$W(PBz, Qx_{2n+1}, t) = M^2(ABBz, STx_{2n+1}, t) \star M^2(PBz, ABBz, t) \star M^2(Qx_{2n+1}, STx_{2n+1}, t) \star M^2(PBz, STx_{2n+1}, t)$

As $BP=PB$ and $AB=BA$, so we have

$P\,Bz = B\,Pz = Bz$ and $AB\,Bz = BA\,Bz = B\,ABz = Bz$.

Taking $n\to\infty$ and using 2.2(ii) we get

$$\int_0^{M^2(PBz,Qx_{2n+1},kt)} \xi(\upsilon)d\upsilon \geq \int_0^{W(PBz,Qx_{2n+1},t)} \xi(\upsilon)d\upsilon$$

$$\int_0^{M^2(Bz,z,kt)} \xi(\upsilon)d\upsilon \geq \int_0^{W(Bz,z,t)} \xi(\upsilon)d\upsilon$$

$W(Bz, z, t) = M^2(Bz, z, t) \star M^2(Bz, Bz, t) \star M^2(z, z, t) \star M^2(Bz, z, t)$

$= M^2(Bz, z, t)$

Therefore,

$$\int_0^{M^2(Bz,z,kt)} \xi(\upsilon)d\upsilon \geq \int_0^{M^2(Bz,z,t)} \xi(\upsilon)d\upsilon$$

So we have $M^2(Bz, z, t) \geq M^2(Bz, z, t)$

That is $M(z, z, kt) \geq M(Bz, z, t)$

Therefore by Lemma 1.1.14 we have

$Bz = z$

And also we have $ABz = z$ implies $Az = z$

Therefore $Az = Bz = Pz = z$ 2.2 (v)

Step – 5: As $P(X) \subset ST(X)$ there exists $u \in X$ such that

$z = Pz = STu$

Putting $x = x_{2n}$ and $y = u$ in 2.2(e) we get

$$\int_0^{M^2(Px_{2n},Qu,kt)} \xi(\upsilon)d\upsilon \geq \int_0^{W(Px_{2n},Qu,t)} \xi(\upsilon)d\upsilon$$

$W(Px_{2n}, Qu, t) = M^2(ABx_{2n}, STu, t) \star M^2(Px_{2n}, ABx_{2n}, t) \star M^2(Qu, STu, t) \star M^2(Px_{2n}, STu, t)$

Taking $\to\infty$ and using 2.2(ii) and 2.2(iii) we get

$$\int_0^{M^2(z,Qu,kt)} \xi(\upsilon)d\upsilon \geq \int_0^{W(z,Qu,t)} \xi(\upsilon)d\upsilon$$

$W(z, Qu, t) = M^2(z, STu, t) \star M^2(z, z, t) \star M^2(Qu, STu, t) \star M^2(z, STu, t)$

$= M^2(Qu, z, t)$

So we have $M^2(z, Qu, kt) \geq M^2(Qu, z, t)$. That is $M(z, Qu, kt) \geq M(z, Qu, t)$

Therefore by using Lemma 1.1.14 we have $Qu = z$

Hence $STu = z = Qu$.

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Hence (Q, ST) is weak compatible, therefore, we have

$QSTu$=STQ Thus Qz=STz.

Step – 6: Putting x=x_{2n} and y=z in 2.2(e) we get

$$\int_0^{M^2(Px_{2n},Qz,kt)} \xi(\upsilon)d\upsilon \geq \int_0^{W(Px_{2n},Qz,t)} \xi(\upsilon)d\upsilon$$

W (Px_{2n},Qz,t) =M^2(ABx_{2n},STz,t) ⋆M^2(Px_{2n},ABx_{2n},t)⋆M^2(Qz, STz, t)⋆M^2(Px_{2n},STz,t)

Taking n→∞ and using 2.2(iii) and step 5 we get

$$\int_0^{M^2(z,Qz,kt)} \xi(\upsilon)d\upsilon \geq \int_0^{W(z,Qz,t)} \xi(\upsilon)d\upsilon$$

W (z, Qz, t) =M^2(z, STz,t) ⋆M^2(z, z, t)⋆M^2(Qz, STz, t) ⋆M^2(z, STz, t)

=M^2(z, Qz, t)

Therefore,

$$\int_0^{M^2(z,Qz,kt)} \xi(\upsilon)d\upsilon \geq \int_0^{W(z,Qz,t)} \xi(\upsilon)d\upsilon$$

$$=\int_0^{M^2(z,Qz,t)} \xi(\upsilon)d\upsilon$$

That is M^2(z,Qz, kt) ≥ M^2(z,Qz,t). And hence M(z, Qz, kt) ≥ M(z,Qz,t)

Therefore by using Lemma 1.1.14 we get Qz=z.

Step – 7: Putting x=x_{2n} and y =Tz in 2.2(e) we get

$$\int_0^{M^2(Px_{2n},QTz,kt)} \xi(\upsilon)d\upsilon \geq \int_0^{W(Px_{2n},QTz,t)} \xi(\upsilon)d\upsilon$$

W (Px_{2n},QTz,t) =M^2(ABx_{2n}, STTz, t)⋆M^2(Px_{2n},ABx_{2n},t)⋆M^2(QTz, STTz, t)⋆M^2(Px_{2n}, STTz, t)

As QT =$TQandST$ =TS we have QTz =TQz =Tz

And (Tz) = () =TQz =Tz.

Taking n→∞ we get

$$\int_0^{M^2(z,Tz,kt)} \xi(\upsilon)d\upsilon \geq \int_0^{W(z,Tz,t)} \xi(\upsilon)d\upsilon$$

W (z, Tz, t) =M^2(z,Tz,t) ⋆M^2(z, z, t)⋆M^2(Tz, Tz, t)⋆M^2(z, Tz, t)

=M^2(z,Tz,t)

Therefore

$$\int_0^{M^2(z,Tz,kt)} \xi(\upsilon)d\upsilon \geq \int_0^{W(z,Tz,t)} \xi(\upsilon)d\upsilon$$

$$=\int_0^{M^2(z,Tz,t)} \xi(\upsilon)d\upsilon$$

And hence M^2(z, Tz, kt) ≥ M^2(z,Tz, t).Therefore M(z, Tz, kt) ≥ M(z,Tz,t)

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Therefore by Lemma 1.1.14 we have $Tz=z$

Now $STz=Tz=z$, im$plies\ Sz=z$.

Hence $=Tz=Qz=z$ 2.2(vi)

Combining 2.2(v) and 2.2(vi) we have

$Az=Bz=Pz=Sz=Tz=Qz=z$

Hence z is the common fixed point of A, B, S, T, P and Q.

Case – II: suppose P is continuous

As P is continuous

$P^2x_{2n}\to Pza(AB)x_{2n}\to Pz$

As (P,AB) is compatible pair of type (β),

$M(PPx_{2n},(AB)(AB)x_{2n},t) = 1\ forall t>0$

Therefore $PPx_{2n}\to (AB)z$ and $(AB)(AB)x_{2n}\to z$

Step -8: Putting $x = ABx_{2n} and y=x_{2n+1}$ in 2.2(e) then we get

$\int_0^{M^2(P(AB)x_{2n},Qx_{2n+1},kt)} \xi(v)\ dv \geq \int_0^{W(P(AB)x_{2n},Qx_{2n+1},t)} \xi(v) dv$

$W(P(AB)x_{2n}, Qx_{2n+1}, t)=M^2(AB(AB)x_{2n},STx_{2n+1},t) \star M^2(P(AB)x_{2n},AB(AB)x_{2n}, t)$

$\star M^2(Qx_{2n+1}, STx_{2n+1},)\star M^2(P(AB)x_{2n},STx_{2n+1},t)$

Taking $n\to\infty$, get

$M^2(Pz,z,t)=M^2(Pz,z,t)\star M^2(Pz,Pz,t)\star M^2(z,z,t)\star M^2(Pz,z,t)$

$=M^2(Pz, z, t)$

Therefore we have

$M^2(Pz, z, kt) \geq M^2(Pz,z,t)$. Hence $(Pz, z, kt) \geq (Pz,,)$

Therefore by Lemma 1.1.14 we get $Pz=z$

Step- 9: Put $x = Px_{2n}$ and $y=x_{2n+1}$ in 2.2(e) then we get

$\int_0^{M^2(PPx_{2n},Qx_{2n+1},kt)} \xi(v)\ dv \geq \int_0^{W(PPx_{2n},Qx_{2n+1},t)} \xi(v) dv$

$W(PPx_{2n}, Qx_{2n+1},t)=M^2(AB\ Px_{2n},STx_{2n+1},t)\star M^2(PPx_{2n},AB\ Px_{2n}, t)\star(Qx_{2n+1}, STx_{2n+1},t)\star M^2(PP\ x_{2n},STx_{2n+1},t)$

Taking $n\to\infty$ we get

$\int_0^{M^2(ABz,z,kt)} \xi(v)\ dv \geq \int_0^{W(ABz,z,t)} \xi(v) dv$

$W(ABz, z, t)=M^2(ABz,z,t) \star M^2(ABz, ABz, t)\star M^2(z, z,t) \star M^2(ABz, z,t)$

$= M^2(ABz, z, t)$

Therefore $M^2(ABz, z, kt) \geq M^2(ABz, z, t)$. And hence $(ABz, z, kt) \geq (ABz,,)$

we get $ABz = z$

By applying step 4, 5, 6, 7, 8 we get

$Az = Bz = Sz = Tz = Pz = Qz = z$.

That is z is a common fixed point of A, B, S, T, P, Q in X.

Uniqueness: Let u be another common fixed point of A, B, S, T, P and Q. Then $Au=Bu=Su=Tu=Pu=Qu = u$ Putting $x = u$ and $y=z$ in 2.2(e) then we get

$\int_0^{M^2(Pu,Qz,kt)} \xi(v)\, dv \geq \int_0^{W(Pu,Qz,t)} \xi(v) dv$

$W(Pu, Qz, t) = M^2(ABu, STz, t) \star M^2(Pu, ABu, t) \star M^2(Qz, STz, t) \star M^2(Pu, STz, t)$

$\int_0^{M^2(u,z,kt)} \xi(v)\, dv \geq \int_0^{W(u,z,t)} \xi(v) dv$

$W(u, z, t) = M^2(u,z,t) \star M^2(u, u, t) \star M^2(z, z, t) \star M^2(u, z, t)$

$= M^2(u,z,t)$

That is $M^2(u, z, kt) \geq M^2(u, z, t)$

And hence $(u, z, kt) \geq (u, z, t)$ we get $z = u$.

That is z is a unique common fixed point of A, B, S, T, P and Q in X.

Remark: 2.3 If we take $B=T=I$ identity map on X in Theorem 2.2.then condition 2.2(b) is satisfy trivially and we get following Corollary

Corollary: 2.4 Let $(X, M, \star)$ be a complete fuzzy metric space and let A, S, P and Q be mappings from X into itself such that the following conditions are satisfied:

2.4(a) $(X) \subset (X)$ and $(X) \subset (X)$,

2.4 (b) either P or AB is continuous,

2.4 (c) (P, AB) is compatible of type (β) and (Q, ST) is weak compatible,

2.4 (d) there exists $k \in (0, 1)$ such that for every $x, y \in X$ and $t > 0$

$$\int_0^{M^2(Px,Qy,kt)} \xi(v) dv \geq \int_0^{W(Px,Qy,t)} \xi(v) dv$$

$W(Px,Qy,t) = M^2(Ax, Sy, t) \star M^2(Px,Ax,t) \star M^2(Qy, Sy, t) \star M^2(Px, Sy, t)$

where ξ: $[0, +\infty] \rightarrow [0, +\infty]$ is a lebgesgueintegrable mapping which is summable on each compact subset of $[0, +\infty]$, non negative, and such that, $\forall \varepsilon > 0$, $\int_0^{\varepsilon} \xi(v) dv > 0$. Then A, B, S, T, P and Q have a unique common fixed point in X.

Remark: 2.5: If we take the pair P, is weakly compatible in place of compatible type of (β) in Theorem 2. 2 then we get the following result.

Corollary: 2.6 Let $(X, M, \star)$ be a complete fuzzy metric space and let A, B, S, T, P and Q be mappings from X into itself such that the following conditions are satisfied:
2.6 (a) $P(X) \subset (X)$ and $(X) \subset AB(X)$,

2.6 (b) $AB=BA$, $=TS$, $PB=BP$, $QT=TQ$,

2.6 (c) either P or AB is continuous,

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2.6 (d) (P,) and (Q,) are weak compatible,

2.6 (e) there exists $k\in$ (0, 1) such that for every x, $y \in X$ and $t > 0$

$\int_0^{M^2(Px,Qy,kt)} \xi(v)dv \geq \int_0^{W(Px,Qy,t)} \xi(v)dv$

W(Px, Qy, t) = M^2(ABx, STy, t)$\star M^2$(Px, ABx, t) $\star M^2$(Qy, STy, t)$\star M^2$ (Px, STy, t)

where ξ:[0 ,+∞] → [0 ,+∞] is a lebgesgueintegrable mapping which is summable on each compact subset of [0,+∞], non negative, and such that, $\forall \varepsilon > 0$, $\int_0^{\xi} \xi(v)dv$>0. Then A, B, S, T, P and Q have a unique common fixed point in X.

Remark: 2.7 If we take B=T=I identity map on X in Corollary 2.6 then condition 2.6(b) is satisfy trivially and we get following Corollary

Corollary: 2.8 Let (X, M, $\star$)be a complete fuzzy metric space and let A, S, P and Q be mappings from X into itself such that the following conditions are satisfied:

2.8 (a) (X) ⊂S (X) and Q(X) ⊂(X),

2.8 (b) either P or AB is continuous,

2.8 (c) (P, A) and (Q, S) are weak compatible,

2.8 (d) there exists $k\in$ (0, 1) such that for every x, $y\in X$ and $t > 0$

$\int_0^{M^2(Px,Qy,kt)} \xi(v)dv \geq \int_0^{W(Px,Qy,t)} \xi(v)dv$

W(Px,Qy,t) = M^2(Ax, Sy, t) $\star M^2$(Px,Ax,t)$\star M^2$(Qy, Sy, t) $\star M^2$(Px,Sy,t)

where ξ:[0 ,+∞] → [0 ,+∞] is a lebgesgueintegrable mapping which is summable on each compact subset of [0 ,+∞], non negative, and such that, $\forall \varepsilon$> 0, $\int_0^{\xi} \xi(v)dv$>0. Then A, S, P and Q have a unique common fixed point in X.

Now following results are also equivalent to Theorem 2.2

Theorem: 2.9 Let X,, $\star$ be a complete fuzzy metric space and let A,B,S,T, P and Q be mappings from X into itself such that the following conditions are satisfied:

2.9(a) (X) ⊂(X) and Q(X)⊂AB(X),

2.9 (b) AB=BA, =TS, PB=BP, T=TQ,

2.9(c) either P or AB is continuous,

2.9(d) (P, AB) is compatible of type (α) and (Q, ST) is weak compatible,

2.9 (e) there exists $k\in$ (0, 1) such that for every x, $y\in X$ and $t > 0$

$\int_0^{M^2(Px,Qy,kt)} \xi(v)dv \geq \int_0^{W(Px,Qy,t)} \xi(v)dv$

W(Px,Qy,t) = M^2(ABx, STy, t) $\star M^2$(Px,ABx,t)$\star M^2$(Qy, STy, t) $\star M^2$(Px,STy,t)

where ξ: [0, +∞] → [0, +∞] is a lebgesgueintegrable mapping which is summable on each compact subset of [0, +∞], non negative, and such that, $\forall \varepsilon > 0$, $\int_0^{\xi} \xi(v)dv > 0$. Then A, B, S, T, P and Q have a unique common fixed point in X.

Proof: Form the definition 1.1.8 and proof of the Theorem 2.2, we get the result.

Remark: 2.10 If we take B=T=I identity map on X in Theorem 2.9 then condition 2.9(b) is satisfy trivially and we get following Corollary

Theorem: 2.11 Let $(X,, \star)$be a complete fuzzy metric space and let $A,S,P\ and\ Q$ be mappings from X into itself such that the following conditions are satisfied:
2.11(a) $(X) \subset (X)\ and\ Q(X) \subset$A$(X)$,

2.11 (b) either P or AB is continuous,

2.11 (c) $(P$, AB) is compatible of type (α) and $(Q$, ST) is weak compatible,

2.11 (d) there exists $k\in$ (0, 1) such that for every $x, y \in X and\ t > 0$

$$\int_0^{M^2(Px,Qy,kt)} \xi(v)dv \geq \int_0^{W(Px,Qy,t)} \xi(v)dv$$

$$W(Px, Qy, t) = M^2(Ax,Sy, t) \star M^2(Px,Ax,t) \star M^2(Qy, Sy, t) \star M^2(Px,Sy,t)$$

where ξ:[0 ,+∞] → [0 ,+∞] is a lebgesgueintegrable mapping which is summable on each compact subset of [0 ,+∞], non negative, and such that, $\forall\varepsilon > 0$, $\int_0^{\varepsilon} \xi(v)dv > 0$. Then A, S, P and Q have a unique common fixed point in X

REFERENCES

1. S. Banach, Theorie les operations Lineaires, ManograieMathematyezne Warsaw Poland, 1932

2. A. Branciari, A fixed point theorem for mappings satisfying a general contractive condition of integral type, Int. J. Math. Math. Sci. **29** (9) (2002), 531-536.

3. S.H., Cho, On common fixed point theorems in fuzzy metric spaces, J. Appl. Math. & Computing Vol. 20 (2006), No. 1 -2, 523-533.

4. Y.J. Cho, Fixed point in Fuzzy metric space, J. Fuzzy Math.

5. (1997), 949-962. 5. Y.J. Cho, H.K. Pathak, S.M. Kang and J.S. Jung, Common fixed points of compatible mappings of type (β) on fuzzy metric spaces, Fuzzy sets and systems, 93 (1998), 99-111.

6. A. George and P. Veeramani, On some results in Fuzzy metric spaces, Fuzzy Sets and Systems 64 (1994), 395-399.

7. M. Grebiec, Fixed points in Fuzzy metric space, Fuzzy sets and systems, 27(1998), 385-389.

8. A. Jain and B. Singh, A fixed point theorem for compatible mappings of type (A) in fuzzy metric space, ActaCienciaIndica, Vol. XXXIII M, No. 2 (2007), 339-346.

9. A. Jain, M. Sharma and B. Singh, Fixed point theorem using compatibility of type (β) in Fuzzy metric space, Chh. J. Sci. & Tech., Vol. 3 & 4, (2006- 2007), 53-62.

10. G. Jungck, P.P. Murthy and Y.J. Cho, Compatible mappings of type (A) and common fixed points, Math. Japonica, 38 (1993), 381-390.

11. E.P. Klement, R. Mesiar and E. Pap, Triangular Norms, Kluwer Academic Publishers.

12. I. Kramosil and J. Michalek, Fuzzy metric and statistical metric spaces, Kybernetica 11 (1975), 336-344.

13. S.N. Mishra, N. Mishra and S.L. Singh, Common fixed point of maps in fuzzy metric space, Int. J. Math. Math. Sci. 17(1994), 253-258.

14. B. Singh and M.S. Chouhan, Common fixed points of compatible maps in Fuzzy metric spaces, Fuzzy sets and systems, 115 (2000), 471-475.

15. L. A. Zadeh, Fuzzy sets, Inform and control 89(1965), 338-353.

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