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DISPERSION OF CONTAMINANT IN UNSTEADY OSCILLATORY FLOW BOUNDED BY POROUS LAYER

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ABSTRACT

The paper presents the study of Dispersion of contaminant in unsteady oscillatory flow of an incompressible fluid between two infinite parallel plates bounded by porous layer under the influence of pressure gradient in the axial direction. Analytical solution of unsteady convective diffusion based on Aris-Barton method of moments is presented up to the second moments about the mean and then a finite difference implicit scheme based on the crank-Nicholson method has been adopted to solve the remaining equation up to fourth moments.

Keywords: *Dispersion, porous layer, unsteady, Contaminant, Gaussianity, Hermite polynomial.*

1. INTRODUCTION

The longitudinal dispersion of a soluble matter in fluid flow has a wide application in the field of environmental fluid mechanics. The first analytical description of the dispersion of soluble material in fluid flow was given by Taylor (1953) and is applicable at asymptotically large times. Aris (1956) extended Taylor's theory to include longitudinal diffusion. Gill and sankarasubramanian (1970) proposed a different method and obtained a model that is valid for all times. Considerable amount of theoretical and experimental work has been done on the dispersion in the Newtonian fluids using the aforementioned approaches. The results obtained by using the Newtonian fluid models are restrictive. The study of dispersion through non-Newtonian fluids has abundant application in polymer and bio-chemical processing and in the cardiovascular system. Chatwin (1970) analyzed the normality of the concentration distribution of solute in a solvent flowing along a straight pipe. Gupta and Gupta (1972) Studied the phenomenon of dispersion of reactive contaminants in a liquid flowing through a channel in the presence of a first order heterogeneous chemical reaction using Taylors conceptual model for asymptotically large time. Smith (1982, 1983a) analyzed the variance and the longitudinal dispersion coefficient during the initial time in the oscillatory current. He showed that the dispersion coefficient was sometimes negative due to the reversing flow in the velocity profile. Which means that the concentration of contaminant may contract and explained periodically and he also pointed out the sensitivity of time of release of contaminant during a cycle. Barton (1983) resolved certain technical difficulties in the Aris method of moments and obtain a dispersion coefficient that is valid for all times. Barton (1984) explain the dispersion of reactive contaminants in pipe poiseuille flow in the presence of the boundary reaction for asymptotically large time. Chatwin and Allen (1985) These two flow situations may be realized, as first approximations, in the mathematical models of oscillatory laminar flows in a tube and between parallel plates. We have applied our general principle to these two models for providing the theoretical frame works for the interpretations of contaminant spreading in the more intricate dispersive systems. It has been assumed in both cases that the fluid is subjected to the combined influence of a mean pressure gradient and an imposed perturbation. Barton and stokes (1986) studied computed shear dispersion in parallel flow numerically. Yasuda (1989) presented a method to escape the negative dispersion coefficient during the initial to stationary stages in both steady and oscillatory current by proposing a new definition of the vertical average of the dispersion. Mandal and Mazumder (2005) have studied the effect of wall absorption on solute dispersion in a pulstile flow through a channel by using the Aris-Barton method of moments.

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Motivated by the above analysis in the present paper, an attempt is made to study the contaminant dispersion of a unsteady laminar flow of an incompressible fluid between two infinite parallel plates bounded by porous layer, under the influence of pressure gradient. In contrast of the work of Bandyopadhya and Mazumder (1999). The present work considers the effect of porous media. The governing partial differential equations are dimensionlised by using non-dimensionlisation. The purpose of the present study is to explore the dispersion phenomenon of the injected solute in the velocity distribution of this type of flow situation that in an unsteady generalized couette flow, initially the problem has been solved analytically upto the second moment about the mean by using the method of moments based on the technique due to Aris-Barton and latter modified by Mukherjee and Mazumder owing to the complexity of the analytical solution of the moment equation is obtaining the higher order moments, it has been solved numerically by adopting a finite difference scheme based on the crank-Nicholson implicit method.

2. MATHEMATICAL FORMULATION

Consider an unsteady fully developed laminar flow of an incompressible porous media between two infinite parallel plates separated by 'd'. Here the upper plate oscillates in its own plane with a constant velocity 'u_0' and the lower plate is stationary. The flow is influenced by an application pressure gradient following form

$$\frac{-1}{\rho}\frac{\partial p^*}{\partial x^*} = \text{Constant} = p \tag{2.1}$$

Where ρ is the density of the fluid assumed to be homogeneous, p^* is the fluid pressure. As the flow is influenced by the combined action of the plate oscillation and a constant pressure gradient along axial direction, the flow will be unidirectional. The velocity has only axial component $u^*(y^*, t^*)$ satisfies the Navier-Stokes equation

$$\frac{\partial u^*}{\partial t^*} = p + \gamma\left(\frac{\partial^2 u^*}{\partial y^{*2}}\right) - \frac{\mu}{\kappa}u^* \tag{2.2}$$

Subjected to the boundary conditions

$$u^* = u_0(1+\varepsilon\cos(\omega t^*)) \text{ for } y^* = d$$
$$u_0\varepsilon = 0 \text{ for } y^* = 0 \tag{2.3}$$

where and ω are respectively the amplitude and frequency of the plate oscillation, γ is the Kinematics viscosity of the fluid and $\frac{\mu}{\kappa}$ is the porosity of the fluid.

It is assumed that the motion of the solute is governed by both convection and molecular diffusion. When a slug of solute with a constant molecular diffusivity D is injected into the above time-dependent flow, the concentration $c(x, y, t)$ satisfies the dimensionless convective-diffusion equation of the form

$$\frac{\partial c}{\partial t} + p_e u(y,t)\frac{\partial c}{\partial x} = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)c \tag{2.4}$$

Where the dimensionless quantities are given by

$$x = \frac{x^*}{d} \quad y = \frac{y^*}{d} \quad t = \frac{Dt^*}{d^2} \quad u = \frac{u^*}{u_0} \quad p_e = \frac{u_0 d}{D}$$

The peclet number p_e, introduced here measures the relative characteristic time of the diffusion process $\left(\frac{L^2}{D}\right)$ to the convection process $\left(\frac{L}{u_0}\right)$.

The initial and boundary conditions for the contaminant input are

$$c(x,y,0)=\delta(x)$$
$$\frac{\partial c}{\partial y}=0 \text{ at } y=0,1 \tag{2.5}$$

C is finite at all points

$$x^n c \to 0 \text{ and } x^m \frac{\partial^n c}{\partial x^n} \to 0 \text{ as } |x| \to \infty \text{ for } m,n=0,1,2,\ldots$$
$$\int_0^1 \int_{-\infty}^{\infty} c\,dx\,dy = 1$$

Where $\delta(x)$ is the Dirac delta function.

Introducing the dimensionless parameters

$$\alpha=\frac{\omega d^2}{\gamma},\ s=\frac{\gamma}{D},\ F=\frac{pd^2}{\gamma u_0},\ M=\frac{d^2\mu}{\kappa\gamma}.$$

The solution of equation (2) is given in the dimensionless form as

$$u(y,t)=u_0(y)+u_1(y,t) \tag{2.6}$$

where

$$u_0=\frac{F}{M^2}[1-\cosh My+(\coth M-\sec hM)\sinh My]+\sec hM\sinh My \tag{2.7}$$

$$u_1(y,t)=R\mathrm{e}\left[\frac{\varepsilon\sinh\left(\sqrt{\frac{i\alpha s+M^2}{s}}\right)y}{\sinh\left(\sqrt{\frac{i\alpha s+M^2}{s}}\right)}e^{i\alpha st}\right] \qquad i=\sqrt{-1} \tag{2.8}$$

Using equations (7) and (8) in equation (6) we get

$$u(y,t)=\frac{F}{M^2}[1-\cosh My+(\coth M-\sec hM)\sinh My]$$
$$+\sec hM\sinh My+R\mathrm{e}\left[\frac{\varepsilon\sinh\left(\sqrt{\frac{i\alpha s+M^2}{s}}\right)y}{\sinh\left(\sqrt{\frac{i\alpha s+M^2}{s}}\right)}e^{i\alpha st}\right] \tag{2.9}$$

Where Re stands for real part, α is called the dimensionless frequency parameters, s is the Schmidt number, F is the dimensionless pressure gradient and M is the dimensionless porous parameter. The first term of the RHS of equation (9) corresponds to simple shear flow, the second term corresponds to plane-poiseuille flow and the third term corresponds to the flow due to the plate oscillation. The physical significance is attributed only to the real part $u(y,t)$ which is given by

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$$u_1(y,t)=\frac{F}{M^2}[1-\cosh My+(\coth M-\sec hM)\sinh My]+\sec hM\sinh My+\frac{\varepsilon}{k_1^{\,2}+k_2^{\,2}}$$

$$\left[\begin{array}{l}\left\{k_1\sinh(r^{\frac{1}{2}}\cos\frac{\theta}{2}y)\cos(r^{\frac{1}{2}}\sin\frac{\theta}{2}y+k_2\cosh(r^{\frac{1}{2}}\cos\frac{\theta}{2}y)\sin(r^{\frac{1}{2}}\sin\frac{\theta}{2}y)\right\}\cos\alpha st\\ +\left\{k_2\sinh(r^{\frac{1}{2}}\cos\frac{\theta}{2}y)\cos(r^{\frac{1}{2}}\sin\frac{\theta}{2}y)-k_1\cosh(r^{\frac{1}{2}}\cos\frac{\theta}{2}y)\sin(r^{\frac{1}{2}}\sin\frac{\theta}{2}y)\right\}\sin\alpha st\end{array}\right] \tag{2.10}$$

where
$$k_1=\cosh(r^{\frac{1}{2}}\cos\frac{\theta}{2})\cos(r^{\frac{1}{2}}\sin\frac{\theta}{2})$$
$$k_2=\sinh(r^{\frac{1}{2}}\cos\frac{\theta}{2})\sin(r^{\frac{1}{2}}\sin\frac{\theta}{2})$$

Following Aris we define the n th moment of the concentration distribution through y at time *t* as

$$c_n(y,t)=\int_{-\infty}^{\infty}x^n c(x,y,t)\,dx \tag{2.11}$$

and the n th moment of the concentration over the cross section of the channel as

$$M_n(t)=\int_0^1 c_n(y,t)dy=\overline{c_n} \tag{2.12}$$

Using equation (2.11) and (2.12) in equation (2.4) and (2.5) we have the following moment equation

$$\frac{\partial c_n}{\partial t}=\frac{\partial^2 c_n}{\partial y^2}+n(n-1)c_{n-2}+np_e u(y,t)c_{n-1} \tag{2.13}$$

with $c_n(y,0)=1,\ \frac{\partial c_n}{\partial y}=0$ at $y=0$ and 1 (2.14)

and $$\frac{\partial M_n}{\partial t}=np_e x\overline{c_{n-1}}+n(n-1)M_{n-2} \tag{2.15}$$

with $M_n(0)=1$ for $n=0$,

$M_n(0)=0$ for $n>0$ (2.16)

Here over bar denotes the cross sectional mean. Also $M_0(t)=1$, since *c* has cross -sectional mean unity for all time (see (equation2.5)). Equation (2.13) subject to the conditions equation (14) for $n=0,1,2,3,...$ form a sequence of homogeneous equations which can be solved for any c_n. Once c_n is found, M_n can be obtained from equation (2.15). According to Aris the above sequence of equation might be solved, in principle to a sufficiently large value of n but the first three or four moments are sufficient to describe the process of the dispersion.

The n th central moment of the concentration distribution can be defined as

$$v_n(t)=\frac{1}{M_0}\int_0^1\int_{-\infty}^{\infty}\left(x-\overline{x_g}\right)^n c\,dxdy \tag{2.17}$$

where $\overline{x_g}=\frac{1}{M_0}\int_0^1\int_{-\infty}^{\infty}xc\,dxdy=\frac{M_1}{M_0}$ can be regarded as the centroid or first moment of the contaminant distribution which measures the location of centre of gravity of the cloud movement with the mean velocity of the fluid, initially located at the source. Then the following central moments can be obtained

$$v_2(t)=\frac{M_2}{M_0}-\overline{x}_g^{\,2}\ ,$$

$$v_3(t) = \frac{M_3}{M_0} - 3\bar{x}_g v_2(t) - \bar{x}_g^{\,3}$$

$$v_4(t) = \frac{M_4}{M_0} - 4\bar{x}_g v_3(t) - 6\bar{x}_g^{\,2} v_2 - \bar{x}_g^{\,4} \tag{2.18}$$

The second central moment (v_2) i.e, variance can be related to the dispersion of diffusing substance about its mean position and (v_3) and (v_4) are third and fourth central moments respectively.

The coefficients of skewness (β_2) and kurtosis (β_3) are defined as follows:

$$(\beta_2) = \frac{v_3}{v_2^{\frac{3}{2}}}, \qquad (\beta_3) = \frac{v_4}{v_2^{\,2} - 3} \tag{2.19}$$

These two coefficients are important criteria to measure the degree of symmetry and peakedness respectively, of the concentration distribution and are used to examine the nature of the concentration distribution and the deviation from Gaussianity. These two coefficients will be zero if the distribution is exactly Gaussian.

HERMITE POLYNOMIAL REPRESENTATION

The method of moments does not give any direct expression for the concentration distribution but it is possible to approximate the mean distribution on $c_m(x,t)$ within a channel by using Hermite polynomials for the representation of non- Gaussian curves ([2.18,2.19]). The concentration distribution can be expressed as an infinite series, known as Edgeworth's asymptotic series, using the first four central moments, which is expected to give good results in practice, Hermite polynomials are chosen because of $\exp(-x^2)$ as weighting function in the orthogonality property.

$$\int_{-\infty}^{+\infty} H_m(x) H_n(x) \exp(-x^2) dx = 2^n n! \pi^{\frac{1}{2}} \delta_{mn} \tag{2.20}$$

The cross sectional mean concentration $c_m(x,t)$ is defined as

$$c_m(x,t) = M_0(t) e^{-\xi^2} \sum_{n=0}^{\infty} a_n(t) H_n(\xi) \tag{2.21}$$

where $M_0(t) = \int_0^1 \int_{-\infty}^{+\infty} c\,dx\,dy$, $\xi = \dfrac{x - x_g}{(2v_2)^{\frac{1}{2}}}$, $x_g = \dfrac{M_1}{M_0}$

and H_i, the Hermite polynomials, satisfy the recurrance relation with $H_0(\xi) = 1.0$ as

$$H_{i+1}(\xi) = 2\xi H_i(\xi) - 2i H_{i-1}(\xi), i = 0,1,2,3,. \tag{2.22}$$

The coefficients a_i are $a_0 = \dfrac{1}{(2\pi v_2)^{\frac{1}{2}}}$, $a_1 = a_2 = 0$, $a_3 = \dfrac{2^{\frac{1}{2}} a_0 \beta_2}{24}$, $a_4 = \dfrac{a_0 \beta_3}{96}$.

Therefore given the statistical parameters (2.18) the concentration distribution can be estimated from equation (2.21) at any given location in the axial direction and time.

ANALYTICAL SOLUTION

The aim of the analysis is to solve the moment equations (2.13) and (2.15) subject to the initial and boundary conditions (2.14) and (2.16) for n = 0, 1, 2, 3… The method of solution adopted here is the Aris method of moment as modified by Barton for steady flow and later by Mukherjee and Mazumder for oscillatory flow. According to Mukherjee and Mazumder for periodic flow. The method of Mukherjee and Mazumder provides an algorithm for the exact analysis of the subsequent moment equation but our aim is restricted only to the second central moments due to the complexity in obtaining higher moments. The solution was based on the method variables, which depends on a

certain eigen value problem with a discrete set of eigen values. According to Mukherjee and Mazumder we consider an eigen value problem.

$$\left(\frac{\partial^2}{\partial y^2}-\frac{\partial}{\partial t}+\mu_i\right)f_i=0 \tag{2.23}$$

$$\frac{\partial f_i}{\partial y}=o \text{ at } y=0,1 \quad f_i \text{ is finite} \tag{2.24}$$

Where i runs over the positive integral values.

This gives us a discrete set of eigen-values $\mu_i=\frac{(i\pi)^2}{1+\varepsilon}$ and the corresponding eigen function

$f_i(y,t)=\sqrt{2}\cos i\pi y e^{-\mu_i t}$ $\quad i$ = 1, 2, 3…….. so that

$$\overline{f_i}=0$$

$$\overline{f_i f_j}=0 \text{ if } i\neq j$$

$$\overline{f_i f_j}=h_i \text{ if } i=j \tag{25}$$

Where h_i is the function of t alone and $h_i(0)=1$.

To complete the set of eigen functions we augment this set by setting $f_0=1$ corresponding to $\mu_0=0$. Following Mukherjee and Mazumder the expression for variance $v_2(t)$ is given by

$$v_2(t)=2t-2p_e^2\sum_i a'_{0i}(0)\int\overline{uf_i}e^{-\mu_i t}dt+2p_e^2\sum_i\int a'_{0i}\overline{uf_i}dt+2p_e^2\sum_i a'_{0i}(0)\left[\int\overline{uf_i}e^{-\mu_i t}dt\right]_{t=0}-2p_e^2\sum_i\int\left[a'_{0i}\overline{uf_i}dt\right]_{t=0} \tag{2.26}$$

Where $a'_{oi}(t)=e^{-\mu_i t}\int e^{\mu_i t}\overline{uf_i}h^{-1}{}_i(t)dt$.

The corresponding results for the steady generalized coquette flow may be retrieved from the eigen value problem (2.23) by putting $\frac{\partial f_i}{\partial t}$ equal to zero, which is the same as that of Barton. The eigen values and the corresponding normalized eigen function for this steady simple shear flow are given by

$$\mu_i=(i\pi)^2,\ f_i(y)=\sqrt{2}\cos i\pi y, i=1,2,3,...$$

And the corresponding expression for variance v_2 is obtained as

$$v_2(t)=2[1+p_e^2\sum_i\frac{a_i^2}{(i\pi)^2}]t-2p_e^2\sum_i\frac{a_i^2}{(i\pi)^4}[1-e^{-\mu_i t}] \tag{2.27}$$

$$v_2(t)=2[1+p_e^2\sum_i\frac{a_i^2}{(i\pi)^2}]t-2p_e^2\sum_i\frac{a_i^2}{(i\pi)^4}[1-e^{-\mu_i t}]$$

$$b_i=\frac{-M\sinh M(-1)^i}{M^2+(i\pi)^2}$$

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In the case of plane couette flow $[F=0, \varepsilon=0$ in equation (6)] the expression for $v_2(t)$ is in complete agreement with that of Barton.

The rate of growth of variance $\frac{1}{2}\frac{dv_2}{dt}$ is proportional to the sum of the constant quantity one due to the longitudinal diffusion and apparent diffusion coefficient D_a due to the interaction between the convection and lateral diffusion and is given by

$$\frac{1}{2}\frac{dv_2}{dt} = 1 + p^2{}_e D_a$$

Where D_a is the apparent dispersion coefficient depending on parameters $s, \alpha, \varepsilon, F, M$ and t.

The expression for the apparent dispersion coefficient D_a for the present problem is given by

$$\begin{aligned} D_a = &\sum_i R_{1i} + R_{2i}\cos(\alpha st) + R_{3i}\sin(\alpha st) + R_{4i}\sin(2\alpha st) + R_{5i}\cos(2\alpha st) \\ &- \sum_i \{G_{1i} + G_{2i}\cos(\alpha st) + G_{3i}\sin(\alpha st)\}\exp(-i^2\pi^2 t) \end{aligned} \tag{2.28}$$

where $R's$ and $G's$ are some functions of $\alpha, \varepsilon, F, M$ and s but independent of t.The expression for these functions are too long and cumbersome and therefore omitted.

For a large time after the release the expression for D_a equation (28) reduces to the form

$$D_a\big|_{t\to\infty} = p_0 + p_1\cos(\alpha st) + p_2\sin(\alpha st) + p_3\sin(2\alpha st) + p_4\cos(2\alpha st) \tag{2.29}$$

Where $p's$ are some real constants that depend upon $\alpha, \varepsilon, F, M$ and s. This result is more general than that obtained by Erdogan, Chatwin and Barton. If we put $\varepsilon = 0$ in (2.29) in the expression $D_a\big|_{t\to\infty}$ reduce to the Erdogan who studied the dispersion by Taylor's asymptotic model and for $\varepsilon = F = 0$ in equation (2.29) reduce to Barton. From the expression (2.29) it is interesting to note that the dispersion coefficient consists of the steady and fluctuating part that arises out due to the oscillation of the plate.

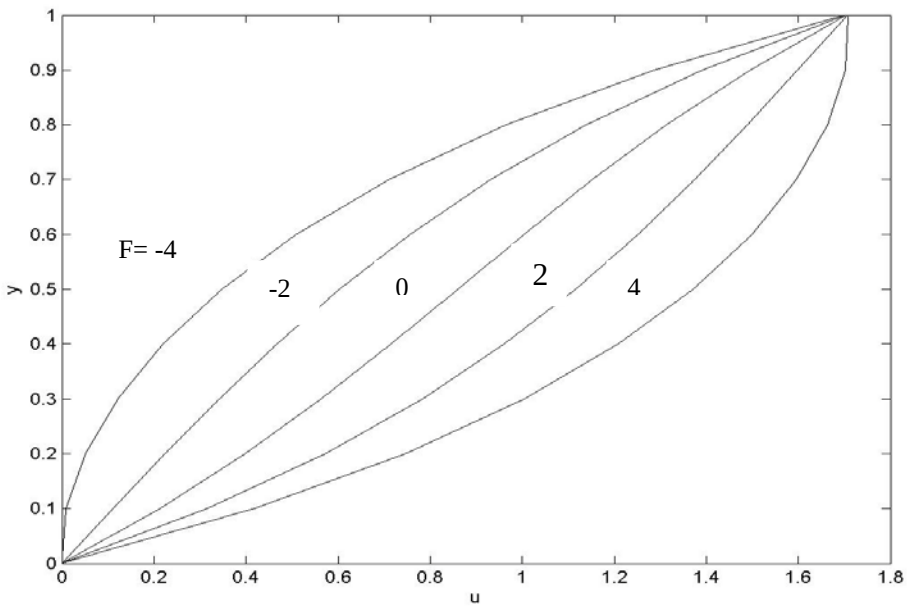

Fig. 1: The velocity distribution (u) against channel width (y) when ε = 1.0, S = 10^3 for αSt=π/2

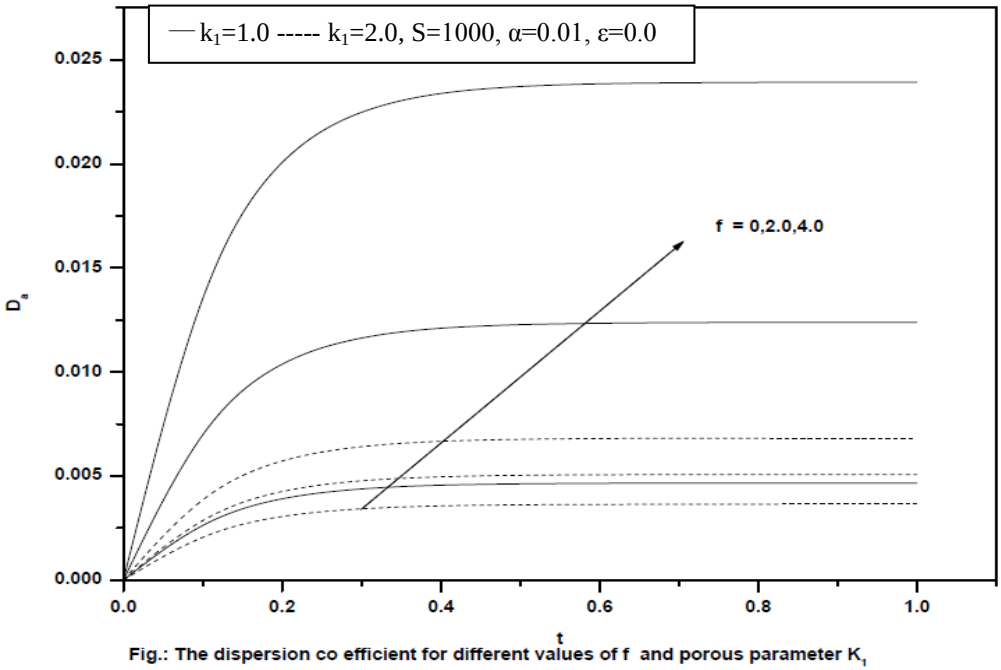

Fig.: The dispersion co efficient for different values of f and porous parameter K_1

Fig. 2:

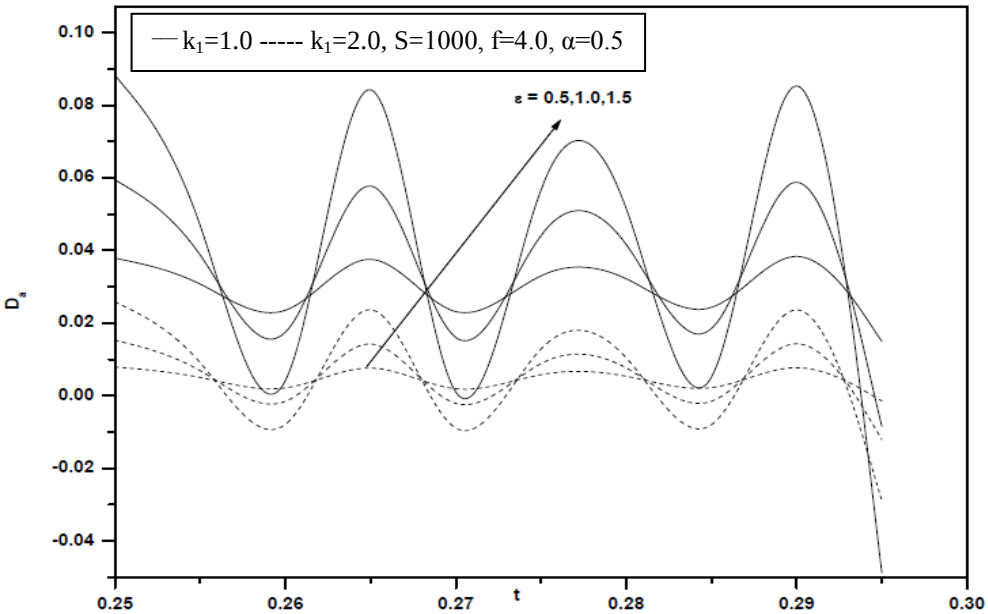

Fig: variation of dispersion co efficient D_a for different values of ε large time when K_1=1.0, and K_1=2.0

Fig. 3:

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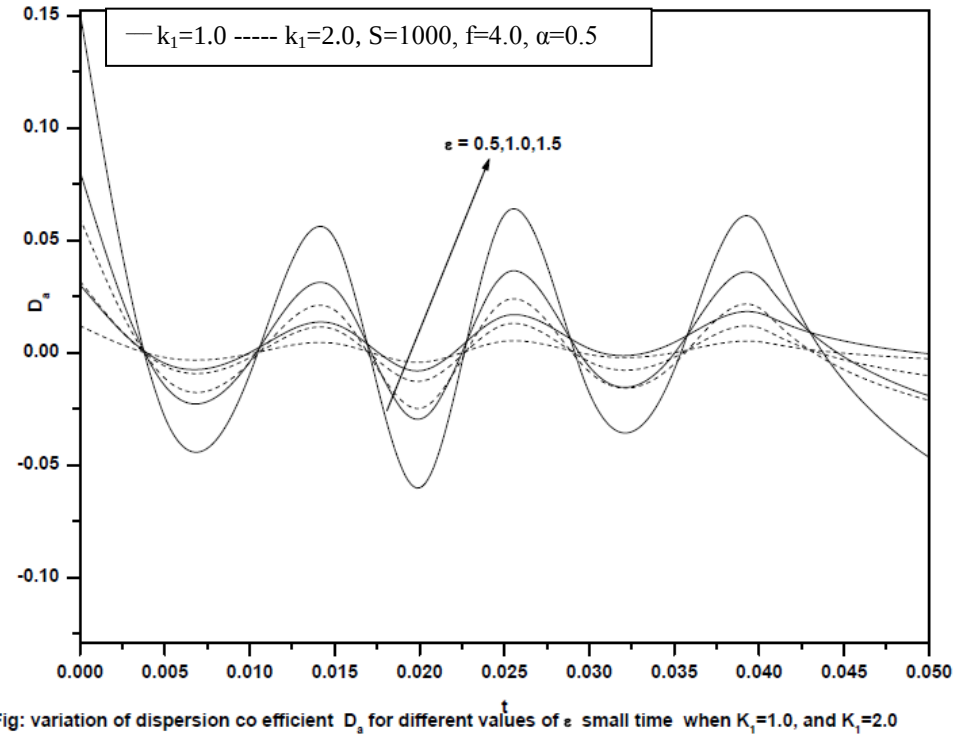

Fig: variation of dispersion co efficient D_a for different values of ε small time when K_1=1.0, and K_1=2.0

Fig .4:

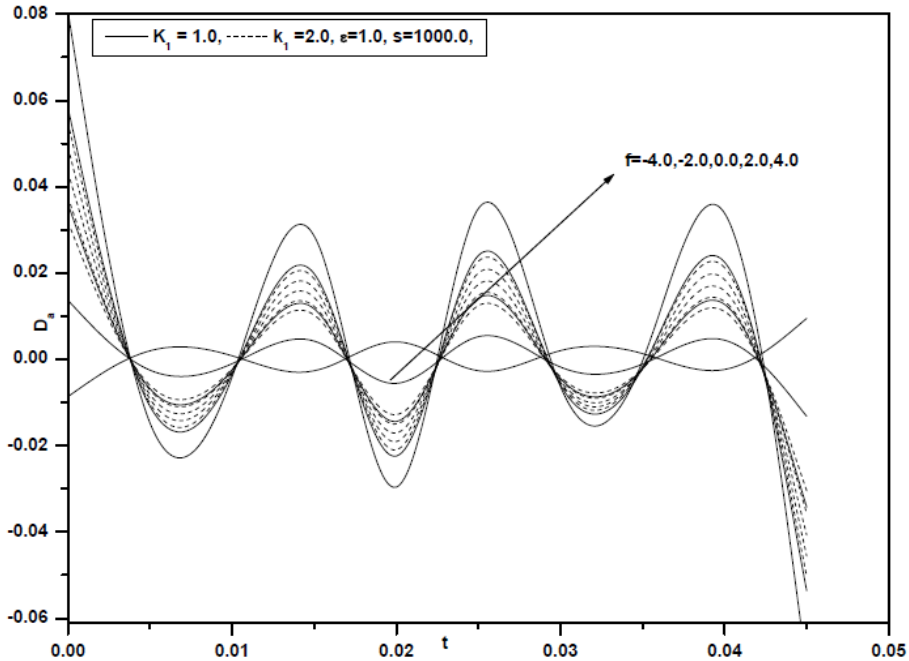

Fig: variation of dispersion co efficient D_a for different values of f small time when K_1=1.0, and K_1=2.0

Fig. 5:

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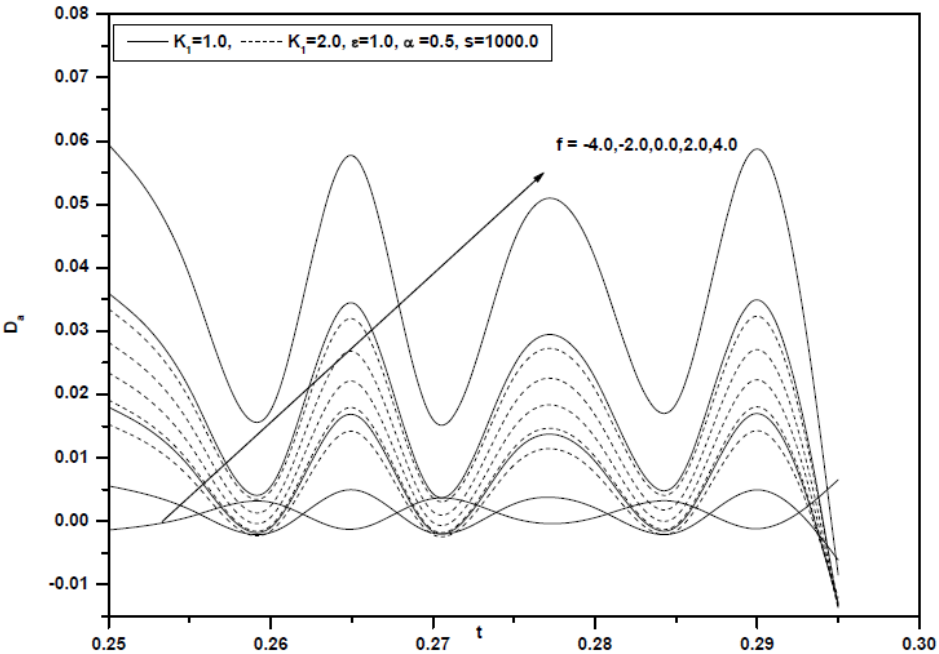

Fig: variation of dispersion co efficient D_a for different values of ε large time when K_1=1.0, and K_1=2.0

Fig. 6:

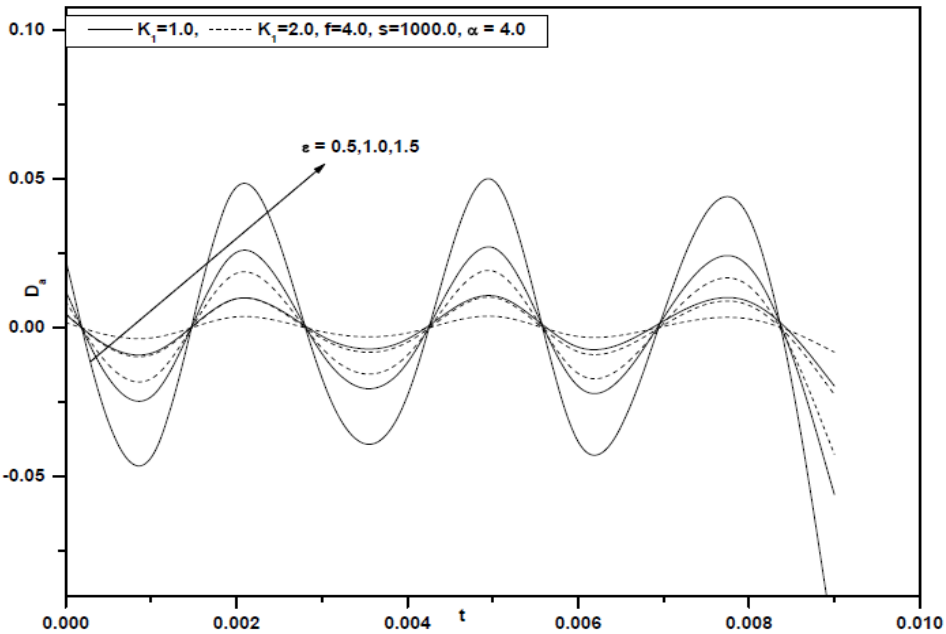

Fig: variation of dispersion co efficient D_a for different values of ε small time when K_1=1.0, and K_1=2.0

Fig. 7:

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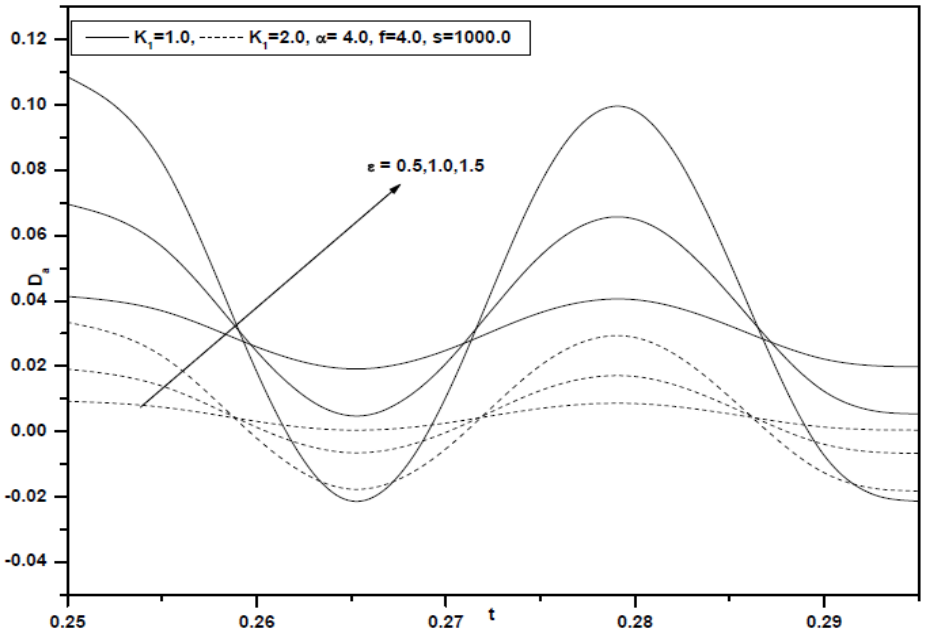

Fig: variation of dispersion co efficient D_a for different values of ε large time when K_1=1.0, and K_1=2.0

Fig. 8:

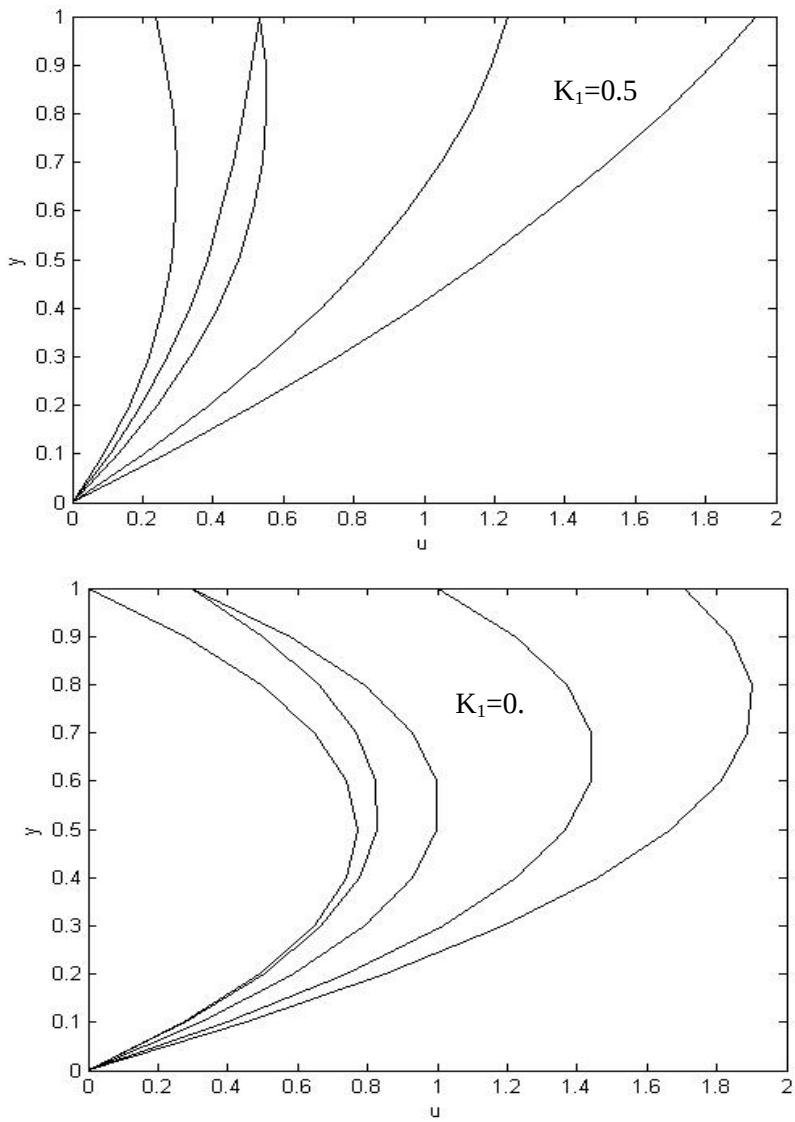

Fig. 9: The Varitation of Velocity u against y for different phases αSt = π/4, αSt = π/2, αSt = 3π/4, αSt = π, αSt = 3π/2, when ε=1.0, α = 1F=2

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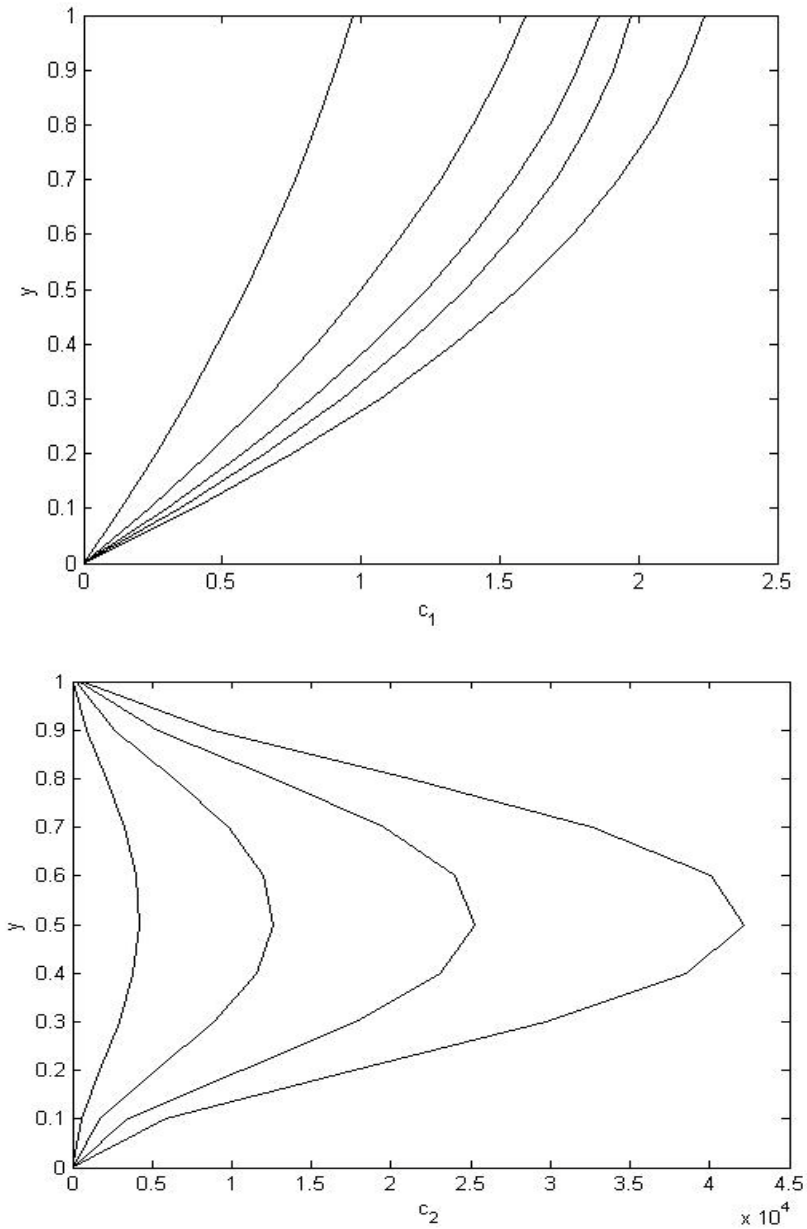

Fig. 10: The variation of first order consentration moment C1 & C2 against y for different phases αSt = π/4, αSt = π/2, αSt = 3π/4, αSt = π, αSt = 3π/2, when ε=1.0, α = 1.0 F=2

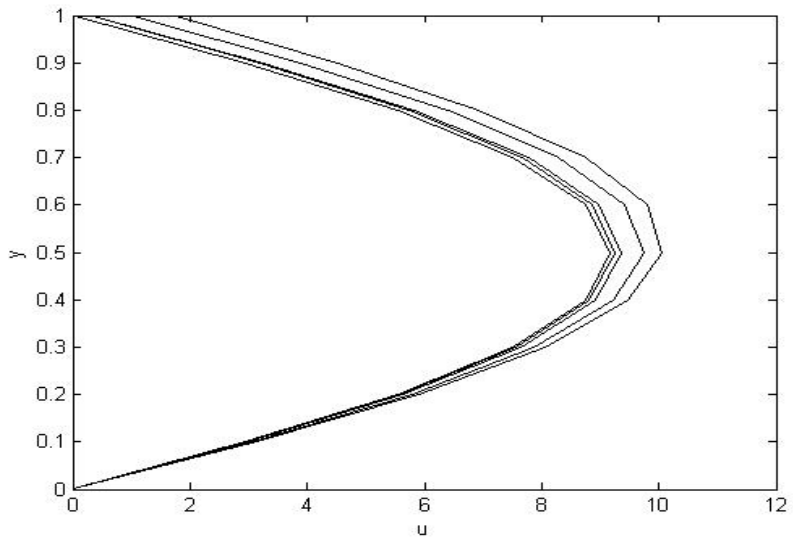

Fig. 11: The variation of velocity u against y for different phases αSt = π/4, αSt = π/2, αSt = 3π/4, αSt = π, αSt = 3π/2, when ε=1.0, α = 1.0 with high pressor

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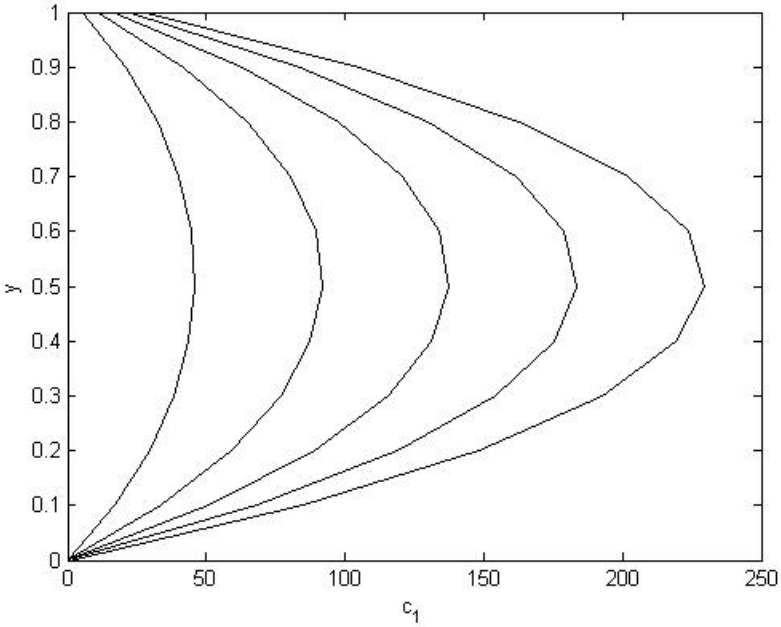

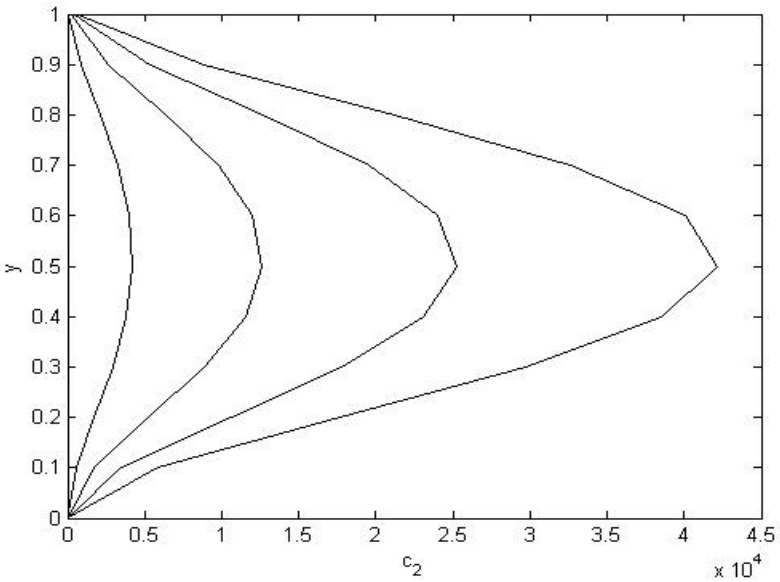

Fig. 12: The variation of first order consentration moment C1 & C2 against y for different phases αSt = π/4, αSt = π/2, αSt = 3π/4, αSt = π, αSt = 3π/2, when ε=1.0, α = 1.0 with high pressor

3. RESULTS AND DISCUSSIONS

The variation of the velocity profile (2.9) depends on the values of dimensionless pressure gradient F, amplitude of oscillation ε, frequency parameter α and Schmidt number S. If the amplitude oscillation ε is zero, the velocity profile(2.9) corresponds to simple shear flow. The shape of the velocity profile is determined by the dimensionless pressure gradient. The velocity is positive over the whole width of the channel for F >0 channel for F >0, where as it becomes negative over a portion of the channel width near the fixed wall for F <0 that is back flow occurs. Fig.1 shows the plot of the profiles for different values of F= -4,-2, 0.2.4 at a particular phase αst=π/2 considering the amplitude of oscillation ε=1.0 and $S=10^3$ and it is found that the magnitude of the velocity u increases with increasing F. In the present problem apparent dispersion coefficient D_a mainly depends on the frequency of oscillation α. the Schmidt number S, the dispersion time 't' and the non-dimensionless pressure gradient F. D_a is only a function of F and t. The dispersion coefficient D_a for this flow initially increases with time and asymptotically reaches steady state for each F when K_1=1.0 and k_1=2.0. From the Fig(2) for a given instant of time t, D_a increases with F and it is clear from the figure that asymptotic values of D_a for simple shear flow are smaller than that for the generlised simple shear flow. Although the non-dimensionless pressure gradient F appears in the expression for D_a, it is interesting to note that the change of sign in F does not affect D_a. Then the results agree well with the asymptotic values of erdogen.

The variation of D_a with time from the initial to be stationary stages are plotted for various values of ε and $S=10^3$ in Fig(3) and Fig(4) for $\alpha=0.5$ $F=4.0.K_1=1.0,K_1=2.0$ .Fig(5) for $\varepsilon=1.0,S=10^3$ F= 0,2,4 $K_1=1.0$ $K_1=2.0$ Fig(6) for $\alpha=0.5$ $\varepsilon=1.0,S=10^3$, F=0,2,4 $K_1=1.0$ $K_1=2.0$ Fig(7) and Fig(8) for $\alpha=F=4.0$ $K_1=1.0$ $K_1=2.0$ amplitude of oscillation in apparent dispersion coefficient increases initially upto certain extent of time then becomes stable for a long time. However the above situation stabilizes after certain time from the figures it is also seen that a fixed instant 't' the amplitude of D_a increases with increase in the amplitude of the plate oscillation clearly we can see that pulsatility of the upper plate contributes to a steady as well as a time dependent part in apparent dispersion coefficient . It is worthwhile to mention here that there is a remarkable differences in the behavior of D_a for small and large values of α.

The first order moment of the concentration $C_1(y, t)$ and $C_2(y, t)$ through y at any time't' can be obtained from the moment equation (2.13) by putting n=1 and n=2 the values of $C_1(y,t)$ and $C_2(y,t)$ calculated from a Finite difference scheme based on Cranck-Nicholson implicit method. The variation of the velocity u(y, t) and concentration $C_1(y,t)$ and $C_2(y, t)$ against y for different phase pressure gradient F=4.0,2.0,0,-2.0,-4.0 and different phase values $\alpha st = \pi/4, \pi/2, 3\pi/4, \pi, 3\pi/2$ and with high pressure gradient have been plotted in figures(9 -12) for $\alpha=1.0$, $\varepsilon=1.0$. The figure indicates that the first order moment $C_1(y, t)$ and $C_2(y, t)$ shifts from the displacement of fluid particles over the whole width of the channel due to the differential concentration and lateral diffusion.

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