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RELIABILITY MEASURES OF A PARALLEL SYSTEM WITH REPAIR BY AN EXPERT SERVER SUBJECT TO MAXIMUM REPAIR AND INSPECTION TIMES OF ORDINARY SERVER

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ABSTRACT

The emphasis of the present work is on the evaluation of reliability measures of a parallel system of two identical units in which repair is done by an expert server subject to maximum repair and inspection times of the ordinary server. Initially, repair of the unit is done by an ordinary server. If ordinary server is unable to do repair the unit in a given maximum time't', then he inspects the unit to see the feasibility of its repair by an expert server. And, if repair of the unit is also not possible by the expert server, it is replaced by new one giving some replacement time. The ordinary server remains idle during repair by an expert server. Both servers visit the system immediately when required. The failure time of the unit and the time to which unit undergoes for inspection follow negative exponential distribution while repair and replacement times are taken as arbitrary with different probability density functions. All random variables are statistically independent. The expressions for some important performance measures of the system are derived in steady state using semi-Markov process and regenerative point technique. The graphical behavior of these measures has been observed for particular values of various parameters and costs.

KeyWords: *Parallel System, Maximum Repair Time, Inspection, Two Servers, Replacement and Reliability Measures.*

INTRODUCTION

The technique of parallel redundancy has been used extensively in many industrial systems in order to improve their performance. Therefore, several research papers have been written by scholars including Kishan and Kumar [2009] and Kumar *et al*. [2010] on reliability modeling of parallel systems. They assumed that every server is capable in repairing the faults occurred during operation of the system. But sometimes a system has an ordinary server who is unable to do some complex repairs. In such a situation, the ordinary server may be asked to inspect the unit after a pre-specified time't' to see the feasibility of its repair by an expert server. If unit is also not repairable by an expert server, it can be replaced by new one. Kumar *et al.* [2012] and Malik [2013] developed reliability models of a computer system with maximum operation and repair times.

The purpose of the present study is to determine reliability measures of a parallel system of two identical units in which repair is done by an expert server subject to maximum repair and inspection times of the ordinary server. The unit fails directly from normal mode. Initially, repair of the unit is done by an ordinary server. If ordinary server is not able to repair the unit in a given maximum time, then he inspects the unit to see the feasibility of its repair by an expert server. And, if repair of the unit is also not possible by the expert server, it is replaced by new one giving some replacement time. The ordinary server remains idle with the system during repair by an expert server. Both servers visit the system immediately when required. The failure time of the unit and the time to which unit undergoes for inspection follow negative exponential distribution while repair and replacement time are taken as arbitrarily with different probability density functions. All random variables are statistically independent. The expression for some important performance measures of the system such as mean sojourn times, mean time to system failure (MTSF), availability, busy period of the servers due to repair, replacement and inspection of the unit, expected number of visits by the servers, expected number of replacements of the unit and profit function have been derived in steady state using semi-Markov process and regenerative point technique. The numerical results for MTSF, availability and profit are obtained giving particular values to various parameters and costs.

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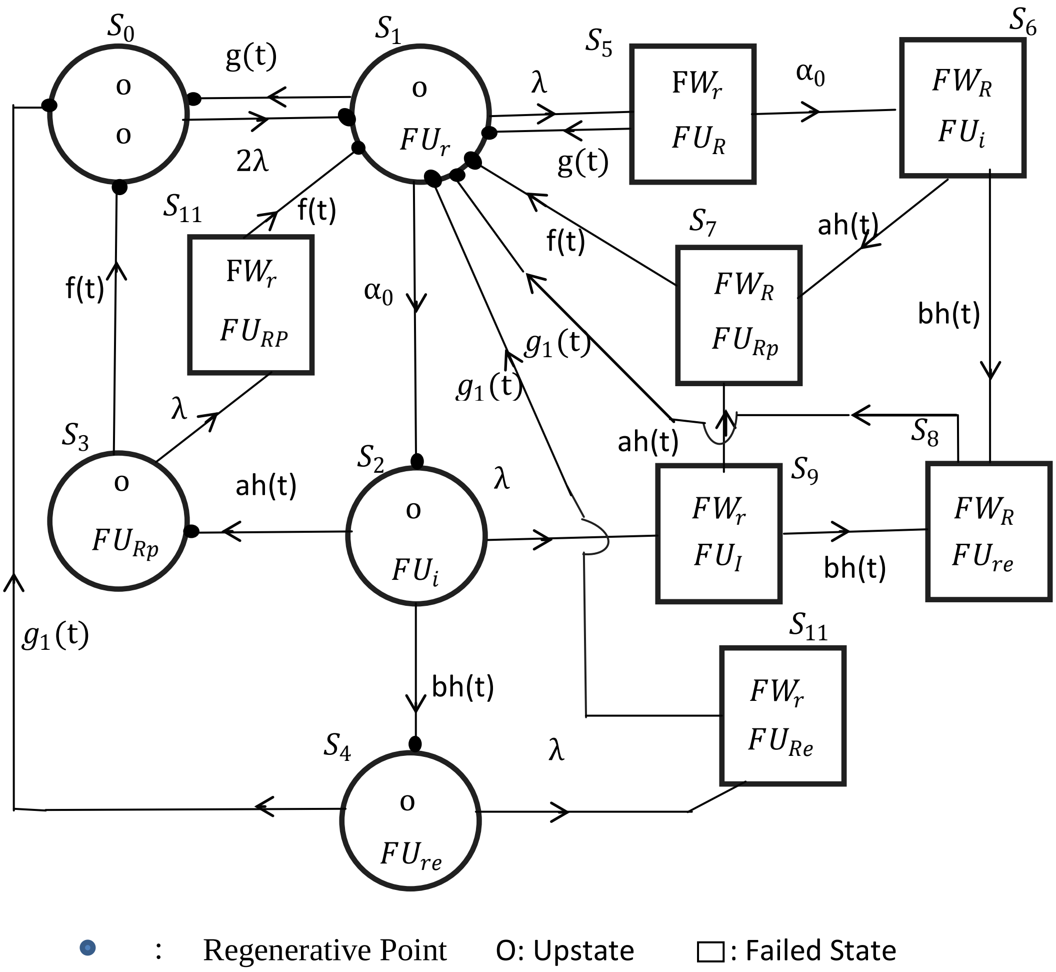
2. NOTATIONS

E	:	Set of regenerative states
O	:	Unit is operative
λ	:	Constant failure rate of the unit
α_0	:	Constant rate by which unit under goes for inspection after a pre-specified time 't' to see the feasibility of repair
a/b	:	Probability that failed unit is not repairable / repairable by an expert server
f(t)/F(t)	:	pdf / cdf of the replacement time of the unit
g(t)/G(t)	:	pdf / cdf of the repair time of the unit taken by ordinary server
$g_1(t)/G_1(t)$	:	pdf / cdf of the repair time of the unit taken by expert serve
$q_{ij}(t)$ / $Q_{ij}(t)$	:	pdf and cdf of direct transition time from a regenerative state S_i to a regenerative state S_j without visiting any other regenerative state.
$q_{ij.k}(t)$, $Q_{ij.k}(t)$	:	pdf and cdf of first passage time from a regenerative state S_i to a regenerative state S_j or to a failed state S_j visiting state k once in (0,t].
$h(t)/H(t)$	:	pdf / cdf of the inspection time of the unit taken by ordinary server
FU_r / FU_R	:	Unit is failed and under repair with ordinary server / under repair continuously from previous state with ordinary server
FW_r/FW_R	:	Unit is failed and waiting for repair / waiting for repair continuously from previous state
FU_i/ FU_I	:	Unit is failed and under inspection with ordinary server / waiting for inspection by ordinary server continuously from previous state
FU_{Rp}/ FU_{RP}	:	Unit is failed and under replacement with ordinary server / waiting for replacement by ordinary server continuously from previous state
FUr_e/FUR_e	:	Unit is failed and under repair with expert server / under repair continuously from previous state with expert server
m_{ij}	:	The conditional mean sojourn time in regenerative state S_i when system is to make transition in to regenerative state S_j. Mathematically, it can be written $m_{ij} = E(T_{ij}) = \int_0^\infty td[Q_{ij}(t)] = -q_{ij}^{*'}(0)$, where T_{ij} is the transition time from state S_i to S_j; S_i , S_j ε E.
μ_i	:	The mean sojourn time in state S_i is given by $\mu_i = E(T_i) = \int_0^\infty P(T_i > t)dt = \sum_j m_{ij}$, where T_i is the sojourn time in state S_i.
~/*	:	Symbol for Laplace Stieltjes transform (LST)/Laplace transform (LT)
Ⓢ /©	:	Symbol for Stieltjes convolution / Laplace convolution.

The possible transitions between states along with transitions rates for the system model are shown in figure 1.The states S_0, S_1, S_2, S_3 and S_4 are regenerative while the other states are non-regenerative.

Fig. 1: State Transition Diagram



TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

Simple probabilistic considerations yield the following expressions for the non-zero elements

$p_{ij} = Q_{ij}(\infty) = \int q_{ij}(t)\,dt$ as

$p_{01} = 1,\ p_{10} = g^*(\lambda+\alpha_0),\ p_{12} = \frac{\alpha_0}{\alpha_0+\lambda}[1-g^*(\lambda+\alpha_0)],$

$p_{15} = \frac{\lambda}{\alpha_0+\lambda}[1-g^*(\lambda+\alpha_0)],\ p_{23} = ah^*(\lambda), p_{24} = bh^*(\lambda),\ p_{29} = \left(1-h^*(\lambda)\right),\ \ p_{30} = f^*(\lambda),$

$p_{3,10} = \left(1-f^*(\lambda)\right),\ p_{40} = {g_1}^*(\lambda),\ p_{4,11} = (1-{g_1}^*(\lambda))\ \ p_{51} = g^*(\alpha_0),\ \ p_{56} = \left(1-g^*(\alpha_0)\right),$

$p_{67} = a,\ p_{68} = b,\ \ p_{71} = f^*(0),\ p_{81} = {g_1}^*(0),\ p_{97} = a, p_{98} = b, p_{10,1} = f^*(0),\ \ \ p_{11,1} = {g_1}^*(0)$

$p_{11.5} = \frac{\lambda}{\alpha_0+\lambda}[1-g^*(\lambda+\alpha_0)]g^*(\alpha_0), p_{11.567} = \frac{a\lambda}{\alpha_0+\lambda}[1-g^*(\lambda+\alpha_0)]\left(1-g^*(\alpha_0)\right)$

$p_{11.568} = \frac{b\lambda}{\alpha_0+\lambda}[1-g^*(\lambda+\alpha_0)]\left(1-g^*(\alpha_0)\right),\ p_{21.97} = a\left(1-h^*(\lambda)\right), p_{21.98} = b$

$p_{31.10} = \left(1-f^*(\lambda)\right),\ p_{41.11} = 1-{g_1}^*(\lambda)$ (1)

It can easily be verified that

$p_{01} = p_{10}+p_{12}+p_{15} = p_{10}+p_{12}+p_{11.5}+p_{11.567}+p_{11.568} = p_{23}+p_{24}+p_{29} = p_{23}+p_{24}+p_{21.97}+p_{21.98} = p_{30}+p_{3,10} = p_{30}+p_{31.10} = p_{40}+p_{4,11} = p_{40}+p_{41.11} = p_{51}+p_{56} = p_{67}+p_{68} = p_{71} = p_{81} = p_{97}+p_{98} = p_{10,1} = p_{11,1} = 1$ (2)

The mean sojourn times μ_i in state S_i is given by

$\mu_0 = \int_0^\infty P(T>t)\,dt = \mathrm{m}_{01} = \frac{1}{2\lambda},\ \ \ \mu_1 = \mathrm{m}_{10} + \mathrm{m}_{12} + \mathrm{m}_{15} = \frac{1}{\alpha_0+\lambda}[1-\mathrm{g}^*(\alpha_0+\lambda)],\ \mu_2 = \mathrm{m}_{23}+\mathrm{m}_{24}+\mathrm{m}_{29} = \frac{1}{\lambda}\left(1-h^*(\lambda)\right),\ \ \mu_3 = \mathrm{m}_{30}+\mathrm{m}_{3,10} = \frac{1}{\lambda}\left(1-f^*(\lambda)\right), \mu_4 = \mathrm{m}_{40}+\mathrm{m}_{4,11} = \frac{1}{\lambda}\left(1-{g_1}^*(\lambda)\right),$

$\mu_1{}' = \mathrm{m}_{10} + \mathrm{m}_{12} + \mathrm{m}_{11.5} + \mathrm{m}_{11.567} + \mathrm{m}_{11.568} = \frac{[1-g^*(\alpha_0+\lambda)]}{\alpha_0+\lambda}[1 + \lambda(1 - g_1{}^*(\alpha_0)(\frac{1}{\alpha_0} - \mathrm{b}\, g_1{}^{*'}(0) - \mathrm{a}\, f^{*'}(0) - h^{*'}(0))]$ $\mu_2' = \mathrm{m}_{23} + \mathrm{m}_{24} + \mathrm{m}_{21.97} + \mathrm{m}_{21.98} = \left(1 - h^*(\lambda)\right)\left(\frac{1}{\lambda} - bg_1{}^{*'}(0) - af^{*'}(0) - h^{*\prime}(0)\right)$

$\mu_3' = \mathrm{m}_{30} + \mathrm{m}_{31.10} = \left(1 - f^*(\lambda)\right)\left(\frac{1}{\lambda} - f^{*'}(0)\right)$

$$\mu_4' = \mathrm{m}_{40} + \mathrm{m}_{41.11} = \left(1 - g_1{}^*(\lambda)\right)\left(\frac{1}{\lambda} - g_1{}^{*'}(0)\right) \tag{3}$$

MEAN TIME TO SYSTEM FAILURE (MTSF)

Let $\varphi_i(t)$ be the cdf of first passage time from the regenerative state S_i to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for $\varphi_i(t)$:

$$\varphi_i(t) = \sum_j Q_{i,j}(t) ® \varphi_j(t) + \sum_k Q_{i,k}(t) \tag{4}$$

where S_j is an un-failed regenerative state to which the given regenerative state S_i can transit and S_k is a failed state to which the state S_i can transit directly. Taking LST of above relation (4) and solving for $\tilde{\phi}_0(s)$, we get

$$MSTF(T_0) = \lim_{s\to 0}\frac{1-\tilde{\varnothing}_0(s)}{s} = \frac{\mu_0+\mu_1+p_{12}\mu_2+p_{12}p_{23}\mu_3+p_{12}p_{24}\mu_4}{1-p_{10}-p_{12}(p_{23}p_{30}+p_{24}p_{40})} = \frac{N_1}{D_1} \tag{5}$$

where $N_1 = \mu_0 + \mu_1 + p_{12}\mu_2 + p_{12}p_{23}\mu_3 + p_{12}p_{24}\mu_4$ and $D_1 = 1 - p_{10} - p_{12}(p_{23}p_{30} + p_{24}p_{40})$

AVAILABILITY ANALYSIS

Let $A_i(t)$ be the probability that the system is in up-state at instant 't' given that the system entered regenerative state S_i at t = 0. The recursive relations for A_i (t) are given as

$$A_i(t) = M_i(t) + \sum_j q_{i,j}^{(n)}(t) © A_j(t) \tag{6}$$

where S_j is any successive regenerative state to which the regenerative state S_i can transit through n transitions. $M_i(t)$ is the probability that the system is up initially in state $S_i \in E$ up at time t without visiting to any other regenerative state,

where $M_0(t) = e^{-2\lambda t}$, $M_1(t) = e^{-(\lambda+\alpha_0)t}\bar{G}(t)$, $M_2(t) = e^{-\lambda t}(t)\,\bar{H}(t)$,
$M_3(t) = e^{-\lambda t}(t)\,\bar{F}(t)$ *and* $M_4(t) = e^{-\lambda t}(t)\,\overline{G_1}(t)$

Taking $L.T.$ of relation (6) and solving for $A_0^*(s)$, we get steady-state availability as

$$A_0 = \lim_{s\to 0} s\,A_0^*(s) = \frac{\mu_0(p_{10}+p_{12}(p_{23}p_{30}+p_{24}p_{40}))+\mu_1+p_{12}\mu_2+p_{12}p_{23}\mu_3+p_{12}p_{24}\mu_4}{\mu_0(p_{10}+p_{12}(p_{23}p_{30}+p_{24}p_{40}))+\mu_1'+p_{12}\,\mu_2'+p_{12}p_{23}\,\mu_3'+p_{12}p_{24}\mu_4'} = \frac{N_2}{D_2} \tag{7}$$

where $N_2 = \mu_0\left(p_{10} + p_{12}(p_{23}p_{30} + p_{24}p_{40})\right) + \mu_1 + p_{12}\mu_2 + p_{12}p_{23}\mu_3 + p_{12}p_{24}\mu_4$ and
$D_2 = \mu_0\left(p_{10} + p_{12}(p_{23}p_{30} + p_{24}p_{40})\right) + \mu_1' + p_{12}\,\mu_2' + p_{12}p_{23}\,\mu_3' + p_{12}p_{24}\mu_4'$

BUSY PERIOD ANALYSIS OF ORDINARY SERVER DUE TO REPAIR

Let $B_i^R(t)$ be the probability that the ordinary server is busy in repairing the unit at an instant 't'given that the system entered regenerative state S_i at $t = 0$. The recursive relations for $B_i^R(t)$ are given as

$$B_i^R(t) = W_i(t) + \sum_j q_{i,j}^{(n)}(t) © B_j^R(t) \tag{8}$$

where S_j is any successive regenerative state to which the regenerative state S_i can transit through n transitions.

$$\text{where } W_1(t) = e^{-(\lambda+\alpha_0)t}\bar{G}(t) + \left(\lambda e^{-\lambda t} © \mathbf{1} © e^{-\alpha_0 t}\right)\bar{G}(t) \tag{9}$$

Taking LT of relation (8) and solving for$B_0^{R^*}(s)$, we get in the long run the time for which the system is under repair is given by

$$B_0^R = \lim_{s\to 0} s\, B_0^{R^*}(s) = \frac{N_3}{D_2} \tag{10}$$

where $N_3 = W_1^*(0)$ and D_2 is already specified.

BUSY PERIOD ANALYSIS OF ORDINARY SERVER DUE TO RPLACEMENT

Let $B_i^{Rp}(t)$ be the probability that the ordinary server is busy in repairing the unit at an instant 't'given that the system entered regenerative state S$_i$ at $t = 0$.The recursive relations for $B_i^{Rp}(t)$ are given as

$$B_i^{Rp}(t) = W_i(t) + \sum_j q_{i,j}^{(n)}(t) © B_j^{Rp}(t) \tag{11}$$

where S$_j$ is any successive regenerative state to which the regenerative state S$_i$ can transit through n transitions.

where $W_3(t) = e^{-\lambda t}\bar{F}(t) + (\lambda e^{-\lambda t} © \mathbf{1} ©)\bar{F}(t)$ (12)

Taking $L.T.$ of relation (11) and solving for $B_0^{Rp^*}(s)$, we get in the long run the time for which the system is under replacement is given by

$$B_0^{Rp} = \lim_{s\to 0} s\, B_0^{Rp^*}(s) = \frac{N_4}{D_2} \tag{13}$$

where $N_4 = \mathrm{p}_{12}\mathrm{p}_{23} W_3^*(0)$ and D_2 is already specified.

BUSY PERIOD ANALYSIS OF ORDINARY SERVER DUE TO INSPECTION

Let $B_i^i(t)$ be the probability that the ordinary server is busy in inspection of the unit at an instant 't'given that the system entered regenerative state S$_i$ at $t = 0$. The recursive relations for $B_i^i(t)$ are given as

$$B_i^i(t) = W_i(t) + \sum_j q_{i,j}^{(n)}(t) © B_j^i(t) \tag{14}$$

where S$_j$ is any successive regenerative state to which the regenerative state S$_i$ can transit through n transitions.

where $W_2(t) = e^{-\lambda t}\bar{H}(t) + (\lambda e^{-\lambda t} © \mathbf{1})\bar{H}(t)$ (15)

Taking $L.T.$ of relation (14) and solving for$B_0^{i^*}(s)$, we get in the long run the time for which the system is under inspection is given by

$$B_0^i = \lim_{s\to 0} s\, B_0^{i^*}(s) = \frac{N_5}{D_2} \tag{16}$$

where $N_5 = \mathrm{p}_{12} W_2^*(0)$ and D_2 is already specified.

BUSY PERIOD ANALYSIS OF EXPERT SERVER DUE TO REPAIR

Let $B_i^e(t)$ be the probability that the expert sever is busy in repairing the unit at an instant 't'given that the system entered regenerative state S$_i$ at $t = 0$. The recursive relation for $B_i^e(t)$ are given by:

$$B_i^e(t) = W_i(t) + \sum_j q_{i,j}^{(n)}(t) © B_j^e(t) \tag{17}$$

where S$_j$ is any successive regenerative state to which the regenerative state S$_i$ can transit through n transitions.

where $W_4(t) = e^{-\lambda t}\overline{G_1}(t) + (\lambda e^{-\lambda t} © \mathbf{1})\overline{G_1}(t)$ (18)

Taking $L.T.$ of relation (17) and solving for $B_0^{e*}(s)$, we get the time for which the system is under repair done by expert server is given by

$$B_0^e = \lim_{s\to 0} s\, B_0^{e*}(s) = \frac{N_6}{D_2} \tag{19}$$

where $N_6 = \mathrm{p}_{12}\mathrm{p}_{24} W_4^*(0)$ and D_2 is already specified.

EXPECTED NUMBER OF VISITS BY ORDINARY SERVER

Let $N_i(t)$ be the expected number of visits by the ordinary server in $(0, t]$ given that the system entered the regenerative state S_i at $t = 0$. The recursive relation for $N_i(t)$ are given by

$$N_i(t) = \sum_j Q_{i,j}^{(n)}(t) \circledR \left[\delta_j + N_i(t) \right] \tag{20}$$

Where S_j is any regenerative state to which the given regenerative state S_i transits and δj =1, if S_j is the regenerative state where the server does job afresh, otherwise δj = 0. Taking LST of relation (20) and solving for $\widetilde{N}_0(s)$, we get the expected number of visits by ordinary server per unit time as

$$N_0 = \lim_{s\to 0} s\,\widetilde{N}_0(s) = \frac{N_7}{D_2} \tag{21}$$

where $N_7 = p_{10} + p_{12}(p_{23}p_{30} + p_{33}p_{40})$ and D_2 is already specified.

EXPECTED NUMBER OF VISITS BY EXPERT SERVER

Let $N_i^e(t)$ be the expected number of visits by expert server $(0, t]$ given that the system entered the regenerative state S_i at $t = 0$. The recursive relation for $N_i^e(t)$ are given by:

$$N_i^e(t) = \sum_j Q_{i,j}^{(n)}(t) \circledR \left[\delta_j + N_j^e(t) \right] \tag{22}$$

where S_j is any regenerative state to which the given regenerative state S_i transits and δj =1, if *j* is the regenerative state where the server does job afresh, otherwise δj = 0. Taking LST of relation (22) and solving for $\widetilde{N}_0^e(s)$, we get the expected number of visits by expert server per unit time as

$$N_0^e = \lim_{s\to 0} s\,\widetilde{N}_0^e(s) = \frac{N_8}{D_2} \tag{23}$$

where $N_8 = p_{11.568} + p_{12}(p_{21.8,10} + p_{24})$ and D_2 is already specified.

EXPECTED NUMBER OF REPLACEMENTS OF UNIT

Let $R_i(t)$ be the expected number of replacement of unit in $(0, t]$ given that the system entered the regenerative state S_i at $t = 0$. The recursive relation for $R_i(t)$ are given by

$$R_i(t) = \sum_j Q_{i,j}^{(n)}(t) \circledR \left[\delta_j + R_j(t) \right] \tag{24}$$

where S_j is any regenerative state to which the given regenerative state S_i transits and δj =1, if *j* is the regenerative state where the server does job afresh, otherwise δj = 0. Taking LST of relation (24) and solving for $\tilde{R}_0(s)$, we get the expected number of replacements of the unit time as

$$R_0 = \lim_{s\to 0} s\,\tilde{R}_0(s) = \frac{N_9}{D_2} \tag{25}$$

where $N_9 = p_{11.567} + p_{12}(p_{21.8,10} + p_{24})$ and D_2 is already specified.

COST-BENEFIT ANALYSIS

Profit incurred to the system model in steady state is given by

$$P = K_1A_0 - K_2B_0^R - K_3B_0^{Rp} - K_4B_0^e - K_5B_0^i - K_6N_0 - K_7N_0^e - K_8R_0$$

where

K_1 = Revenue per unit uptime of the system
K_2 = Cost per unit time for which ordinary server is busy due to repair
K_3 = Cost per unit time for which ordinary server is busy due to replacement

K_4 = Cost per unit time for which expert server is busy due to repair
K_5 = Cost per unit time for which ordinary server is busy due to inspection
K_6 = Cost per unit visits by the ordinary server
K_7 = Cost per unit visits by the expert server
K_8 = Cost per unit time replacement of the unit

CONCLUSION

While considering $g(t) = \theta e^{-\theta t}$, $g_1(t) = \theta_0 e^{-\theta_0 t}$, $h(t) = \gamma e^{-\gamma t}$, the numerical results for MTSF, availability and profit of the system are obtained giving arbitrary values to various parameters and costs. The graphical behavior of these measures with respect to replacement rate (β) for K2>K4 is shown respectively in figures 1, 2 and 3. It is observed that all these measures go on increasing with the increase of replacement rate (β), repair rate of ordinary server (θ), repair rate of expert server(θ_0) and inspection rate (γ) while they decline with the increase of failure rate (λ). And, the effect of repair rate of ordinary server (θ) on these measures is more as compared to the other parameters. Further, results indicate that profit of the system decreases with the increase of the constant rate (α_0) by which unit under goes for inspection. However, MTSF and availability keep on increasing. Hence study reveals that a parallel system of two-identical units can be made more reliable and profitable to use by calling an expert server immediately when ordinary server fails to repair the system in a pre-specified repair time.

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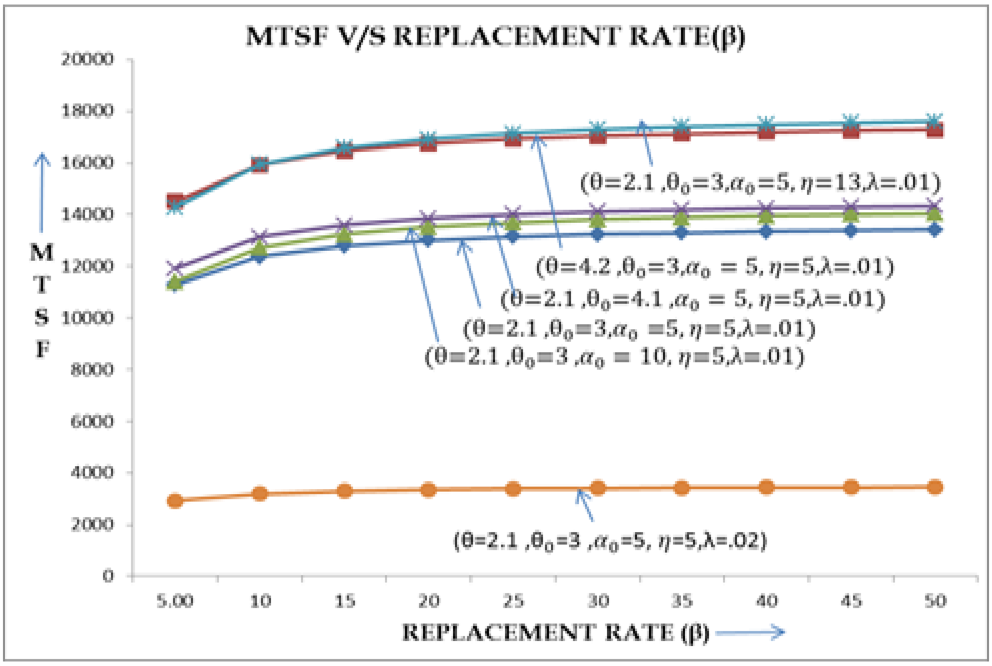

Figure 2: MTSF V/S Replacement Rate

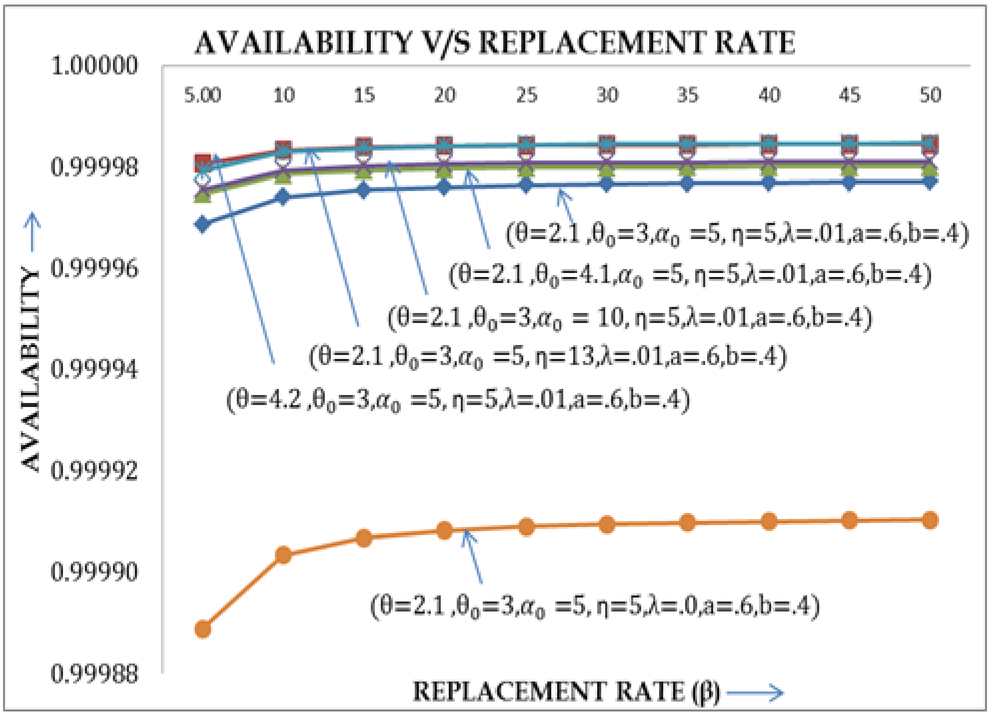

Figure 3: Availability V/S Replacement Rate

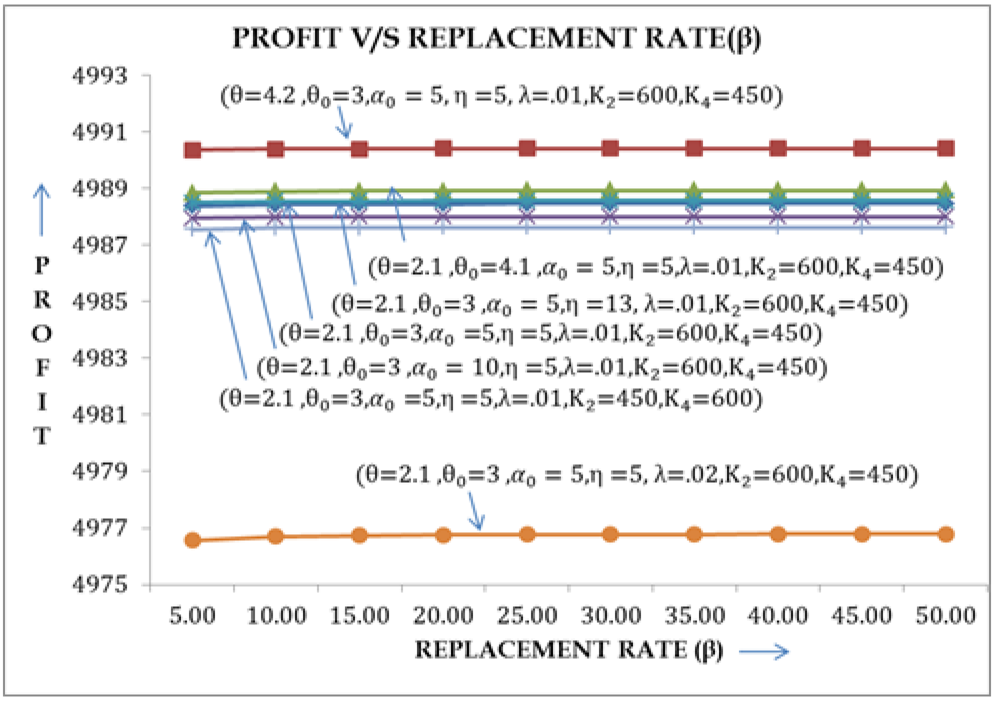

Figure 4: Profit V/S Replacement Rate

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