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Geodomination, g-independence and g-irredundance

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ABSTRACT

The concept of geodetic set was introduced by F. Buckley and F.Harary in [1] and G.Chartrand , F.Harary and P.Zhang in [2]. In [4], the geodetic number was defined by F.Harary, E.Loukakis and C.Tsouros. In this paper, we introduce the concept of geodetic neighbourhood(g-neighborhood) and closed geodetic neighbourhood sets of a pair of vertices of a connected graph G with atleast two vertices. And, we define the geodetic number of a graph using g-neighbourhood sets. Further, we introduce some new concepts such as g-isolated vertices, g-independence, g-independence number, g-connectedness of a graph, g-independent geodetic set, g-independent geodetic number, g-irredundance number etc. Results connecting the above defined parameters are developed.

Keywords: *g-neighborhood , g-independence, g-connectedness, g-irredundance.*

I. INTRODUCTION

Through out this paper, we consider only finite, undirected, connected graphs with at least two vertices and with out loops and multiple edges. For graph theoretic representations, we refer [5]. Let G=(V,E) be any graph and u,v $\in$V(G) such that u$\neq$v. d(u,v) is the length of the shortest path connecting u and v. A u-v geodesic is a u-v path of length d(u,v). For a pair u,v of vertices in G, the closed interval I[x,y] is defined as I[x,y] = {x,y} $\cup$ {v : v is an internal vertex of an x-y geosic in G}. In this paper, we define the geodetic neighborhoood(g-neighborhood) set of a pair x,y of vertices in G as the open interval I(x,y) = {v: v is an internal vertex of an x-y geodesic in G}. We denote it by $N_g(x,y)$. Correspondingly, we call I[x,y] as the closed geodetic neighborhood set of x,y and we denote it as $N_g[x,y]$. That is, $N_g[x,y] = N_g(x,y) \cup \{x,y\}$. For S $\subseteq$ V, I[S] is defined as $I[S] = \bigcup_{x,y\in S} I[x,y] = \bigcup_{x,y\in S} N_g[x,y]$. We denote it as $N_g[S]$. A set S of vertices in G is called a geodominating (or geodetic) set of G if I[S] = V. Equivalently, a set S of vertices in G is a geodetic set of G if $N_g[S]$ = V. A geodetic set of minimum cardinality is called a minimum geodetic set in [2] (or a geodetic basis in [1]). A geodetic set S is said to be a minimal geodetic set of G if no proper subset of S is a geodetic set of G. The minimum cardinality of all minimal geodetic sets of G is the geodomination(or geodetic) number of G. It is denoted as g(G)[1,2,4].

Definition: 1.1 Let G=(V,E) be any graph and v$\in$V(G). The neighborhoood of v, written as $N_G(v)$ or N(v) is defined by N(v)= {x$\in$V(G):x is adjacent to v}. The closed neighborhood of of v is defined as N[v]=N(v) $\cup${v}.

Definition: 1.2 A vertex v in G is an extreme vertex of G if the sub graph induced by its neighbors is complete.

Definition: 1.3 Let G=(V,E) be any graph. Then, G^+ is the graph obtained from G by attaching a pendant vertex to each vertex of G.

Definition: Generalized Hajo's graph: 1.4[6] For k$\geq$3, the generalized Hajos graph H_k is a graph on $n=k+\binom{k}{2}$ vertices with vertex set $V(H_k)=\{x_1,x_2,\ldots,x_k\}\cup\{y_{i,j}:1\leq i<j\leq k\}$ where $<\{x_1,x_2,\ldots,x_k\}> = K_k$ and each $y_{i,j}$ has degree two with $N(y_{i,j})=\{x_i,x_j\}$, $deg(x_i)=2k-2$.

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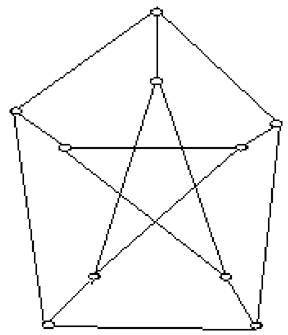
Definition: Peterson graph: 1.5



2. GEODETIC NEIGHBORHOOD SETS

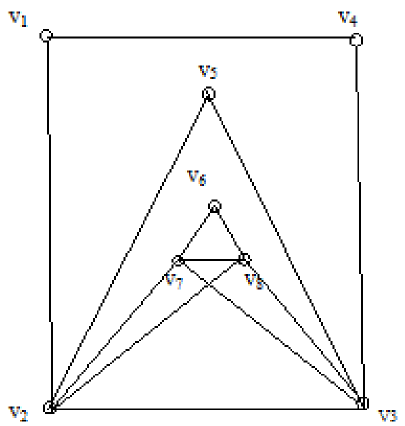
Definition: 2.1 Let G=(V,E) be any graph and $x,y\in V(G)$ such that $x\neq y$. The geodetic neighborhood(g -neighborhood) set of the pair x,y is defined as $N_g(x,y)=$ {v : v is an internal vertex of an x-y geodesic in G} and the closed geodetic neighborhood set of x,y is defined as $N_g[x,y]=N_g(x,y)\cup\{x,y\}$.

Remark: 2.2
1.If x and y are adjacent, then $N_g(x,y)=\phi$. In particular, if $y\in N(x)$, then $N_g(x,y)=\phi$.
2.Let G be a complete graph. Then, $N_g(x,y)=\phi$ for every $x,y\in V(G)$.

Definition: 2.3 A pair of vertices $x,y\in G$ is said to geodominate a vertex $v\in G$ if $v\in N_g[x,y]$. Similarly, a set $S\subseteq V(G)$ is said to geodominate a vertex $v\in V(G)$ if $v\in N_g[x,y]$ for some $x,y\in S$.

Definition: 2.4 A vertex $v\in V(G)$ is said to be a geodetically isolated(g-isolated) vertex of G if $v\notin N_g(x,y)$ for every pair of vertices $x,y\in G$.
Extreme vertices are g-isolated vertices of G.

Example: 2.5



G Figure 2.1

The vertices v_5 and v_6 are g-isolated vertices of G, as they do not belong to the open g-neighborhood set of any pair of vertices of G.

Remark: 2.6
1.The set of all g-isolated vertices of G is denoted by $I_g(G)$. The set of all extreme vertices of G is a subset of $I_g(G)$.
2.If G is a connected graph without g-isolates, then G has no extreme vertices and $\delta(G)\geq 2$.

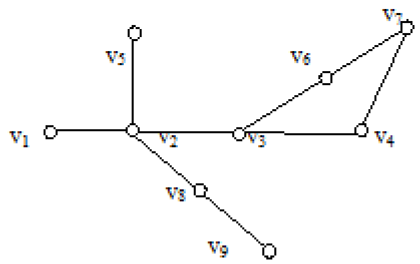
Proposition: 2.7 For any graph G, $I_g(G)$ is equal to the set of all extreme vertices of G.

Proof: Let S=The set of all extreme vertices of G. By Remark 2.6, $S\subseteq I_g(G)$. Let $x\in V$-S. Then, N(x) contains at least two non-adjacent vertices, say, u,v. Further, the uxv geodesic from u to v contains v. So, $x\notin I_g(G)$. Hence the result.

Definition: 2.8 Let $S \subseteq V(G)$. A vertex $v \in S$ is said to be a geodetically isolated(g-isolated) vertex of S if $v \notin N_g(x,y)$ for every pair x,y of vertices in S.

That is, v is not geodominated by the vertices of $S-\{v\}$.

Example: 2.9



G Figure 2.2

Consider $S=\{v_1,v_4,v_5,v_7,v_9\}$. Every vertex of S other than v_4 is a g-isolated vertex of S. For $S'=\{v_2,v_3,v_4\}$, v_2 and v_4 are g-isolated vertices.

Definition: 2.10 Let $S \subseteq V(G)$ and $v \in S$. A vertex $w \in V(G)$ is said to be a private g-neighbor of v with respect to S if w is geodominated by the vertices of S and it is not geodominated by the vertices of $S-\{v\}$.

Definition: 2.11 Let $S \subseteq V(G)$ and $v \in S$. The private g-neighbor set of v with respect to S is defined as $p_{gn}(v,S)=\{w$: w is a private g-neighbor of v with respect to S$\}$

Example: 2.12 Condider G in figure 2.2. Let $S=\{v_1,v_5,v_7,v_9\}$. The vertices v_3,v_4,v_6 and v_7 are g-private neighbors of v_7 and v_8,v_9 are g-private neighbors of v_9. The vertex v_1 is the unique g-private neighbor of itself. Similarly, v_5 is a only g-private neighbor of v_5. Therefore, $p_{gn}(v_7,S)=\{v_3,v_4,v_6,v_7\}$, $p_{gn}(v_9,S)=\{v_8,v_9\}$, $p_{gn}(v_1,S)=\{v_1\}$ and $p_{gn}(v_5,S)=\{v_5\}$.

Remark: 2.13 If v is a geodetically isolated(or g-isolated) vertex of S, then $v \in p_{gn}(v,S)$ and $p_{gn}(v,S) \neq \phi$.

Definition: 2.14 Let $S \subseteq V(G)$. The g-private neighbour set of S denoted by $p_{gn}(S)$ is defined as $p_{gn}(S)=\{v \in S : p_{gn}(v,S) \neq \phi\}$.

Definition: 2.15 The g-private neighbour count of S is the cardinality of the g-private neighbour set of S. It is denoted as $p_{gnc}(S)$. That is, $p_{gnc}(S)=|p_{gn}(S)|$.

Remark: 2.16 For $S=\{v_1,v_5,v_7,v_9\}$ in example 2.12, $p_{gn}(S)=S$ and $p_{gnc}(S)=|S|=4$.

Definition: 2.17 [1,2,3] A set S of vertices of G is said to be a geodominating(or geodetic) set of G if every vertex of G is geodominated by the vertices of S.

Equivalently, a set S of vertices of G is a geodetic set of G if for every $v \in V-S$, $v \in N_g(x,y)$ for some pair of vertices $x,y \in S$ or $V= I[S] = \bigcup_{x,y \in S} I[x,y] = \bigcup_{x,y \in S} N_g[x,y] = N_g[S]$.

A geodetic set S is said to be a minimal geodetic set of G if no proper subset of S is a geodetic set of G.

A geodetic set of minimum cardinality is called a minimum geodetic set (or a geodetic basis) of G and the minimum cardinality of a minimal geodetic set of G is called the geodetic number of G and it is denoted as g(G).

The maximum cardinality of a minimal geodetic set of G is called its upper geodetic number and it is denoted as $g^+(G)$.

Remark: 2.18 The set $I_g(G)$ is a subset of every geodetic set of G.

3. g-INDEPENDENT SETS

Definition: 3.1 A subset S of V(G) is said to be a g-independent set of G if every $v \in S$ is such that $v \notin N_g(x,y)$ for every $x,y \in S-\{v\}$.

If S is g-independent then every vertex of S is a g-isolate of S.

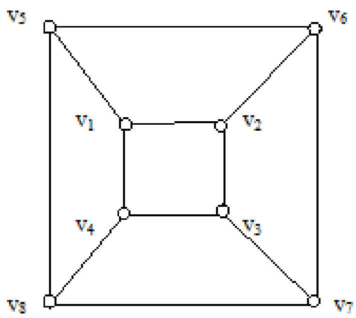
Observation: 3.2
1. For any graph G, every two element subset of V(G) is a g-independent set.
2. For the complete graph G=(V,E), V(G) is a g-independent set.
3. Let G be a non-complete connected graph. If S is a g-independent set of G, then $2 \le |S| \le p-1$.
4. If G is any graph, then the set of all end vertices of G^+ is a g-independent set of G^+.

Definition:3.3 A g-independent set S of G is said to be maximal if no super set of S is a g-independent set of G.

Example: 3.4
1. For C_5, the set $S=\{v_1,v_3\}$ is a g-independent set whereas $S'=\{v_1,v_3,v_4\}$ is a maximal g-independent set.

Definition: 3.5 The maximum cardinality of all maximal g-independent sets of G is called its g-independence number. It is denoted as $\beta_g(G)$. A g-independent set of cardinality $\beta_g(G)$ is called a β_g-set of G.

Example: 3.6



G figure 3.1

1. For G, $\{v_1,v_3,v_6,v_8\}$, $\{v_2,v_4,v_5,v_7\}$, $\{v_1,v_7\}$ and $\{v_3,v_5\}$ are maximal g-independent sets.

Observation: 3.7
1. For a complete graph G on p vertices, $\beta_g(G)=p$.
2. For a non-complete connected graph G, $2 \le \beta_g(G) \le p-1$.
3. For $n \ge 2$, $\beta_g(P_n) = 2$.
4. For $n \ge 4$, $\beta_g(C_n) = \begin{cases} 2 & if\ n\ is\ even \\ 3 & otherwise \end{cases}$
5. $\beta_g(K_{m,n}) = \begin{cases} 2 & if\ m=n=1 \\ \max\{m,n\} & otherwise \end{cases}$
6. For any graph G on p vertices, $\beta_g(G^+) = p$.

Definition: 3.8 A geodetic set S is said to be a g-independent geodetic set of G if S is g-independent. A g-independent geodetic set S is maximal if no super set of S is a g-independent geodetic set of G. That is, $S \cup \{v\}$ is not a g-independent geodetic set for every $v \in V-S$. The minimum cardinality of all maximal g-independent geodetic sets is called g-independent geodetic number of G. It is denoted as gig(G). A g-independent geodetic set of cardinalilty gig(G) is called a gig-set of G.

Observation:3.9
1. For a complete graph on p vertices, gig(G)=p, as the unique geodetic set S=V(G) is g-independent.
2. For a non-complete connected graph G, $2 \le gig(G) \le p-1$.
3. For $n \ge 2$, $gig(P_n) = 2$, as the set of two end vertices of P_n is a g-independent geodetic set of P_n.
4. For $n \ge 4$, $gig(C_n) = \begin{cases} 2 & if\ n\ is\ even \\ 3 & otherwise \end{cases}$

Proposition: 3.10 $gig(K_{m,n}) = \begin{cases} 2 & if\ m=n=1 \\ \min\{m,n\} & if m,n \ge 2 \\ \max\{m,n\} & otherwise \end{cases}$

Proof: Let U,W be the vertex partition of $K_{m,n}$ with |U|=m and |W|=n.

When m=n=1, $V(K_{m,n})$ is the unique g-independent geodetic set. So, $gig(K_{m,n})=2$.

If m=1 and $n\geq 2$(or n=1 and $m\geq 2$) , then W (or U) is the unique g-independent geodetic set of G. Therefore, $gig(K_{m,n}) = \max\{m,n\}$.

When $m,n \geq 2$, U and W are the only minimal g-independent geodetic set. Hence, $gig(K_{m,n}) = \min\{m,n\}$.

Proposition:3.11 For any graph G on p vertices, $gig(G^+) = p$.

Proof: In G^+, the set of all end vertices attached to the vertices of G is the unique g-independent geodetic set. Hence the result.

Proposition: 3.12 Let G=(V,E) be any graph. A g-independent set S of G is maximal if and only if it is g-independent and geodetic.

Proof: Let S be a g-independent set of G. Suppose S is maximal. Then, for every $v\in V-S$, $S\cup\{v\}$ is not g-independent. So, for every $v\in V-S$, $v\in N_g(x,y)$ for some $x,y\in S$. Therefore, S is a geodetic set of G. Conversely, Suppose, S is both g-independent and geodetic. Since S is geodetic, retracing the above steps we find, $S\cup\{v\}$ is not g-independent for every $v\in V-S$. But, S is g-independent. Hence, S is a maximal g-independent set of G.

Proposition: 3.13 Every maximal g-independent set in a graph G is a minimal geodetic set of G.

Proof: Let S be a maximal g-independent set of G. Proposition 3.12 asserts that S is a geodetic set. If S is not a minimal geodetic set, then $S-\{v\}$ is a geodetic set for some $v\in S$. So, for every $u\in V-(S-\{v\})$, $u\in N_g(x,y)$ for some $x,y\in S-\{v\}$. In particular, $v\in N_g(x,y)$ for a pair of vertices $x,y\in S-\{v\}$. This contradicts our assumption that S is g-independent. Therefore, S is minimal geodetic set of G.

Corollary: 3.14 For any graph G, $g(G) \leq gig(G) \leq \beta_g(G) \leq g^+(G)$.

Proof: We prove the theorem in three steps.

(i) $g(G) \leq gig(G)$.

Let S be a gig-set of G. Since every g-independent geodetic set is a geodetic set of G, $g(G)\leq|S|=gig(G)$.

(ii) $gig(G) \leq\beta_g(G)$.

Let S be a β_g-set of G. That is, S is a maximal g-independent set. By proposition 3.13, S is a geodetic set of G.

Therefore, S is a g-independent geodetic set of G.But, gig-set is a g-independent geodetic set of minimum cardinality.

So, $gig(G)\leq \beta_g(G)$.

(iii) $\beta_g(G)\leq g^+(G)$.

Let S be a β_g-set of G. By Proposition 3.13, S is a minimal geodetic set of G. Any g+-set is a minimal geodetic set of maximum cardinality. Therefore, $\beta_g(G) \leq g^+(G)$.

The following theorem gives a necessary and sufficient condition for a geodetic set of a graph G to be a minimal geodetic set of G.

Theorem: 3.15 A geodetic set D of G is a minimal geodetic set of G if every $u\in D$ satisfies one of the following two conditions.
(i) u is a g-isolated vetex of D.
(ii) There exists at least one vertex $w\in V-D$ such that w is geodominated by D and it is not geodominated by $D-\{u\}$.

Proof: Assume that D is a minimal geodetic set of G. Then, for every vertex $u\in D$, $D-\{u\}$ is not a geodetic set of G. Therefore, there exists a vertex $w\in V-(D-\{u\})$ such that w is not geodominated by the vertices of $D-\{u\}$.

Case 1: w=u.
So, u is not geodominated by the vertices of $D-\{u\}$.That is, $u\notin Ng[x,y]$ for every $x,y\in D-\{u\}$. That is, $u\notin N_g(x,y)$ for every $x,y\in D$. That is, u is a g-isolated vertex of D. So, u satisfies (i).

Case 2: $w \neq u$.
Now, $w \in V-D$ and it is not geodominated by the vertices of D-{u}. But, as D is a geodetic set of G, w is geodominated by the vertices of D. So, u satisfies (ii).

Conversely, assume that every $u \in D$ satisfies (i) or (ii). Let C_1 and C_2 denote the set of all vertices of D satisfying (i) and (ii) respectively.

case a: Let $u \in C_1$. Then, u is a g-isolated vertex of D. Therefore, $u \notin N_g(x,y)$ for every pair x,y of vertices of D. That is, $u \in V-(D-\{u\})$ is not geodominated by the vertices of D-{u}.

Therefore, D-{u} is not a geodetic set of G.

case b: Let $u \in C_2$. Then, there exists $w \in V-D$ such that w is not geodominated by the vertices of D-{u}. So, D-{u} is not a geodetic set of G.

By assumption, $D = C_1 \cup C_2$. Therefore, D-{u} is not a geodetic set of G for every $u \in D$.
Therefore, D is a minimal geodetic set of G.

Corollary: 3.16 A geodetic set D of G is a minimal geodetic set of G if and only if $p_{gn}(v,S) \neq \phi$ for every $v \in D$.

4. g-IRREDUNDANT SETS

The irredundance and upper irredundance number were first defined by Cockayne, Hedetnienmi and Miller[3]. In this section, we extend these parameters with respect to the geodetic concept. Theorem 2.15 can be restated as "A geodetic set S of G is minimal if and only if for every vertex $v \in S$, there exists a vertex $w \in V-(S-\{v\})$ which is not geodominated by S-{v}---(1). That is, for every vertex $v \in S$, $p_{gn}(v,S) \neq \phi$.---(2). We call a set S of vertices is g-irredundant if condition (2) is satisfied.

Definition: 4.1 A set $S \subseteq V(G)$ is said to be a g-irredundant set of G if $p_{gn}(v,S) \neq \phi$ for every $v \in S$.

Remark: 4.2 If S is a g-irredundant set, then $p_{gnc}(S) = |S|$.

Proposition: 4.3 A geodetic set S is a minimal geodetic set if and only if it is geodetic and g-irredundant.

Proof: Let S be a geodetic set of G. Suppose S is minimal. By corollary 3.16, S is g-irredundant. Conversely, if a set S is both geodetic and g-irredundant. Let $v \in S$. As S is g-irredundant, $p_{gn}(v,S) \neq \phi$. Let $w \in p_{gn}(v,S)$. Then, $w \in V-(S-\{v\})$ is not geodeominated by the vertices of S-{v}. Therefore, S-{v} is not geodetic. As $v \in S$ is arbitrary, S is a minmal geodetic set of G.

Definition: 4.4 A g-irredundant set S is said to be maximal g-irredundant if no super set of S is a g-irredundant set of G.

That is, $S \cup \{v\}$ is not g-irredundant for every vertex $v \in V-S$.

Remark: 4.5 By definition 4.1, S is a maximal g-irredundant set of G if and only if for every vertex $w \in V-S$, there exists a vertex $v \in S \cup \{w\}$ for which $p_{gn}(v, S \cup \{w\}) = \phi$.

Proposition: 4.6 Let G=(V,E) be a connected graph and $S \subseteq V(G)$ is a g-irredundant set of G. Then, the following are equivalent.
(i). For every vertex $w \in V-S$, there exists a vertex $v \in S \cup \{w\}$ for which $p_{gn}(v, S \cup \{w\}) = \phi$.
(ii). For every vertex $w \in V-S$, $p_{gnc}(S \cup \{w\}) \leq p_{gnc}(S)$.

Proof: (i) $\Rightarrow$ (ii)

By definitions 2.14 and 2.15, $p_{gnc}(S) = |p_{gn}(S)| = |\{v \in S : p_{gn}(v,S) \neq \phi\}|$. As S is g-irredundant, by

Remark 4.2, $p_{gnc}(S) = |S|$.

By (i), for every $w \in V-S$, $|p_{gn}(S \cup \{w\})| < |S \cup \{w\}| = |S|+1$. Therefore, $p_{gnc}(S \cup \{w\}) = |p_{gn}(S \cup \{w\})| \leq |S| = p_{gnc}(S)$.

(ii) $\Rightarrow$(i)
Since S is g-irredundant, $p_{gnc}(S) = |S|$. That is, for every $v \in S$, $p_{gn}(v,S) \neq \phi$.

(ii) implies for every $w\in V-S$, $|p_{gn}(S\cup\{w\})|<|S\cup\{w\}|$. Therefore, for every $w\in V-S$, there exists atleast one vertex v in $S\cup\{w\}$ such that $p_{gn}(v,S\cup\{w\})=\phi$.

Hence the result.

Remark: 4.7 Let G be a connected graph. A g-irredundant set S of G is a maximal g-irredundant set of G if and only if for every vertex $w\in V-S$, $p_{gnc}(S\cup\{w\})\leq p_{gnc}(S)$.

Definition: 4.8 Let G be a connected graph. The minimum cardinality of a maximal g-irredundant set of G is called the g-irredundance number of G. It is denoted as gir(G). The maximum cardinality of a mamimal g-irredundant set is called its upper g-irredundance number. It is denoted by GIR(G).

Example: 4.9 For the graph G in figure 3.1 $\{v_3,v_6,v_8\}$ is a g-irredundant set whereas $\{v_1,v_3,v_6,v_8\}$ is a maximal g-irredundant set.

Proposition: 4.10 Every minimal geodetic set in a connected graph G is a maximal g-irredundant set of G.

Proof: Let S be a minimal geodetic set of G. By Proposition 4.3, S is g-irredundant. Suppose it is not maximal g-irredundant. Then, there exists a vertex $u\in V-S$ such that $S\cup\{u\}$ is g-irredundant. So, for every $v\in S\cup\{u\}$, $p_{gn}(v, S\cup\{u\})\neq\phi$. In particular, $p_{gn}(u,S\cup\{u\})=\phi$. Let $w\in p_{gn}(u,S\cup\{u\})$. Therefore, w is a private neighbor of u in $S\cup\{u\}$ and w is not geodominated by the vertices of S. This is a contradiction to S is a geodetic set of G. Hence S is a maximal g-irredundant set of G.

Theorem: 4.11 For any graph G,

$$gir(G)\leq g(G)\leq gig(G)\leq \beta g(G)\leq g^{+}(G)\leq GIR(G).$$

Proof: By Theorem 3.14, it is enough to prove $gir(G) \leq g(G)$ and $g^{+}(G)\leq GIR(G)$.

(i) $gir(G)\leq g(G)$.

Let S be a g-set of G. By proposition 4.10, S is a maximal g-irredundant set of G. So, by definition of gir(G), we get $gir(G)\leq|S|=g(G)$.

(ii) $g^{+}(G)\leq GIR(G)$.

Let S be a g^{+}-set of G. Then, S is a minimal geodetice set of G. Therefore, by 3.10, S is a maximal g-irredundant set of G. Further, GIR(G) is the maximum cardinality of all maximal g-irredundant sets of G. Therefore, $g^{+}(G)\leq GIR(G)$.

Hence the result.

Remark: 4.12 For a graph G, the above inequality is called the geodetic chain of G.

5. g-CONNECTIVITY OF A GRAPH

Definition: 5.1 A graph G=(V,E) is said to be geodetically connected(or g-connected) if every vertex $u\in V(G)$ is an internal vertex of an x-y geodesic for some $x,y\in V(G)$.

Remark: 5.2 If a graph G is g-connected then
1. G contains no extreme vertices.
2. $\delta(G)\geq 2$.

Examples: 5.3
1. Every cycle C_p is a g-connected graph for $p\geq 4$.
2. Peterson graph is a g-connected graph.

Remark: 5.4 The following graphs are not g-connected.
1. Any complete graph is not g-connected.
2.The Hajo's graphs H_k are not g-connected for $k\geq 2$.
3. P_n is not g-connected for $n\geq 2$.
4. Any graph G with atleast one extreme vertex is not g-connected.

Remark: 5.5 A graph G contains no extreme vertices is not a sufficent condition for G is g-connected. For example, the generalised Hajo's graphs H_k (see definition 1.4)contain no extreme vertices. It is not g-connected.

Problem: 5.6 Find out a sufficient condition for g-connectedness of a graph.

Conjecture: 5.7 Suppose G is a g-connected graph and S is a geodetic set of G with $|S| \leq n/2$ then V-S contains a geodetic set of G.

Remark: 5.8 In the above conjecture, it is evident that the condition $|S| \leq n/2$ cannot be ommited.

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